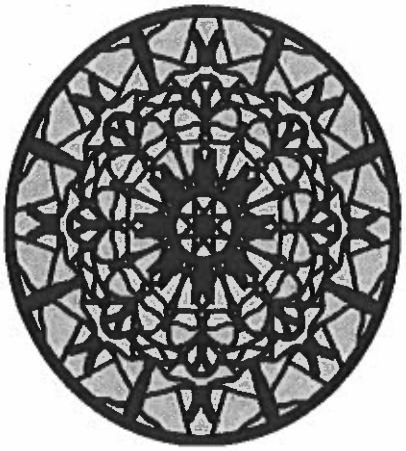
SOLUTIONS



GCSE Mathematics

2019 Predicted Paper 1 (Non-Calculator) except Questions 12, 13 & 18

1MA1

Higher Tier (1hr 40mins)

Remember: *These questions are just a guide. There are no guarantees that these questions/topics will come up! So, revise all you can before the calculator exams!*

Instructions

- Use **black** ink or ball-point pen.
- Answer **all** questions.
- Answer the questions in the spaces provided
  – *there may be more space than you need*

- You must show all your working
- **Calculators may be used** ~~EXCEPT~~ ONLY ON QUESTIONS 12, 13 & 18.
- Diagrams are **NOT** accurately drawn, unless otherwise indicated

Information

- The total mark for this paper is **110**.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

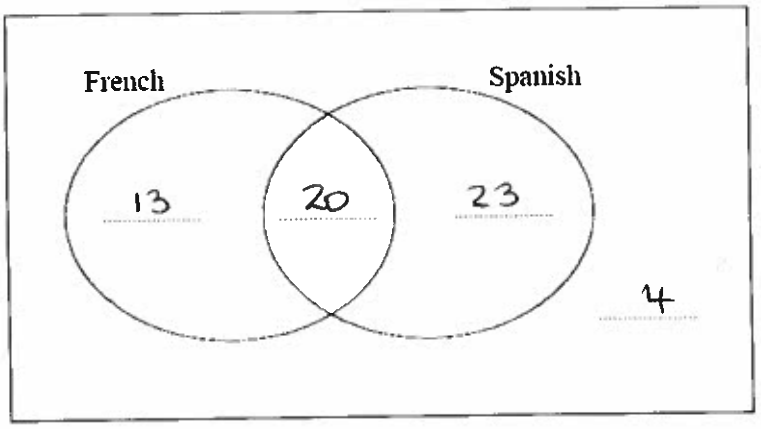
Advice

- Read each question carefully before you start to answer it.
- Keep an eye on time.
- Try to answer every question.
- Check your answers if you have time at the end.

1. There are 60 students at a college.

20 students study both French and Spanish.
13 students study French but not Spanish.
A total of 43 students study Spanish.

(a) Complete the Venn diagram for this information.



(3)

One of the students at the college is to be selected at random.

(b) Write down the probability that this student studies neither French nor Spanish.

$\frac{4}{60}$

(1)

(Total for Question 1 is 4 marks)

2. There are only blue counters, green counters, red counters and yellow counters in a bag.
Olga is going to take at random a counter from the bag.

The table shows the probability that Olga will take a blue counter and the probability that she will take a yellow counter.

Colour	blue	green	red	yellow
Probability	0.4	0.09	0.36	0.15

The number of red counters in the bag is 4 times the number of green counters in the bag.

Complete the table.

green $= x$
red $= 4x$

$x + 4x + 0.4 + 0.15 = 1$

$5x = 1 - 0.55$

$5x = 0.45$

$x = \frac{0.45}{5} = 0.09$ and $4x = 4 \times 0.09 = 0.36$

(Total for Question 2 is 3 marks)

3. (a) Solve $3(x + 2) = 4$

$3x + 6 = 4$

$3x = -2$

$x = -\frac{2}{3}$

$x =$ $-\frac{2}{3}$

(2)

(b) Solve $\frac{3x}{2} - 5 = 7$

$\frac{3x}{2} = 12$

$3x = 24$

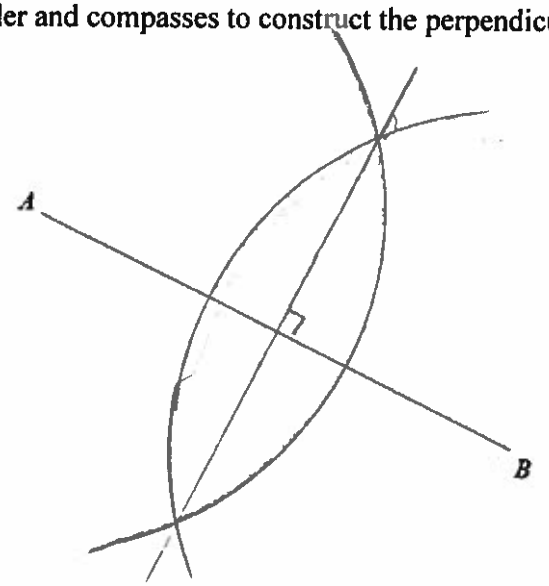
$x = \frac{24}{3} = 8$

$x =$ 8

(3)

(Total for Question 3 is 5 marks)

4. In the space below, use ruler and compasses to construct the perpendicular bisector of line AB.



(Total for Question 4 is 2 marks)

5. The table shows some information about the prices of 64 second-hand cars that are for sale.

Price (£x)	Frequency	Midpt (x)	$F \times x$
$0 < x \leq 2000$	8	1,000	8,000
$2000 < x \leq 4000$	14	3,000	42,000
$4000 < x \leq 6000$	28	5,000	140,000
$6000 < x \leq 8000$	10	7,000	70,000
$8000 < x \leq 10\,000$	4	9,000	36,000

TOTAL = 296,000

(a) Calculate an estimate for the mean price.

$$\text{Mean} = \frac{296,000}{64}$$

$$\text{estimated mean} \approx \frac{300,000}{60} = £5,000$$

£ 5,000

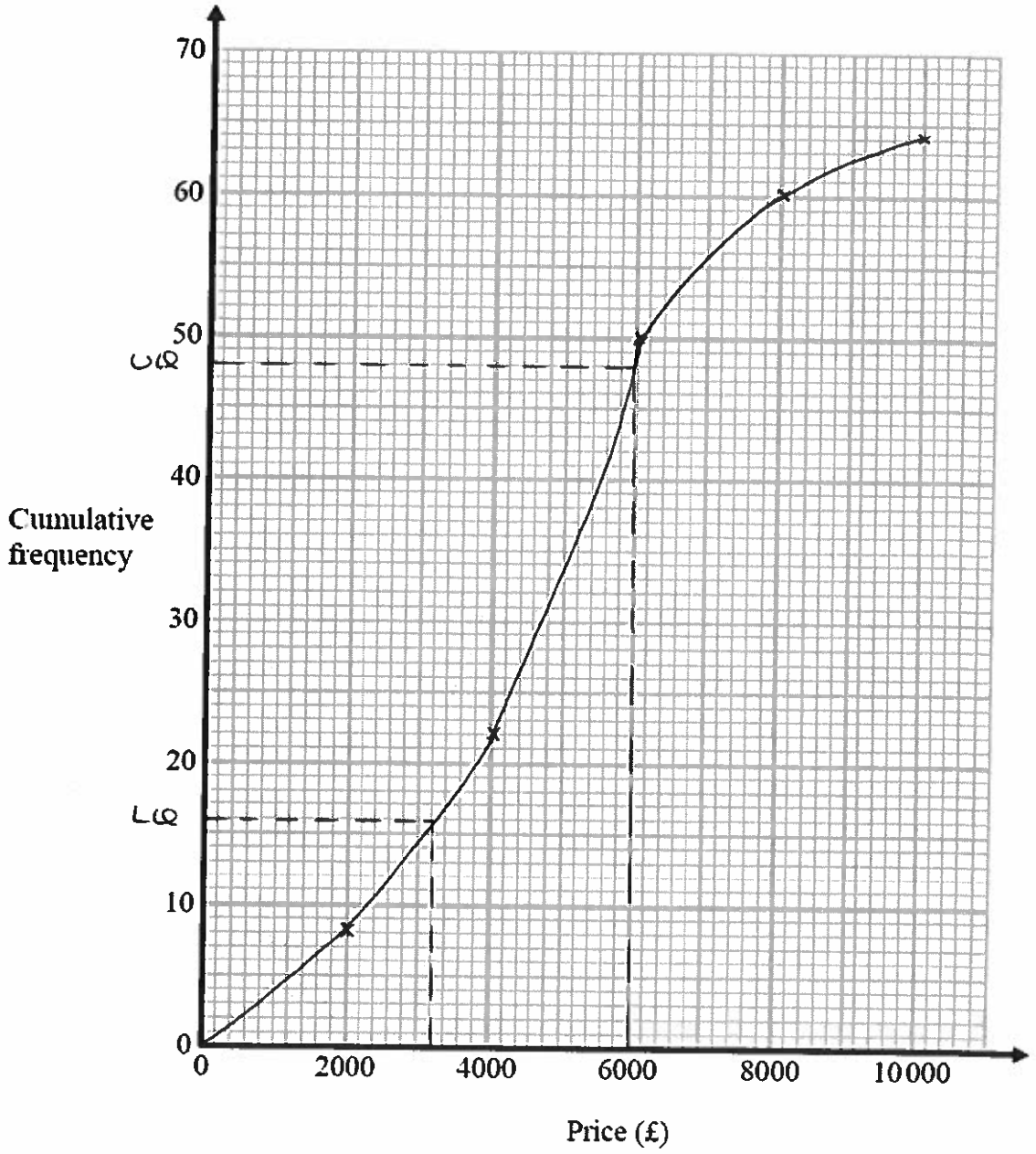
(3)

(b) Complete the cumulative frequency table.

Price (£x)	Cumulative frequency
$0 < x \leq 2000$	8
$0 < x \leq 4000$	22
$0 < x \leq 6000$	50
$0 < x \leq 8000$	60
$0 < x \leq 10\,000$	64

(1)

(*c*) On the grid, draw a cumulative frequency graph for your table.



(2)

(*d*) Find an estimate for the interquartile range.

IQR = UQ − LQ

= £6,000 − £3,200

= £2,800

£ 2,800

(2)

(Total for Question 5 is 8 marks)

6. Expand and simplify $(x+2)(x+8)(x-4)$

$(x+2)(x+8) = x^2+8x+2x+16$

$= x^2+10x+16$

$= (x^2+10x+16)(x-4)$

$= x^3-4x^2+10x^2-40x+16x-64$

$= x^3+6x^2-24x-64$

$x^3+6x^2-24x-64$

(Total for Question 6 is 3 marks)

7. There are 24 girls and 12 boys in a club.

One girl and one boy are going to be chosen to go to a meeting.

Work out the total number of ways of choosing a girl and a boy.

$24 \times 12 = 288$ ways

$$\begin{array}{r} 24 \\ \times 12 \\ \hline 48 \\ 240 \\ \hline 288 \end{array}$$

288

(Total for Question 7 is 2 marks)

8. Solve the simultaneous equations

$4x + 2y = 7$ (1) $\times 5$

$3x - 5y = -24$ (2) $\times 2$

$20x + 10y = 35$ (3)

$6x - 10y = -48$ (4)

(3)+(4) $26x = -13$

$x = -\frac{13}{26} = -\frac{1}{2}$

Sub into (1) $4(-\frac{1}{2}) + 2y = 7$

$-2 + 2y = 7$

$2y = 9$

$y = \frac{9}{2}$

$x =$ $-\frac{1}{2}$ $(= -0.5)$

$y =$ $\frac{9}{2}$ $(= 4.5)$

(Total for Question 8 is 4 marks)

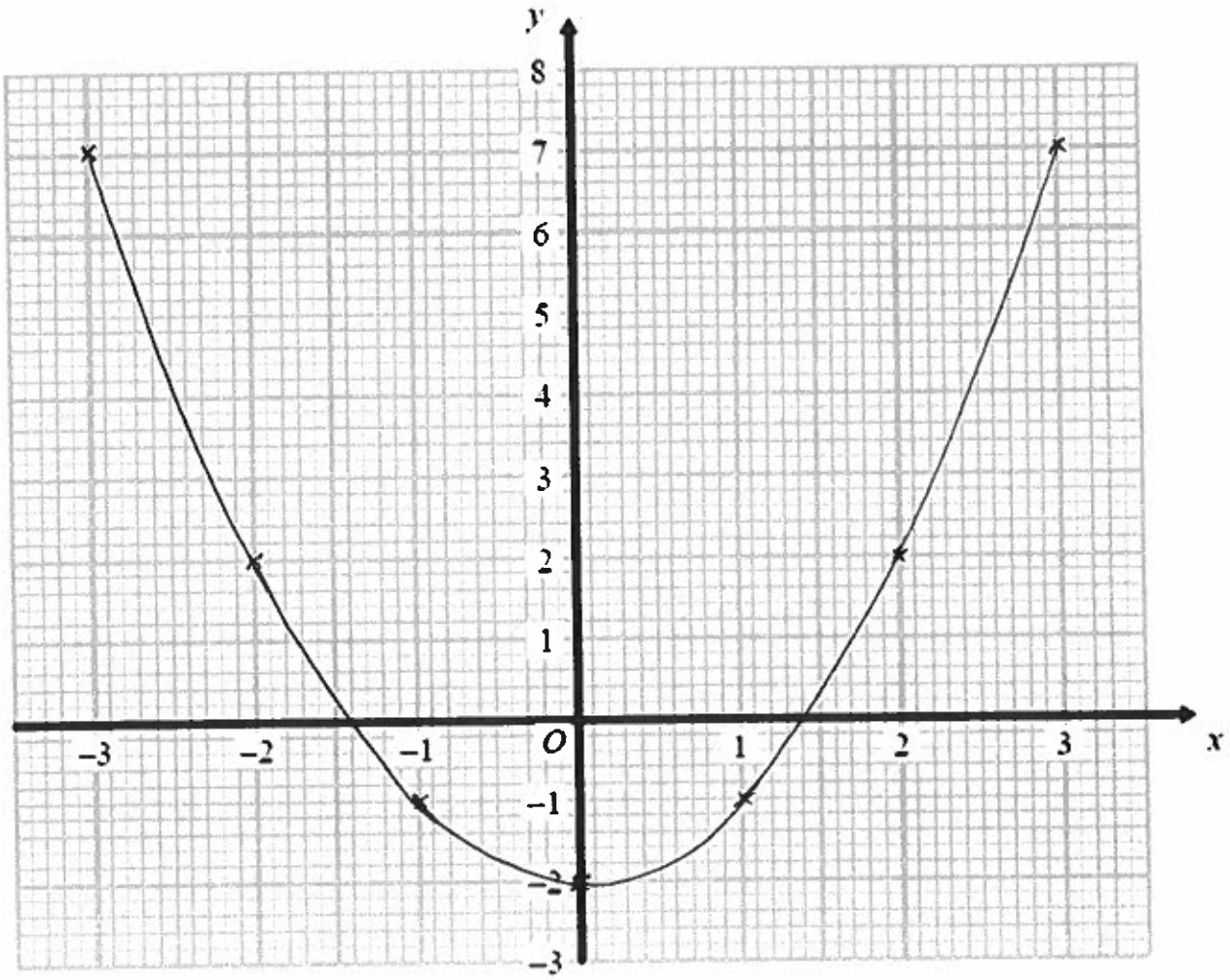
9.

(a) Complete the table of values for $y = x^2 - 2$

x	−3	−2	−1	0	1	2	3
y	7	2	−1	−2	−1	2	7

(2)

(b) On the grid, draw the graph of $y = x^2 - 2$ for values of x from −3 to 3



(2)

(Total for Question 9 4 marks)

10

Work out $2\frac{3}{5} - 1\frac{5}{6}$

$= \frac{13}{5} - \frac{11}{6}$

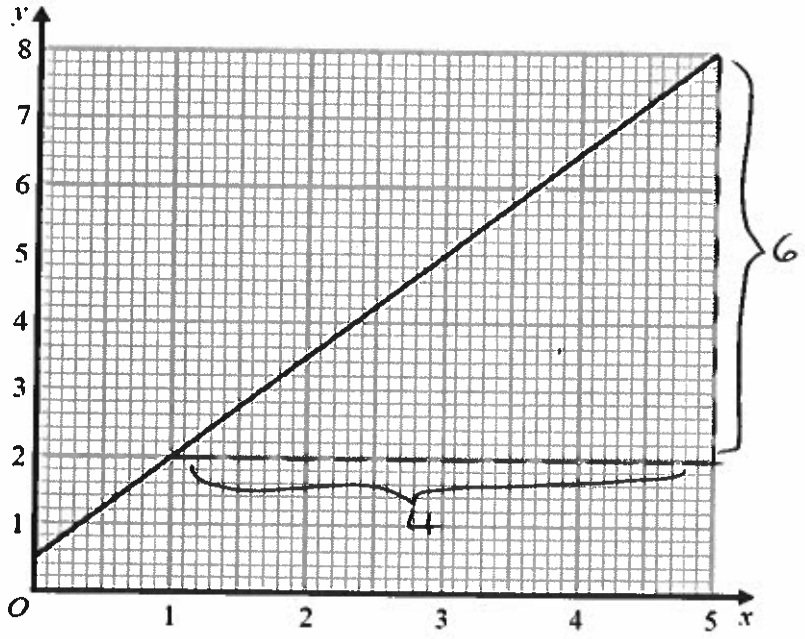
$= \frac{78}{30} - \frac{55}{30}$

$= \frac{23}{30}$

$\frac{23}{30}$

(Total for Question 10 is 2 marks)

11



Phone calls cost £ y for x minutes.

The graph gives the values of y for values of x from 0 to 5

(a) (i) Give an interpretation of the intercept of the graph on the y-axis.

It is a fixed charge (of 60p) for using the (mobile) phone.

(ii) Give an interpretation of the gradient of the graph.

It is the cost per minute of using the phone.

(2)

(b) Find the equation of the straight line in the form $y = mx + c$

gradient, $m = \frac{6}{4} = £1.5$ $c = £0.6$

$y = 1.5x + 0.6$

(2)

(Total for Question 11 is 4 marks)

12 Liquid **A** has a density of 1.42 g/cm^3 CALC

7 cm^3 of liquid **A** is mixed with 125 cm^3 of liquid **B** to make liquid **C**.

Liquid **C** has a density of 1.05 g/cm^3

$D = \frac{M}{V}$ or $M = DkV$

Find the density of liquid **B**.

Give your answer correct to 2 decimal places.

Liquid A: $M = D \times V = 1.42 \times 7 = 9.94$ g

Liquid C: Volume $= 7 + 125 = 132$ cm^3

$M = DkV = 1.05 \times 132 = 138.6$ g

ie, Liquid B: $M = 138.6 - 9.94 = 128.66$ g

$D = \frac{M}{V} = \frac{128.66}{125} = 1.02928$ g/cm^3

$= 1.03$ g/cm^3 (2 dp)

1.03 ..... g/cm^3

(Total for Question 12 is 3 marks)

13 In a sale, the price of a TV is reduced by 25%. CALC

A week later, the sale price of the TV is reduced by 15%.

The price of the TV is now £293.25.

What was the price of the TV before the sale? (3C)

$100\% - 25\% = 75\% = 0.75$

and $100\% - 15\% = 85\% = 0.85$

$x \times 0.75 \times 0.85 = £293.25$

$x \times 0.6375 = £293.25$

$x = \frac{£293.25}{0.6375} = £460$

£ 460

(Total for Question 13 is 3 marks)

14 Make m the subject of

$$f = \frac{4-3m}{5+m} \qquad (m)$$

$f(5+m) = 4-3m$

$5f + fm = 4-3m$

$fm + 3m = 4-5f$

$m(f+3) = 4-5f$

$m = \frac{4-5f}{f+3}$

$m = \frac{4-5f}{f+3}$

(Total for Question 14 is 3 marks)

15 Here are the first four terms of a quadratic sequence.

3 (+5) 8 (+7) 15 (+9) 24

(+2, +2) → ÷2 = $1n^2$

Find an expression, in terms of n, for the nth term of this sequence.

n	1	2	3	4
n^2	1	4	9	16
Sequence	3	8	15	24
Diff	+2	+4	+6	+8

(+2, +2, +2) ← n^{th} term of this line $= 2n + 0$

ie n^{th} term $= n^2 + 2n$

$n^2 + 2n$

(Total for Question 15 is 3 marks)

16 For all values of x,

$$f(x) = x^2 + 1 \qquad g(x) = 3x - 4$$

(a) Find $g^{-1}(x)$.

$y = 3x - 4$

$y + 4 = 3x$

$\frac{y+4}{3} = x$

ie, $g^{-1}(x) = \frac{x+4}{3}$

(2)

(b) Solve $fg(x) = gf(x)$.

$fg(x) = (3x-4)^2 + 1$

$= (3x-4)(3x-4) + 1$

$= 9x^2 - 12x - 12x + 16 + 1$

$= 9x^2 - 24x + 17$

$gf(x) = 3(x^2+1) - 4$

$= 3x^2 + 3 - 4$

$= 3x^2 - 1$

ie, $9x^2 - 24x + 17 = 3x^2 - 1$

$6x^2 - 24x + 18 = 0$

$\div 6 \quad x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

ie, $x = 1$ or $x = 3$

(3)

(Total for question 16 is 5 marks)

17 Anna and Bill share some money in the ratio 2 : 5

Anna gets £A

Bill gets £B

assume A and B share £70

A : B = TOTAL

2 : 5 = 7

£20 : £50 = £70

Carl and Donna share twice as much money as Anna and Bill share.

They share the money in the ratio 3 : 1

so C and D share £140

C : D = TOTAL

3 : 1 = 4

x35 ↓ £105 : £35 = £140 ↓ x35

Carl gets £C

Donna gets £D

Find $A : B : C : D$

Give your answer in its simplest form.

ie, $A : B : C : D$

$\div 5$ ↓ $20 : 50 : 105 : 35$

$4 : 10 : 21 : 7$ ↓ $\div 5$

4 : 10 : 21 : 7

.....

(Total for Question 17 is 3 marks)

18 A bonus of £4200 is shared by ~~10~~ 20 people who work for a company. CALC
80% of the bonus is shared equally between 6 managers.
The rest of the bonus is shared equally between 14 salesmen.

One of the salesmen says,

"If the bonus is shared equally between all 20 people I will get 50% more money."

Is the salesman correct?
You must show how you get your answer

If shared equally: share $= \frac{£4200}{20} = £210$ each person

OR 80% of £4,200 $= \frac{80}{100} \times £4,200 = £3,360$ (shared between 6 managers)

Balance $= £4,200 - £3,360 = £840$ between 14 salesmen

$\frac{£840}{14} = £60$ to each salesman

additional bonus $= £210 - 60 = £150$

% increase $= \frac{£150}{£60} = 250\%$

ie, salesman is wrong, he will get 250% more money.

(Total for Question 18 is 5 marks)

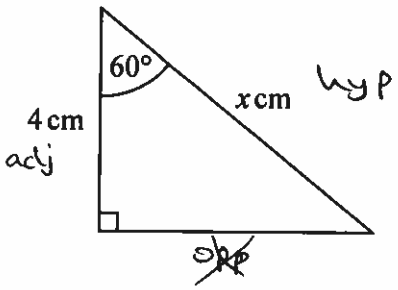
19 (*a*) Write down the exact value of Cos 45°
NON-CALC

$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$

.....

(1)

Here is a right-angled triangle.



$\cos 60 = \frac{4}{x}$

$x \cos 60 = 4$

$x = \frac{4}{\cos 60} = \frac{4}{0.5} = 8$

cos 60° = 0.5

(*b*) Work out the value of x.

(from over leaf)

8

.....

(2)

(Total for Question 19 is 3 marks)

20 (*a*) Simplify $\dfrac{x-1}{5(x-1)} = \dfrac{1}{5(x-1)}$

$\dfrac{1}{5(x-1)}$

.....

(1)

(*b*) Factorise fully $98 - 2y^2$

$= 2(49 - y^2)$

$= 2(7+y)(7-y)$

$2(7+y)(7-y)$

.....

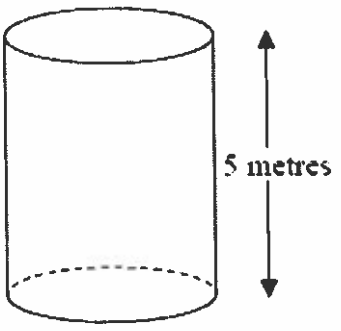
(2)

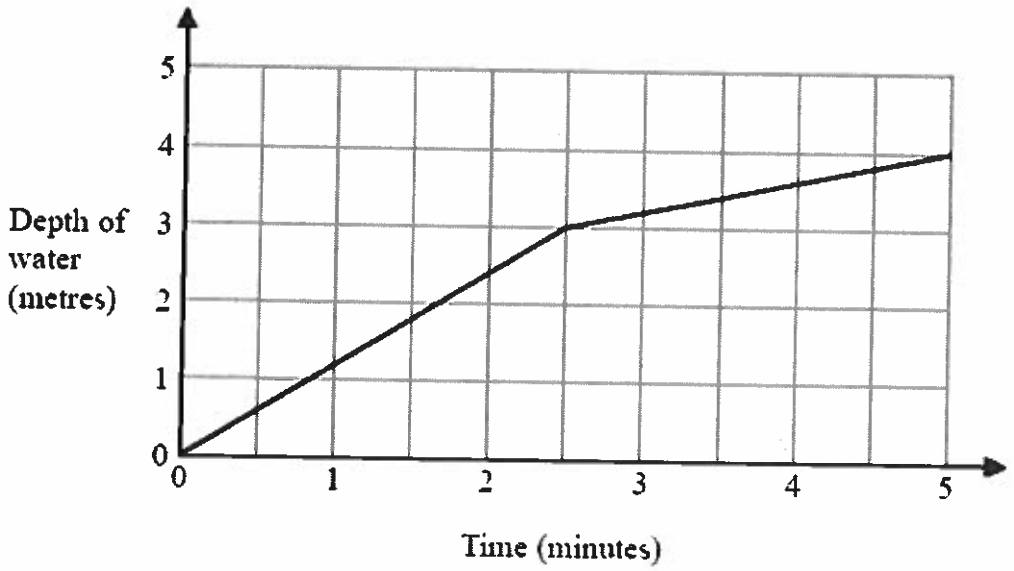
(Total for Question 20 is 3 marks)

21 An empty tank is a cylinder of height 5 metres.

Water is poured into the tank at a constant rate.
It takes 4 minutes to fill the tank completely with water.

Malcolm draws this graph to show the depth of water in the tank as water is poured into the tank.





Write down two things wrong with this graph.

1 The graph should be a straight line as the water is poured into the tank at a constant rate.

2 The graph should end at (4mins, 5metres) and NOT at (5mins, 4 metres)

(Total for Question 21 is 2 marks)

22 Cars are made in a factory for 24 hours every day. NON-CALC

In the factory a car is made every 209 seconds.

(a) Work out an estimate for the number of cars made in the factory in one year.
You must show how you get your answer.

Total hours in one year = 24×365 hours

Total seconds in one year = $24 \times 365 \times 60 \times 60$ seconds

No. of cars made = $\frac{24 \times 365 \times 60 \times 60}{209}$

$= \frac{20 \times 400 \times 60 \times 60}{200}$

$= 20 \times 2 \times 60 \times 60$

$= 40 \times 60 \times 60$

$= 144{,}000$

144,000 (cars)

(4)

(b) Is your answer to part (a) an underestimate or an overestimate?
Give a reason for your answer.

It's an underestimate because the denominator was rounded down (and the numerator up).

.....

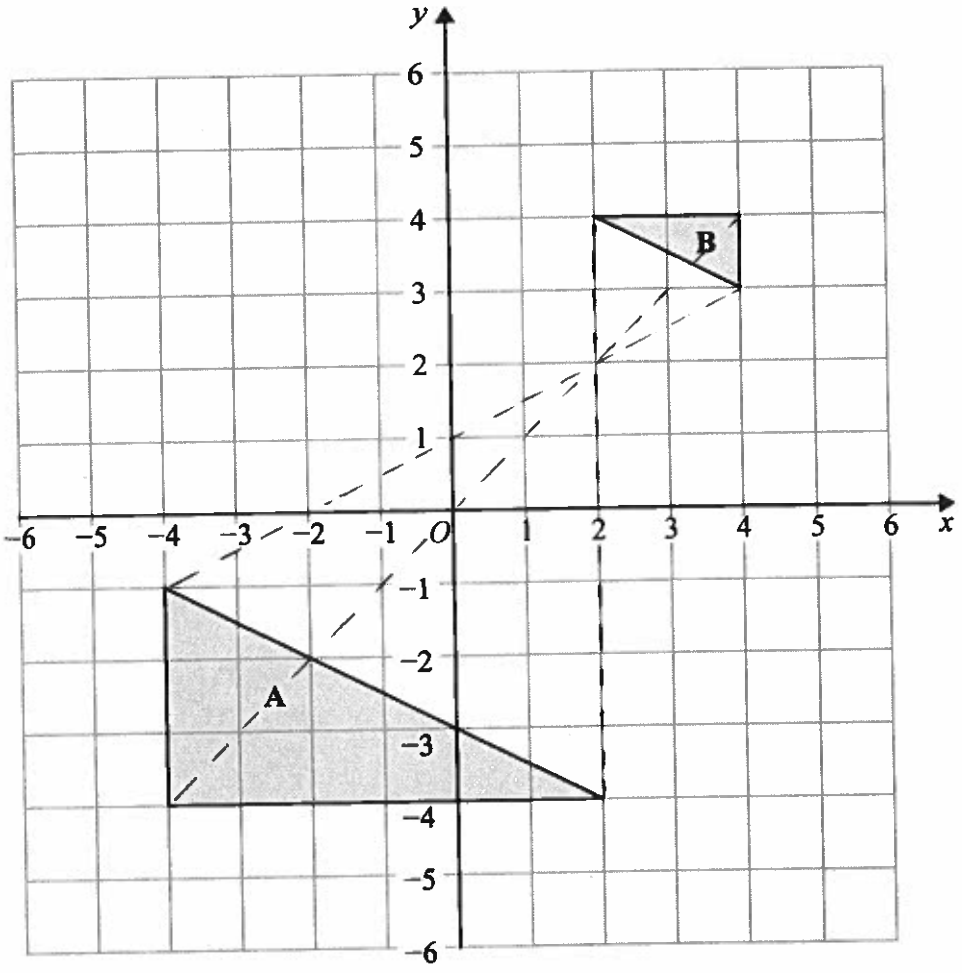
.....

.....

(1)

(Total for Question 22 is 5 marks)

23



Describe fully the single transformation that maps triangle **A** onto triangle **B**.

Enlarge by scale factor $-\frac{1}{3}$, centre of enlargement (2,2).

(Total for Question 23 is 2 marks)

24 Ben fills a glass with orange juice and lemonade in the ratio 1 : 4 by volume.
He mixes the liquid that is in the glass.

Ben drinks $\frac{1}{4}$ of this liquid.

He then fills the glass using orange juice.
Work out the ratio of orange juice to lemonade, by volume, that is now in the glass.
Give your ratio in its simplest form.

OJ : L = DRINK

1 : 4 = 5 ↓ ÷ 4

$-\frac{1}{4}$: −1 = $-5/4$

$\frac{3}{4}$: 3 = $3\frac{3}{4}$

+ $1\frac{1}{4}$: 0 = $+1\frac{1}{4}$

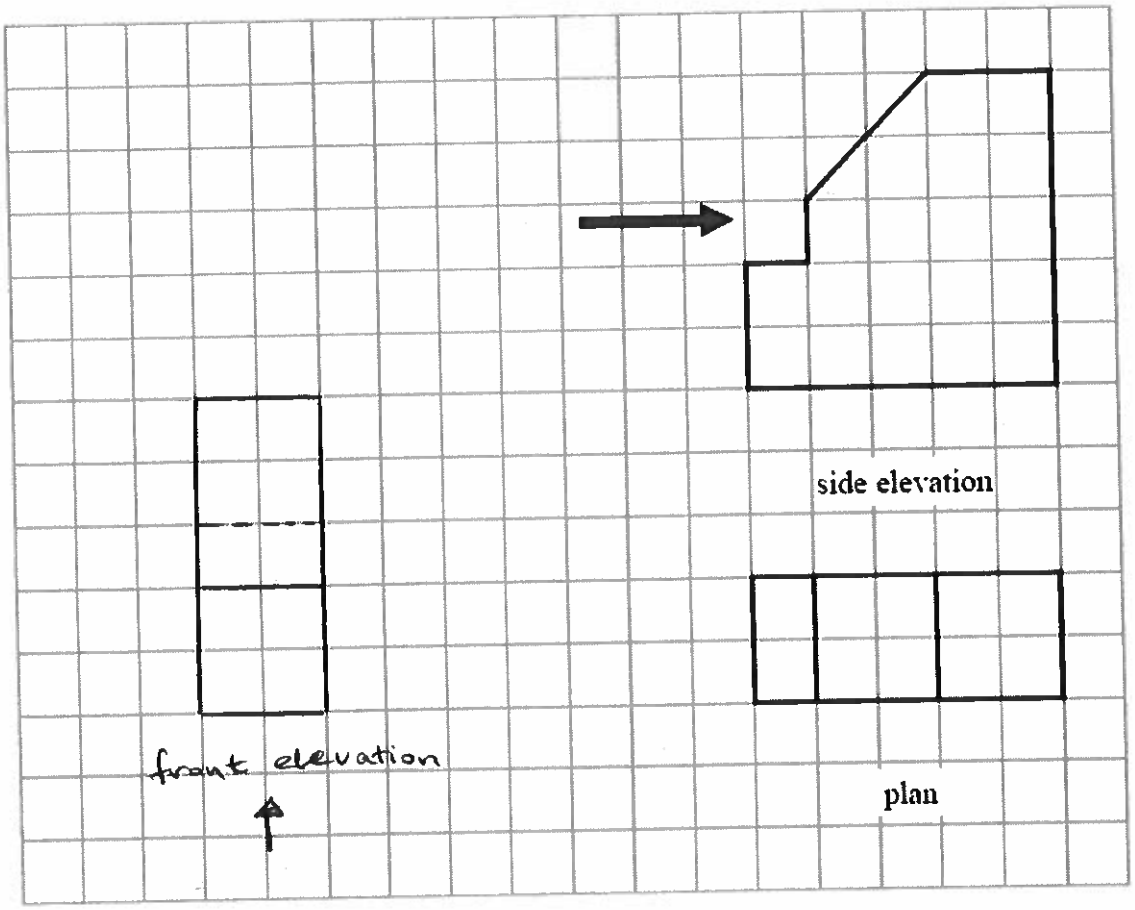
2 : 3 = 5

2 : 3

(Total for Question 24 is 3 marks)

25 The plan and side elevation of a solid prism are shown on the grid.

On the grid, draw the front elevation of the prism from the direction of the arrow.



(Total for Question 25 is 2 marks)

NON-CALC

26 Show that $\frac{12+\sqrt{128}}{1-\sqrt{2}}$ can be written in the form $a+b\sqrt{2}$, where a and b are integers.

$$\frac{12+\sqrt{64}\times\sqrt{2}}{1-\sqrt{2}} = \frac{(12+8\sqrt{2})(1+\sqrt{2})}{(1-\sqrt{2})(1+\sqrt{2})}$$

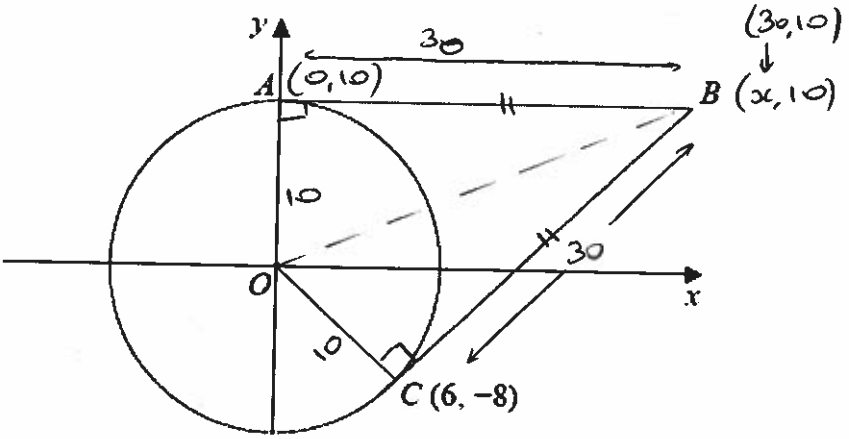
$$= \frac{12+12\sqrt{2}+8\sqrt{2}+16}{1+\sqrt{2}-\sqrt{2}-2}$$

$$= \frac{28+20\sqrt{2}}{-1}$$

$$= -28-20\sqrt{2}$$

27

(Total for Question 26 is 4 marks)



The diagram shows the circle with equation $x^2 + y^2 = 100$ $\Rightarrow$ $r^2 = 100$
The unit of length on both axes is one centimetre. ie, $r = \sqrt{100} = 10$

The circle intersects the positive y-axis at the point A.
The point C on the circle has coordinates (6, −8)
The straight lines AB and CB are tangents to the circle.

Find the area of quadrilateral $ABCO$.

length $BC = \sqrt{(x-6)^2 + (10-(-8))^2}$

$= \sqrt{(x-6)^2 + 324}$

but length $AB = x$

ie, $x = \sqrt{(x-6)^2 + 324}$

ie, $x^2 = (x-6)(x-6) + 324$

$x^2 = x^2 - 6x - 6x + 36 + 324$

$12x = 360$

$x = \frac{360}{12} = 30$

ie, B (30, 10)

(two tangents from a point are equal)

area $OAB = \frac{1}{2} \times 10 \times 30 = 150 \text{ cm}^2$

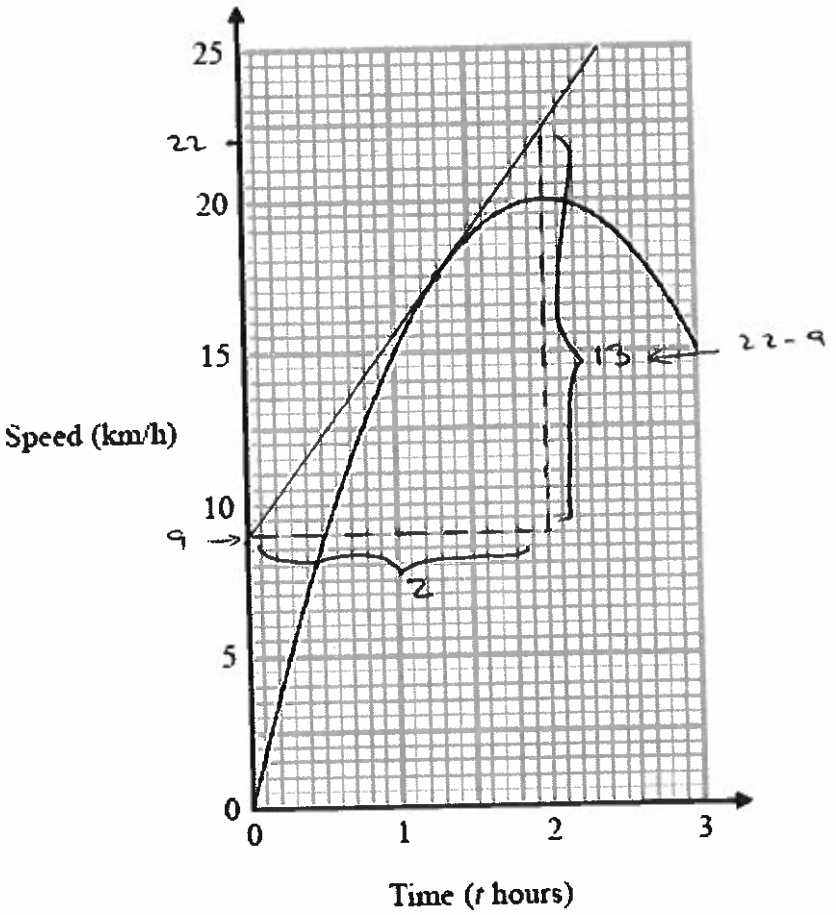
area $OCB = \frac{1}{2} \times 10 \times 30 = 150 \text{ cm}^2$

TOTAL area = 300 cm^2

..... 300 ..... cm^2

(Total for Question 27 is 4 marks)

28 The graph gives the speed, in km/h, of a ship t hours after leaving a port.



(a) Find an estimate of the gradient of the graph when $t = 1.3$

You must show how you get your answer.

gradient $= \frac{13}{2} = 6.5$

(allow 6-7)

6.5

.....

(3)

(b) Interpret your answer to part (a) in the context of the question.
You must give units with your interpretation.

The gradient is the acceleration of the ship of 6.5 km/h^2.

.....

(2)

(Total for Question 28 is 5 marks)

29.

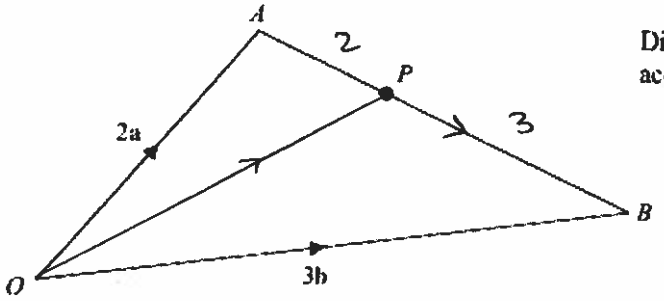


Diagram NOT accurately drawn

OAB is a triangle.

$\overrightarrow{OA} = 2\mathbf{a}$

$\overrightarrow{OB} = 3\mathbf{b}$

(*a*) Find $\overrightarrow{AB}$ in terms of **a** and **b**.

$\overrightarrow{AB} = -2\underline{a} + 3\underline{b}$

$\overrightarrow{AB} =$ $-2\underline{a} + 3\underline{b}$

(1)

P is the point on *AB* such that $AP : PB = 2 : 3$

(*b*) Show that $\overrightarrow{OP}$ is parallel to the vector **a** + **b**.

$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$

$= 2\underline{a} + \frac{2}{5}\overrightarrow{AB}$

$= 2\underline{a} + \frac{2}{5}(-2\underline{a} + 3\underline{b})$

$= 2\underline{a} - \frac{4}{5}\underline{a} + \frac{6}{5}\underline{b}$

$= \frac{6}{5}\underline{a} + \frac{6}{5}\underline{b}$

$\overrightarrow{OP} = \frac{6}{5}(\underline{a} + \underline{b})$ which is parallel to $(\underline{a} + \underline{b})$

(3)

(Total for Question 29 4 marks)

30

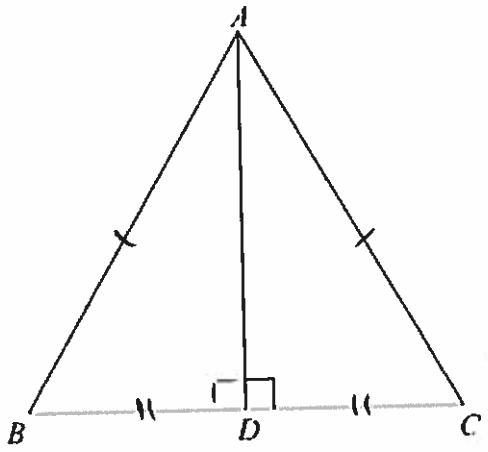


Diagram **NOT** accurately drawn

SSS
SAS
ASA
RHS ✓

ABC is an equilateral triangle.
D lies on *BC*.
AD is perpendicular to *BC*.

(a) Prove that triangle *ADC* is congruent to triangle *ADB*.

AB = AC (equal sides of an equilateral triangle) H

Angle BDA = Angle ADC (both 90°) R

and AD is common to both triangles S

ie, △ ADC is congruent to △ ADB (RHS)

(3)

(b) Hence, prove that $BD = \frac{1}{2}AB$.

BD = DC (two sides in congruent triangles)

ie, $BD = \frac{1}{2}BC$

but $BC = AB$

ie, $BD = \frac{1}{2}AB$

(2)

(Total for Question 30 5 marks)