**Discovering Mathematics**
**Student Books 1B, 2B and 3B**

# Home Learning Pack:
# Problems in Real-world Contexts

**THIS PACK CONTAINS** pages from Discovering Mathematics Student Books 1B, 2B and 3B. It includes questions which apply mathematics knowledge to real-life situations. It also includes fully-worked solutions for all questions. You can find Student Books 1B, 2B and 3B on Amazon.

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*Year 7*

*Year 8*

*Year 9*

# DISCOVERING MATHEMATICS

**1B**

## Victor Chow

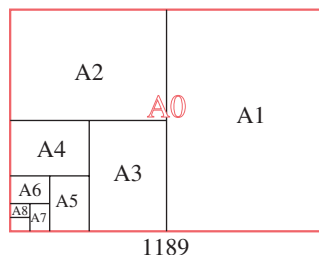UK Consultant: Robert Wilne

Singapore Consultant: Berinderjeet Kaur

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# PROBLEMS IN REAL-WORLD CONTEXTS

## A. Paper Sizes



1189

The diagram shows the dimensions of 'A series' paper sizes. The complete sheet is of size A0 with length 1189 mm and area 1 000 000 mm$^2$ (or 1 m$^2$). The dimensions of this series of paper have the special property that one piece of A0 paper can be divided into two pieces of A1 paper, one piece of A1 paper can be divided into two pieces of A2 paper and so on.

1. Find the width of a piece of A0 paper. Give your answer to the nearest mm.

2. **(a)** Find the area of a piece of A4 paper in m$^2$, expressing your answer as a fraction.
   **(b)** Find the length and width of a piece of A4 paper to the nearest mm.

3. The mass of a piece of A0 paper is 80 grams. Find the mass of a piece of A4 paper of the same thickness and quality.

4. Sam puts some A4 posters in an envelope. The envelope has mass 25 grams. To minimise the cost of the postage, the total mass should not be greater than 100 grams. How many A4 posters can be placed in the envelope?
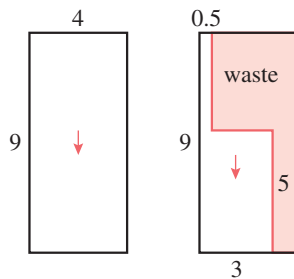
## B. Laying Carpet
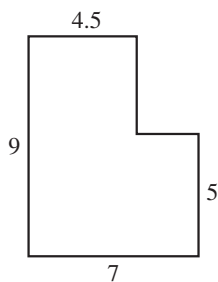


The diagram shows the floor plan of Mr Brown's living room where all the angles are right angles. The dimensions are in metres. Mr Brown wants to lay carpet in the living room. The carpet is sold from a roll that is 4 m wide. Every linear metre (that is 4 m$^2$) cut off the roll costs £80.

1. Find the area of the living room.

**2.** When there is a seam, all carpet must run in the same direction otherwise different pieces of carpet will show different colours under natural light. The diagram below shows one way of laying the carpet for this room.
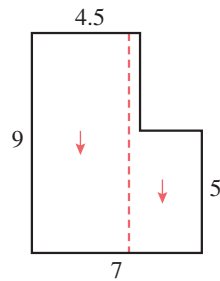


There are two pieces of carpet. The red dotted line is the seam and the arrows show the carpet direction. Here, Mr Brown has to buy two pieces of carpet of 9 m by 4 m. The second piece has to be cut to the shape shown on the right.



(a) Find the total cost of the carpet.
(b) Find the percentage of waste of the carpet bought to the nearest 1%.

**3.** Design another way of laying the carpet that minimises the waste and hence reduces the carpet cost. Find the cost of the carpet and the amount of waste for this design.
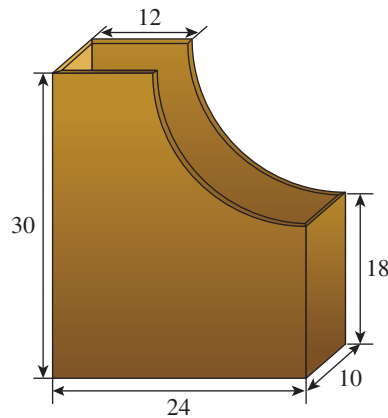
# C. Brick Wall

The standard size of bricks in the UK is 215 mm long, 102.5 mm wide and 65 mm high. When laying bricks, mortar of thickness 10 mm is used to hold the bricks together. One-half running bond is the basic pattern for laying a wall, i.e. approximately half of a brick's length overlaps the brick below.
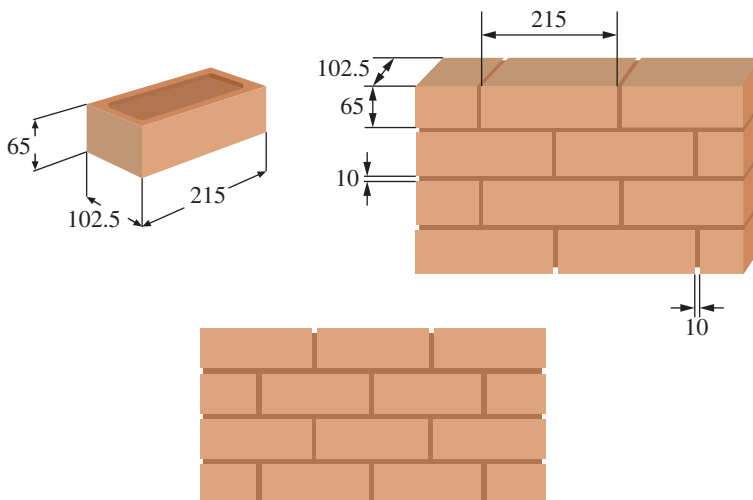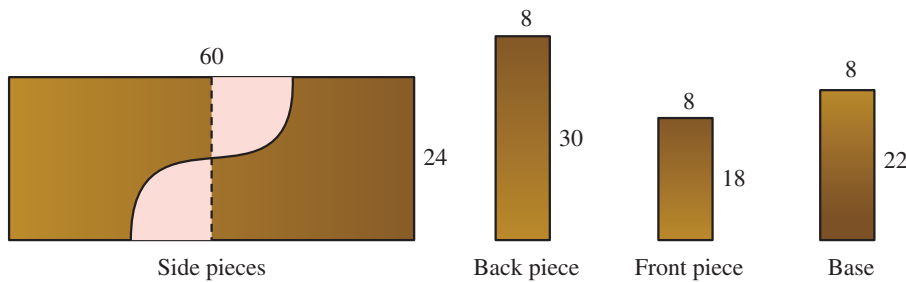
1. In the diagram, the top and the third rows use three whole bricks, while bricks in the second and bottom rows have to be cut at both ends. Find

   (a) the length and height of this arrangement in cm,

   (b) the volume of this arrangement to the nearest $10\,cm^3$ if the thickness of this arrangement is one brick width.

2. Mr Tamworth is going to build a wall $2\,m$ high, $5\,m$ long and $102.5\,mm$ thick. Estimate the number of bricks required. State any assumptions you make in your calculation.

3. Mr Tamworth wants to buy 5% more bricks than the number required to allow for breakage and waste. How many bricks should he buy?

# D.   Magazine Holder



Jenny makes a magazine holder using wooden board $1\,cm$ thick. The two side pieces are cut from a board of $60\,cm$ by $24\,cm$ as shown below. A quarter of a circle of radius $12\,cm$ is cut from each side in the middle.



| Side pieces | Back piece | Front piece | Base |

The back end piece is $30\,cm$ high and $8\,cm$ wide. The front end piece is $18\,cm$ high and $8\,cm$ wide. The base piece is $22\,cm$ long and $8\,cm$ wide. A piece of wood of $90\,cm$ by $24\,cm$ by $1\,cm$ is ordered to make the magazine holder.

1. Find the volume of the piece of wood ordered.

2. Find the perimeter of each side piece to the nearest cm.

3. Find the percentage of wastage from the piece of wood that is ordered to the nearest 1%.

# E.   Clearway Road Sign



The above diagram shows a clearway road sign. It is used to indicate no stopping on the main carriageway. The sign is a circle of diameter 60 cm. The red border and the red cross are 6 cm wide. The blue parts are four congruent shapes.

1.   (a)   State the number of lines of symmetry on the sign.
     (b)   Does the sign possess rotation symmetry? If so, what is the order of the rotation symmetry?

2.   Explain why each blue part is not a quarter of the inner circle.

3.   Estimate the area of the red region of the sign. State any assumptions you make. Give your answer to the nearest cm$^2$.

# F.   London Tower Bridge



Tower Bridge is a bridge crossing the River Thames in London. It was built between 1886 and 1894. One special feature of the bridge is the two bascules (or leaves) in the middle span of the bridge. These can be raised to a maximum angle of 86° and are supported by two piers. Towers 65 m high stand on each pier, connected by a high-level walkway. The original design used steam engines to drive the bascules up and down, but these have now been replaced by an electro-hydraulic system.

1.   The bridge is crossed by over 40 000 people (motorists, cyclists and pedestrians) every day. Estimate the number of people crossing the bridge in a year.

2.   Why do you think the design of the bridge uses bascules in the middle span and has a high-level walkway connecting the two towers?

**3.** The diagram below shows that the two equal bascules $AB$ and $DC$ are rotated 70° about their hinges at $A$ and $D$, where $AD = 61$ m. The end $C$ is at a horizontal distance of $0.34x$ metres from the hinge $D$ and at a vertical height $0.94x$ metres above the level $AD$, where $x$ metres is the length of each bascule.
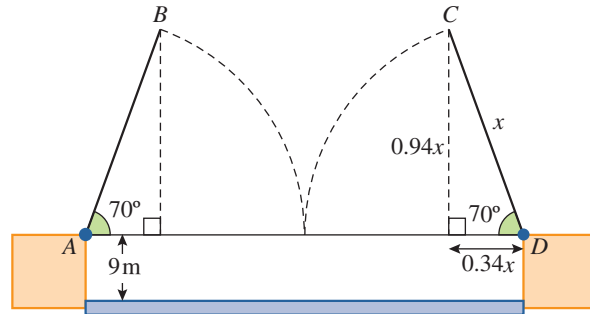


(a) Find the distance $BC$. Give your answer to the nearest metre.
(b) The bridge $AD$ is 9 m above the water level at a particular time. How high is the point $C$ above the water level at this time? Give your answer to the nearest metre.

# G. Population Pyramid for the UK

A population pyramid shows the age and sex of a population. The diagram shows the population pyramid for the UK in 2015.

**Population pyramid for the UK in 2015**
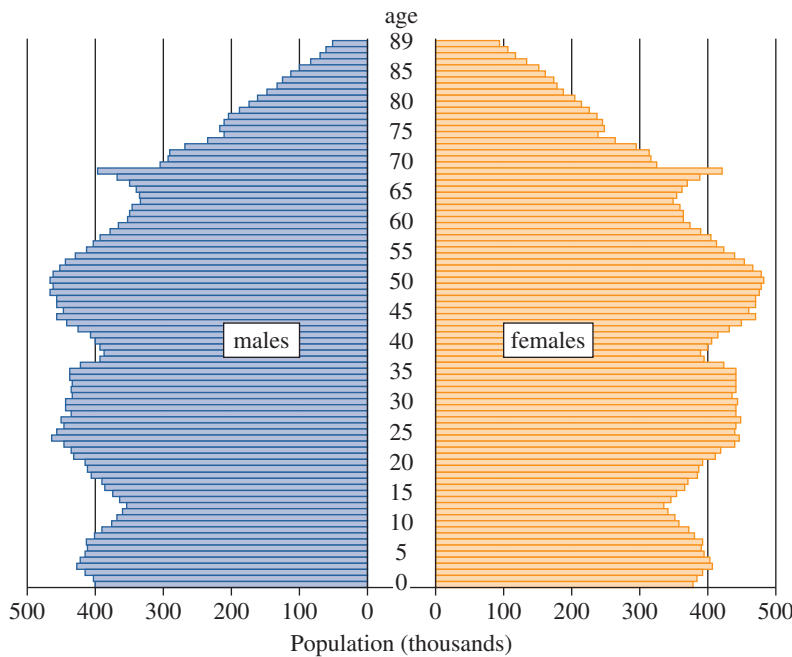


Source: Office for National Statistics

**1.** Assume all Year 7 students are 11 years old. Estimate the total number of Year 7 students from the population pyramid.

**2.** Among the age groups 5 to 10, 25 to 30 and 45 to 50, which age group has the largest number of people? Explain your answer.

**3.** Compare the shape of the population by age for males and females. What can you say about the number of males and females under 20, aged 50 to 60 and over 70?

# Problems in Real-world Contexts

## A. Paper Sizes



1189

The diagram shows the dimensions of 'A series' paper sizes. The complete sheet is of size A0 with length 1189 mm and area 1 000 000 mm² (or 1 m²). The dimensions of this series of paper have the special property that one piece of A0 paper can be divided into two pieces of A1 paper, one piece of A1 paper can be divided into two pieces of A2 paper and so on.

**GCSE Grade 2**

1. Find the width of a piece of A0 paper. Give your answer to the nearest mm.

**GCSE Grade 3+**

2. (a) Find the area of a piece of A4 paper in m², expressing your answer as a fraction.
   (b) Find the length and width of a piece of A4 paper to the nearest mm.

**GCSE Grade 3**

3. The mass of a piece of A0 paper is 80 grams. Find the mass of a piece of A4 paper of the same thickness and quality.

**GCSE Grade 3+**

4. Sam puts some A4 posters in an envelope. The envelope has mass 25 grams. To minimise the cost of the postage, the total mass should not be greater than 100 grams. How many A4 posters can be placed in the envelope?

### Solution

1. Let $x$ mm be the width of a piece of A0 paper.
   As the area of a piece of A0 paper is 1 000 000 mm²,
   $$1189 \times x = 1\,000\,000$$
   $$x = \frac{1\,000\,000}{1189}$$
   $$x = 841.04\ldots$$
   $$x = 841 \text{ (to the nearest mm)}$$
   $\therefore$ the width of a piece of A0 paper is 841 mm.

2. (a) A0 paper = 2 A1 paper
   = 2 × 2 A2 paper
   = 2 × 2 × 2 A3 paper
   = 2 × 2 × 2 × 2 A4 paper
   = 16 A4 paper
   You can see how a piece of A0 paper is divided into 16 pieces of A4 paper in the diagram.
   $\therefore$ area of a piece of A4 paper
   $$= \frac{1}{16} \times \text{area of a piece of A0 paper}$$
   $$= \frac{1}{16} \times 1\,\text{m}^2$$
   $$= \frac{1}{16}\,\text{m}^2$$

   (b) Length of a piece of A4 paper
   $$= \frac{1}{4} \times \text{length of a piece of A0 paper}$$
   $$= \frac{1}{4} \times 1189$$
   $$= 297.25$$
   $$= 297\,\text{mm (to the nearest mm)}$$

   Width of a piece of A4 paper
   $$= \frac{1}{4} \times \text{width of a piece of A0 paper}$$
   $$= \frac{1}{4} \times 841.04$$
   $$= 210.26$$
   $$= 210\,\text{mm (to the nearest mm)}$$

3. Mass of a piece of A4 paper
   $$= \frac{1}{16} \times \text{mass of a piece of A0 paper}$$
   $$= \frac{1}{16} \times 80$$
   $$= 5\,\text{g}$$

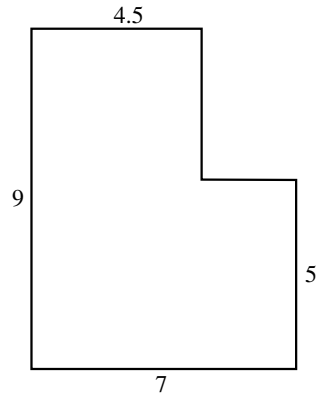4. Let $n$ be the number of A4 posters in the envelope. Consider the total mass of the envelope and the posters:
   $$25 + 5n = 100$$
   $$5n = 75$$
   $$n = \frac{75}{5}$$
   $$n = 15$$
   $\therefore$ 15 A4 posters can be placed in the envelope.

## B. Laying Carpet

4.5

9

5

7

The diagram shows the floor plan of Mr Brown's living room where all the angles are right angles. The dimensions are in metres. Mr Brown wants to lay carpet in the living room. The carpet is sold from a roll that is 4 m wide. Every linear metre (that is $4\,\text{m}^2$) cut off the roll costs £80.

**1.** Find the area of the living room.

| GCSE |
| Grade |
| **3** |

**2.** When there is a seam, all carpet must run in the same direction otherwise different pieces of carpet will show different colours under natural light. The diagram below shows one way of laying the carpet for this room.

| GCSE |
| Grade |
| **3** |
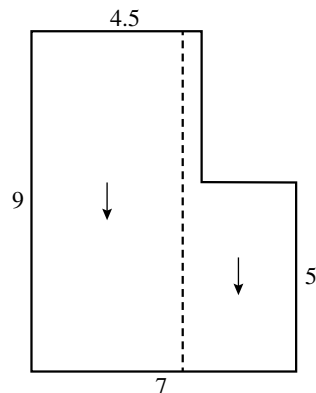
4.5

9

5

7

There are two pieces of carpet. The red dotted line is the seam and the arrows show the carpet direction. Here, Mr Brown has to buy two pieces of carpet of 9 m by 4 m. The second piece has to be cut to the shape shown on the right.

4          0.5

9          9          waste

5

3
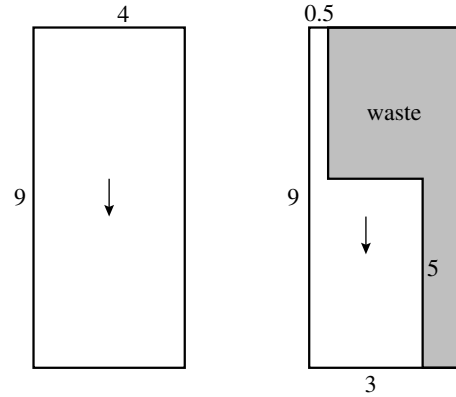
**(a)** Find the total cost of the carpet.
**(b)** Find the percentage of waste of the carpet bought to the nearest 1%.

**3.** Design another way of laying the carpet that minimises the waste and hence reduces the carpet cost. Find the cost of the carpet and the amount of waste for this design.

| GCSE |
| Grade |
| **3+** |

### Solution

**1.** Area of the living room
= area of the rectangle of $9\,\text{m} \times 7\,\text{m}$
  − area of the rectangle of $(9-5)\,\text{m} \times (7-4.5)\,\text{m}$
= $9 \times 7 - 4 \times 2.5$
= $53\,\text{m}^2$

*Alternative Solution 1*
Draw a horizontal line at the height of 5 m to divide the room into two rectangles.
Area of the room
= $4.5 \times 4 + 7 \times 5$
= $53\,\text{m}^2$

*Alternative Solution 2*
Draw a vertical line 4.5 m from the left edge to divide the room into two rectangles.
Area of the room
= $9 \times 4.5 + 5 \times (7 - 4.5)$
= $53\,\text{m}^2$

**2. (a)** Number of linear metres bought = $9 + 9$
                                        = $18\,\text{m}$
Total cost of carpet = £80 × 18
                     = £1440

**(b)** Area of the waste = $(9-5) \times (4-0.5)$
                         $+ 5 \times (4-3)$
                       = $19\,\text{m}^2$
Alternatively,
area purchased − area of room
= $(2 \times 9 \times 4) - 53$
= $72 - 53$
= $19\,\text{m}^2$
Percentage waste = $\frac{19}{72} \times 100\%$
                 = $26\%$ (to the nearest 1%)

**3.**
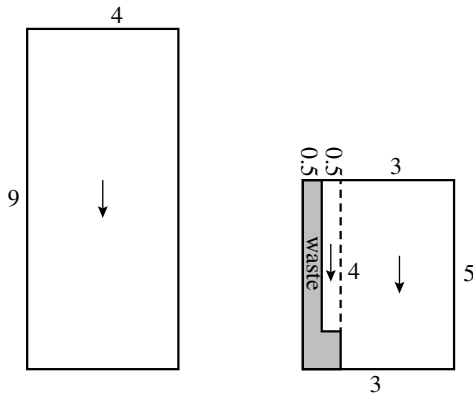


Mr Brown can buy 9 + 5 = 14 linear metres of carpet and cut three pieces of dimensions 9 m × 4 m, 5 m × 3 m and 4 m × 0.5 m from it to carpet the room (see diagram above).
This way,
total cost of carpet = 80 × 14
$$= £1120$$
area of waste = 4 × 0.5 + 1 × 1
$$= 3 \, m^2$$

## C. Brick Wall

The standard size of bricks in the UK is 215 mm long, 102.5 mm wide and 65 mm high. When laying bricks, mortar of 10 mm thick is used to hold the bricks together. One-half running bond is the basic pattern when laying a wall, i.e. approximately half of a brick's length overlaps the brick below.



**1.** In the diagram, the top and the third rows use three whole bricks, while bricks in the second and bottom rows have to be cut at both ends. Find

   **(a)** the length and height of this arrangement in cm,
   **(b)** the volume of this arrangement to the nearest 10 cm³ if the thickness of this arrangement is one brick width.

**2.** Mr Tamworth is going to build a wall 2 m high, 5 m long and 102.5 mm thick. Estimate the number of bricks required. State any assumptions you make in your calculation.

**3.** Mr Tamworth wants to buy 5% more bricks than the number required to allow for breakage and waste. How many bricks should he buy?

### Solution

**1. (a)** Length = 3 × length of brick + 2 × thickness of mortar
    = 3 × 21.5 + 2 × 1 cm
    = 66.5 cm
    Height = 4 × height of brick + 3 × thickness of mortar
    = 4 × 6.5 + 3 × 1 cm
    = 29 cm

   **(b)** Volume = $lhw$
    = 66.5 × 29 × 10.25
    = 19 767.125
    = 19 770 cm³ (to the nearest 10 cm³)

**2.**
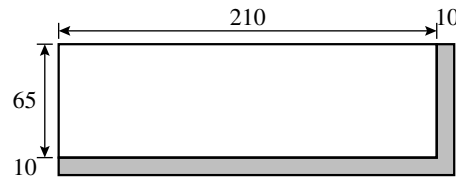


Make the assumption that each brick is a 'nominal brick' with mortar of thickness 10 mm on the bottom and on the right side as shown above i.e. each nominal brick is 225 mm long and 75 mm high.
Area of the front face of a nominal brick
= 0.225 × 0.075 m²
= 0.016 875 m²
Area of the wall = 5 × 2 m²
           = 10 m²
Number of nominal bricks required
= 10 ÷ 0.016 875
= 593 (to the nearest integer)
∴ an estimate of the number of bricks required for the wall is 593.

**3.** Number of bricks that should be bought
= 593 + 593 × 5%
= 623 (to the nearest integer)

## D. Magazine Holder
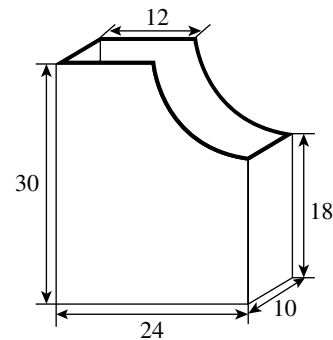
Jenny makes a magazine holder using wooden board 1 cm thick. The two side pieces are cut from a board of 60 cm by 24 cm as shown below. A quarter of a circle of radius 12 cm is cut from each side in the middle.



The back end piece is 30 cm high and 8 cm wide. The front end piece is 18 cm high and 8 cm wide. The base piece is 22 cm long and 8 cm wide. A piece of wood of 90 cm by 24 cm by 1 cm is ordered to make the magazine holder.

**1.** Find the volume of the piece of wood ordered.

**2.** Find the perimeter of each side piece to the nearest cm.

**3.** Find the percentage of wastage from the piece of wood that is ordered to the nearest 1%.

### Solution

**1.** Volume of the piece of wood ordered $= 90 \times 24 \times 1$
$$= 2160 \, \text{cm}^3$$

**2.** Perimeter of each side piece
$= 30 + 24 + (30 - 12) + \frac{1}{4} \times 2 \times \pi \times 12 + (24 - 12)$
$= 84 + 6\pi$
$= 103 \, \text{cm}$ (to the nearest cm)

**3.**
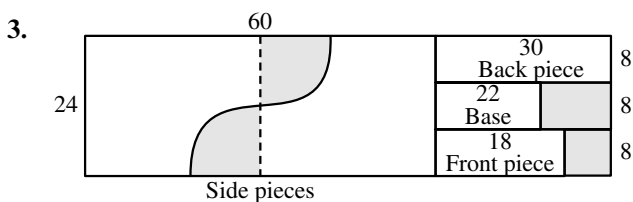


The diagram shows the way to cut the pieces from the piece of wood bought.
The shaded part is the wastage.
Total area of the two quarters cut out $= \frac{1}{2} \times \pi \times 12^2$
$$= 72\pi \, \text{cm}^2$$

Total area of the shaded rectangles
$= (30 - 22) \times 8 + (30 - 18) \times 8$
$= 160 \, \text{cm}^2$

Percentage of wastage $= \frac{72\pi + 160}{90 \times 24} \times 100\%$
$$= 18\% \text{ (to the nearest 1\%)}$$
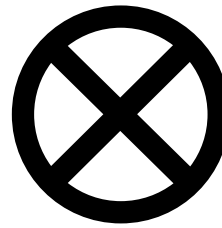
## E. Clearway Road Sign
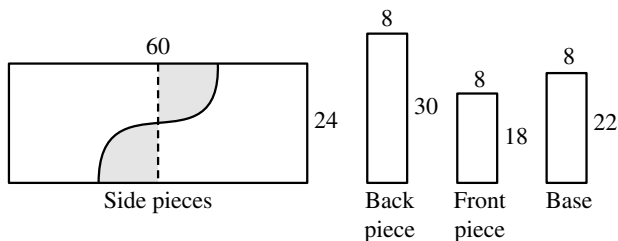


The above diagram shows a clearway road sign. It is used to indicate no stopping on the main carriageway. The sign is a circle of diameter 60 cm. The red border and the red cross are 6 cm wide. The blue parts are four congruent shapes.

**1. (a)** State the number of lines of symmetry on the sign.

**(b)** Does the sign possess rotation symmetry? If so, what is the order of the rotation symmetry?

**2.** Explain why each blue part is not a quarter of the inner circle.

**3.** Estimate the area of the red region of the sign. State any assumptions you make. Give your answer to the nearest cm².

### Solution

**1. (a)**



The sign has 4 lines of symmetry as shown above.

**(b)** The sign has rotation symmetry. It repeats itself 4 times in a full turn about its centre. The order of rotation symmetry is 4.

**2.**
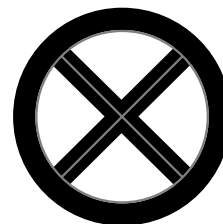


As shown in the above diagram, the two white diameters of the inner circle divide the inner circle into four quarters. This means the curved line of each blue area is part of the circle with the centre at the centre of the road sign, not at the point of intersection of the straight sides. Therefore, each blue area is only a part of a quarter, not a quarter.

**3.** Assume that the red cross is formed by two rectangles measuring $(60 - 12)$ cm by 6 cm. The red part consists of the red cross and the red circular rim.

Area of the red region
≈ area of the red ring + area of two rectangles measuring $(60 - 12)$ cm by 6 cm − area of the common square of the two rectangles at the centre of the cross
$= \pi \times 30^2 - \pi \times (30 - 6)^2 + 2 \times (60 - 12) \times 6 - 6 \times 6$
$= 900\pi - 576\pi + 576 - 36$
$= 1558$ cm$^2$ (to the nearest cm$^2$)

## F. London Tower Bridge

Tower Bridge is a bridge crossing the River Thames in London. It was built between 1886 and 1894. One special feature of the bridge is the two bascules (or leaves) in the middle span of the bridge. These can be raised to a maximum angle of 86° and are supported by two piers. Towers 65 m high stand on each pier, connected by a high-level walkway. The original design used steam engines to drive the bascules up and down, but these have now been replaced by an electro-hydraulic system.

**GCSE Grade 2⁺**
**1.** The bridge is crossed by over 40 000 people (motorists, cyclists and pedestrians) every day. Estimate the number of people crossing the bridge in a year.

**GCSE Grade 3**
**2.** Why do you think the design of the bridge uses bascules in the middle span and has a high-level walkway connecting the two towers?

**GCSE Grade 4⁻**
**3.** The diagram below shows that the two equal bascules $AB$ and $DC$ are rotated 70° about their hinges at $A$ and $D$, where $AD = 61$ m. The end $C$ is at a horizontal distance of $0.34x$ metres from the hinge $D$ and at a vertical height $0.94x$ metres above the level $AD$, where $x$ metres is the length of each bascule.
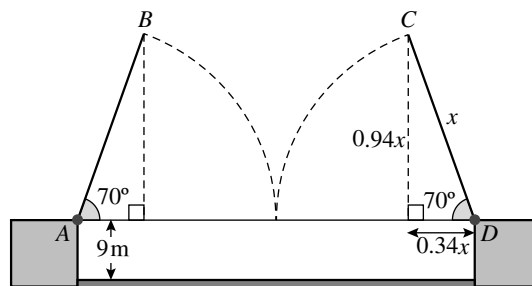


**(a)** Find the distance $BC$. Give your answer to the nearest metre.
**(b)** The bridge $AD$ is 9 m above the water level at a particular time. How high is the point $C$ above the water level at this time? Give your answer to the nearest metre.

### Solution
**1.** Estimate of the number of people crossing the bridge in a year
$= 40\,000 \times 365$
$= 14\,600\,000$

**2.** The design uses bascules in the middle span so that vessels, whose heights are less than the high-level walkway, can pass beneath the bridge. There is a high-level walkway to allow pedestrians to cross the bridge even when the bascules are raised.
**Note:** The bascules are raised about 1000 times each year. Vessels have to give 24 hours' notice but there is no charge for vessels.

**3. (a)** Length of each bascule $= 61 \div 2$
$\qquad\qquad\qquad\qquad\quad = 30.5$ m
Horizontal distance between $C$ and $D$
$= 0.34 \times 30.5$
$= 10.37$ m
Distance $BC = 2 \times (30.5 - 10.37)$
$\qquad\qquad\ = 40.26$
$\qquad\qquad\ = 40$ m (to the nearest metre)

**(b)** Height of $C$ above the water level
$= 0.94 \times 30.5 + 9$
$= 37.67$
$= 38$ m (to the nearest metre)

**Note:** This means that vessels less than 38 m high can pass beneath the bridge.

## G. Population Pyramid for the UK

A population pyramid shows the age and sex of a population. The diagram shows the population pyramid for the UK in 2015.

**Population pyramid for the UK in 2015**
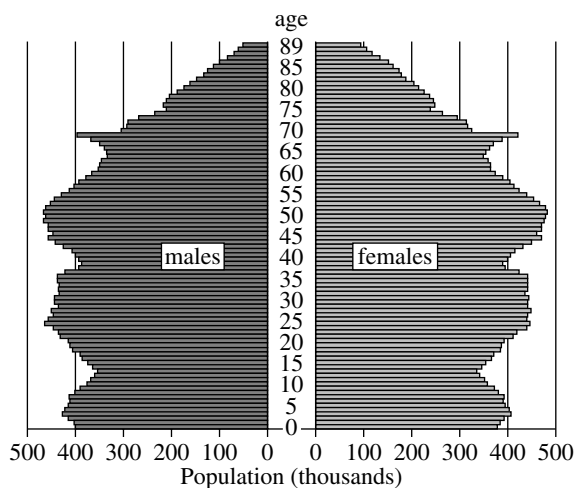


Source: Office for National Statistics

1. Assume all Year 7 students are 11 years old. Estimate the total number of Year 7 students from the population pyramid.
   
   **GCSE Grade 3**

2. Among the age groups 5 to 10, 25 to 30 and 45 to 50, which age group has the largest number of people? Explain your answer.

   **GCSE Grade 3⁺**

3. Compare the shape of the population by age for males and females. What can you say about the number of males and females under 20, aged 50 to 60 and over 70?

   **GCSE Grade 4⁻**

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*Solution*

1. Reading from the graph,
   number of boys at age 11 ≈ 370 000
   number of girls at age 11 ≈ 370 000
   Therefore, the total number of Year 7 students
   = the total number of boys and girls at age 11
   ≈ 370 000 + 370 000
   = 740 000
   **Note:** In fact, most students in Year 7 are age 11 or 12, and most students in Year 8 are age 12 or 13, and so on. You can estimate the total number of students in Year 7 by considering all the children who are 11 or all the children who are 12.

2. The bars for the age group 45 to 50 for both male and female are longer than the bars for the other two age groups.
   Therefore, the age group 45 to 50 has the largest number of people.

3. For people under 20 and aged 50 to 60, the lengths of the bars for males for females are nearly the same for each age. Therefore, the numbers of males and females at these ages are more or less the same. However, the numbers of women over 70 begin to outnumber men. In particular, this is quite obvious for ages over 80. This means, in general, the lifetime of women is longer than the lifetime of men.

# DISCOVERING MATHEMATICS 2B

**Victor Chow**

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OXFORD
UNIVERSITY PRESS

# PROBLEMS IN REAL-WORLD CONTEXTS

## A.    Rate and Discount in a Supermarket

The advertisement shows the price of fresh prawns in a promotion.

**1.** What is the promotional price of the prawns in pounds per kg?

**2.** Find the percentage discount in this promotion, giving your answer to three significant figures.

**3.** Sally bought 1.5 kg of the prawns. How much did she save from the usual price?



Special **Offer**

**Fresh Raw Jumbo King Prawns** **£1.39** per 100g

Usual price £1.79 per 100g

## B.    Top International Meeting Cities

The bar chart from a newspaper shows that Paris was the top international meeting city in 2016.

**1.** Estimate the number of international meetings held in Paris in 2016. Explain to your classmates how you arrived at your answer.

**2.** Estimate how many more international meetings Paris held compared to Seoul in 2016.

**3.** Would a pie chart be suitable to express this set of data? Explain your answer.



**Global hotspots**
Top international meeting cities in 2016

*Source: The Business Times, May 4, 2017*

# C.    Prepaid Mobile Phone Plans

The charges for prepaid mobile phone plans offered by two communications companies are as follows.

| Company A | National calls will be charged at 16 pence per minute during day time and 8 pence per minute during night time. |
| --- | --- |

| Company B | National calls will be charged at 22 pence for the first minute and 8 pence per minute thereafter for the whole day. |
| --- | --- |

Your friend from the United States is visiting the United Kingdom for seven days. He would like you to suggest a phone plan to purchase during his visit.

1.  What would you need to consider when helping your friend decide on which phone plan to purchase?

2.  Assume that he makes 10 phone calls in the day and 5 phone calls at night for each day, and each call lasts about 3 minutes. Calculate his daily call charges for each plan.

3.  Suppose you wish to propose another plan that charges the same rate per minute for each phone call throughout the day. What would be your plan? How would your plan be more economical? Justify your explanation.

# D.    Paper Clip

The diagram shows a paper clip which is made from a circular wire of diameter 1 mm. The dimensions shown are in centimetres. The clip consists of three semicircular arcs and four straight edges.
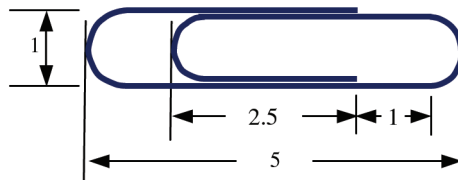


Suppose you are the manufacturer of the paper clip and you would like to know more about the amount of material required to make the clip.

1.  Calculate the total length of the wire needed to make a clip. State the assumption you have to make in your calculation.

2.  What is the volume (in cm³) of the material used for making each clip?

3.  The material of the clips has a density of 7.6 grams/cm³. The clips are packed in a box of 200 pieces. Find the total mass of the clips in a box.

4.  The wire of the clip is opened out and then bent into these shapes

      **(i)**   a square,              **(ii)**   a rectangle,              **(iii)**   a circle.

   **(a)**  Find a possible set of dimension(s) for each shape:

   **(b)**  Find the area of each shape.

   **(c)**  Given the same perimeter (in this case, the length of wire is fixed), which shape gives the largest area?

# E. Medals Won in 2016 Summer Olympics

The table shows the number of medals won by the top 10 countries in the 2016 Summer Olympics in Rio de Janeiro.

| Rank | Country | Gold | Silver | Bronze |
|------|---------|------|--------|--------|
| 1 | United States | 46 | 37 | 38 |
| 2 | Great Britain | 27 | 23 | 17 |
| 3 | China | 26 | 18 | 26 |
| 4 | Russia | 19 | 17 | 20 |
| 5 | Germany | 17 | 10 | 15 |
| 6 | Japan | 12 | 8 | 21 |
| 7 | France | 10 | 18 | 14 |
| 8 | South Korea | 9 | 3 | 9 |
| 9 | Italy | 8 | 12 | 8 |
| 10 | Australia | 8 | 11 | 10 |

1. Represent the data using a graph, showing the gold, silver and bronze medals won by the countries.

   **Hint:** You may use a spreadsheet program to help you draw the appropriate graph.

2. Besides ranking by gold medals, what other ways would you suggest ranking the countries? Will the ranking be different? Which way of ranking do you think is most reasonable? Explain your answer.

# F. Stretching a Metal Spring

A science experiment is being conducted by a class into the stretching of a steel spring by different loads. One end of the spring is suspended from a clamp and various masses are hung from the free end of the spring. A ruler fixed behind the spring is used to measure the change in length of the spring.

The values obtained from the experiment are recorded in the table.

| Mass (g) | 20 | 30 | 50 | 100 | 120 | 150 |
|----------|----|----|----|-----|-----|-----|
| Spring stretch (cm) | 3.2 | 4.8 | 8.0 | 16.0 | 19.2 | 24.0 |

1. Draw a graph of the data with the mass as the $x$-value and stretch as the $y$-value. Use a scale of 1 cm to 10 units on the $x$-axis and 1 cm to 2 units on the $y$-axis.

2. Connect the points with a line and find the gradient of the line. Interpret the meaning of the gradient in this case.

3. If the experiment was repeated with a stronger spring that stretches less, what would happen to the gradient of the graph it made?

# Problems in Real-world Contexts

## A. Rate and Discount in a Supermarket

The advertisement shows the price of fresh prawns in a promotion.

**1.** What is the promotional price of the prawns in pounds per kg?

**2.** Find the percentage discount in this promotion, giving your answer to three significant figures.

**3.** Sally bought 1.5 kg of the prawns. How much did she save from the usual price?

### Solution

**1.** Promotional price in pounds per kg
= £1.39 per 100 g
= £13.90 per 1000 g
= £13.90 per kg

**2.** Percentage discount
$$= \frac{1.79 - 1.39}{1.79} \times 100\%$$
$$= 22.3\% \quad \text{(to 3 sf)}$$

**3.** Amount of savings
$$= £1.5(1.79 - 1.39) \times 10$$
$$= £6.00$$



Special **Offer**

Fresh Raw Jumbo King **Prawns** **£1.39** per 100g

Usual price £1.79 per 100g

## B. Top International Meeting Cities

The bar chart from a newspaper shows that Paris was the top international meeting city in 2016.

**1.** Estimate the number of international meetings held in Paris in 2016. Explain to your classmates how you arrived at your answer.

**2.** Estimate how many more international meetings Paris held compared to Seoul in 2016.

**3.** Would a pie chart be suitable to express this set of data? Explain your answer.

### Solution

**1.** Since 1.3 cm represents 50 meetings and the length of the bar for Paris is 5.1 cm, the estimated number of meetings in Paris

$$= \frac{50}{1.3} \times 5.1$$
$$= 196 \quad \text{(to nearest whole number)}$$

**2.** The difference in the lengths of the bars is 1.5 cm. Thus the estimated number of meetings

$$= \frac{50}{1.3} \times 1.5$$
$$= 58 \quad \text{(to nearest whole number)}$$

Paris held 58 more meetings than Seoul.



**Global hotspots**
Top international meeting cities in 2016

Paris, Vienna, Barcelona, Berlin, London, Singapore, Madrid, Amsterdam, Lisbon, Seoul

■ Number of meetings

0  50  100  150  200  250
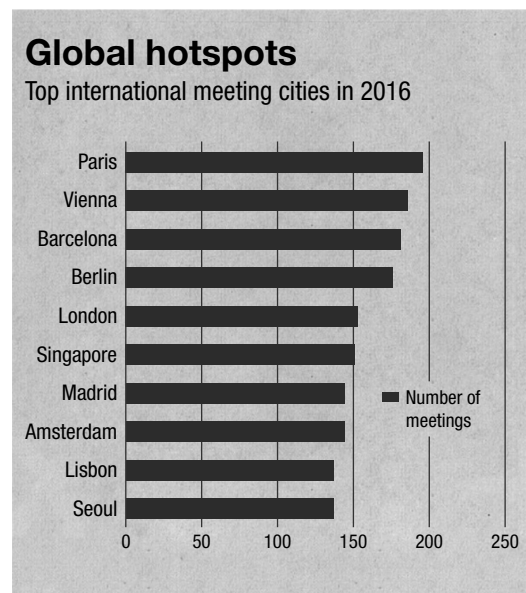
*Source: The Business Times, May 4, 2017*

**3.** A pie chart would not be suitable as there are too many cities.

## C. Prepaid Mobile Phone Plans

The charges for prepaid mobile phone plans offered by two communications companies are as follows.

| Company A | National calls will be charged at 16 pence per minute during day time and 8 pence per minute during night time. |
|---|---|

| Company B | National calls will be charged at 22 pence for the first minute and 8 pence per minute thereafter for the whole day. |
|---|---|

Your friend from the United States is visiting the United Kingdom for seven days. He would like you to suggest a phone plan to purchase during his visit.

**GCSE Grade 3**

**1.** What would you need to consider when helping your friend decide on which phone plan to purchase?

**GCSE Grade 3⁺**

**2.** Assume that he makes 10 phone calls in the day and 5 phone calls at night for each day, and each call lasts about 3 minutes. Calculate his daily call charges for each plan.

**GCSE Grade 4⁻**

**3.** Suppose you wish to propose another plan that charges the same rate per minute for each phone call throughout the day. What would be your plan? How would your plan be more economical? Justify your explanation.

### Solution

**1.** You need to consider the preference of the caller to make his calls during the day or night time and how long each phone call lasts.

**2.**

| Company A | Company B |
|---|---|
| Daily call charge = £0.16 × 10 × 3 + £0.08 × 5 × 3<br>= £4.80 + £1.20<br>= £6.00 | Daily call charge = £0.22 × 15 + £0.08 × 15 × 2<br>= £3.30 + £2.40<br>= £5.70 |

**3.** Answer depends on plan chosen.

## D. Paper Clip

The diagram shows a paper clip which is made from a circular wire of diameter 1 mm. The dimensions shown are in centimetres. The clip consists of three semicircular arcs and four straight edges.
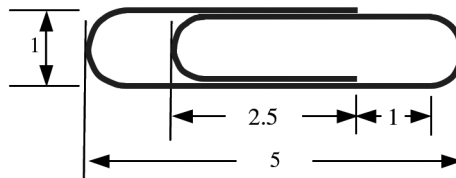


Suppose you are the manufacturer of the paper clip and you would like to know more about the amount of material required to make the clip.

**GCSE Grade 4⁺**

**1.** Calculate the total length of the wire needed to make a clip. State the assumption you have to make in your calculation.

**GCSE Grade 4**

**2.** What is the volume (in cm³) of the material used for making each clip?

**GCSE Grade 4⁺**

**3.** The material of the clips has a density of 7.6 grams/cm³. The clips are packed in a box of 200 pieces. Find the total mass of the clips in a box.

**4.** The wire of the clip is opened out and then bent into these shapes:
  **(i)**   a square,
  **(ii)**  a rectangle,
  **(iii)** a circle.
  **(a)** Find a possible set of dimension(s) for each shape.
  **(b)** Find the area of each shape.
  **(c)** Given the same perimeter (in this case, the length of wire is fixed), which shape gives the largest area?

### Solution

**1.** Assume that all 3 semicircles are of diameter 1 cm.
  Total length of wire $= 3 \times \pi \times 0.5 + 4 + 3 + 3 + 2$
  $$= 16.7123...$$
  $$= 16.7 \text{ cm} \quad \text{(to 3 sf)}$$

**2.** Volume $= \pi \times 0.05^2 \times 16.7123...$
  $$= 0.1312...$$
  $$= 0.131 \text{ cm}^3 \quad \text{(to 3 sf)}$$

**3.** Total mass $= 7.6 \times 0.1312... \times 200$
  $$= 199.512...$$
  $$= 200 \text{ g} \quad \text{(to 3 sf)}$$

**4.**

|  | **(i) Square** | **(ii) Rectangle** | **(iii) Circle** |
|---|---|---|---|
| **(a) Dimension(s)** | Length $= 16.7123... \div 4$ $= 4.1780...$ $= 4.18$ $\therefore$ 4.18 cm by 4.18 cm | Length + Width $= 16.7123... \div 2$ $= 8.3561...$ $\therefore$ 3.36 cm by 5 cm 4.36 cm by 4 cm 5.36 cm by 3 cm etc. | $2\pi r = 16.7123...$ $r = 2.66$ cm |
| **(b) Area** | $4.1780...^2 = 17.4564...$ $= 17.5 \text{ cm}^2$ | $3.36 \times 5 = 16.8 \text{ cm}^2$ $4.36 \times 4 = 17.4 \text{ cm}^2$ $5.36 \times 3 = 16.1 \text{ cm}^2$ | $\pi r^2 = \pi(2.6598...)^2$ $= 22.2 \text{ cm}^2$ |

  **(c)** From the table above, the circle gives the largest area.

## E. Medals Won in 2016 Summer Olympics

The table shows the number of medals won by the top 10 countries in the 2016 Summer Olympics in Rio de Janeiro.

| Rank | Country | Gold | Silver | Bronze |
|---|---|---|---|---|
| 1 | United States | 46 | 37 | 38 |
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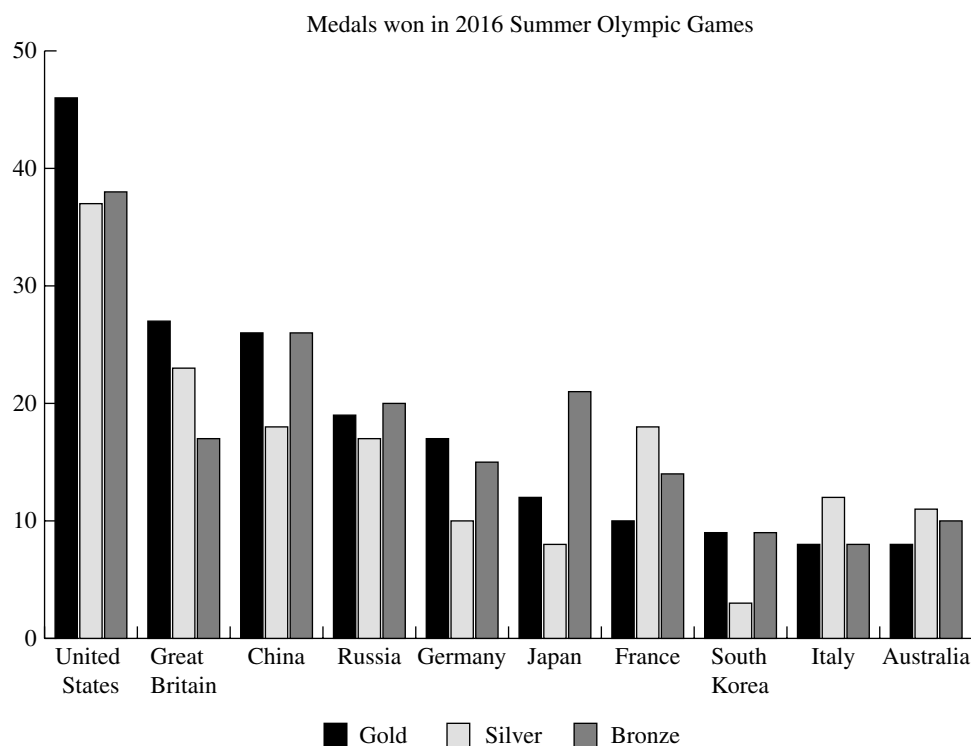**1.** Represent the data using a graph, showing the gold, silver and bronze medals won by the countries.
  **Hint:** You may use a spreadsheet program to help you draw the appropriate graph.

**2.** Besides ranking by gold medals, what other ways would you suggest ranking the countries? Will the ranking be different? Which way of ranking do you think is most reasonable? Explain your answer.

### Solution

**1.**



Medals won in 2016 Summer Olympic Games

**2.** Since countries may rank differently for each type of medal, ranking countries by the total number of medals is one reasonable alternative way to rank them. Another alternative could be to apply a score of 3 for gold, 2 for silver and 1 for bronze and the totals for countries ranked in that way.

## F. Stretching a Metal Spring

A science experiment is being conducted by a class into the stretching of a steel spring by different loads. One end of the spring is suspended from a clamp and various masses are hung from the free end of the spring. A ruler fixed behind the spring is used to measure the change in length of the spring.

The values obtained from the experiment are recorded in the table.



| Mass (g) | 20 | 30 | 50 | 100 | 120 | 150 |
|---|---|---|---|---|---|---|
| Spring stretch (cm) | 3.2 | 4.8 | 8.0 | 16.0 | 19.2 | 24.0 |

**GCSE Grade 4⁻**

**1.** Draw a graph of the data with the mass as the *x*-value and stretch as the *y*-value. Use a scale of 1 cm to 10 units on the *x*-axis and 1 cm to 2 units on the *y*-axis.

**GCSE Grade 4⁺**

**2.** Connect the points with a line and find the gradient of the line. Interpret the meaning of the gradient in this case.

**GCSE Grade 5⁻**

**3.** If the experiment was repeated with a stronger spring that stretches less, what would happen to the gradient of the graph it made?
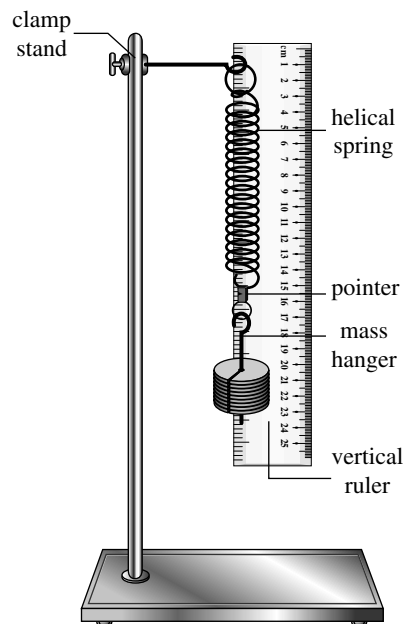
### Solution

**1.**

Graph of mass applied and amount that makes a spring stretch
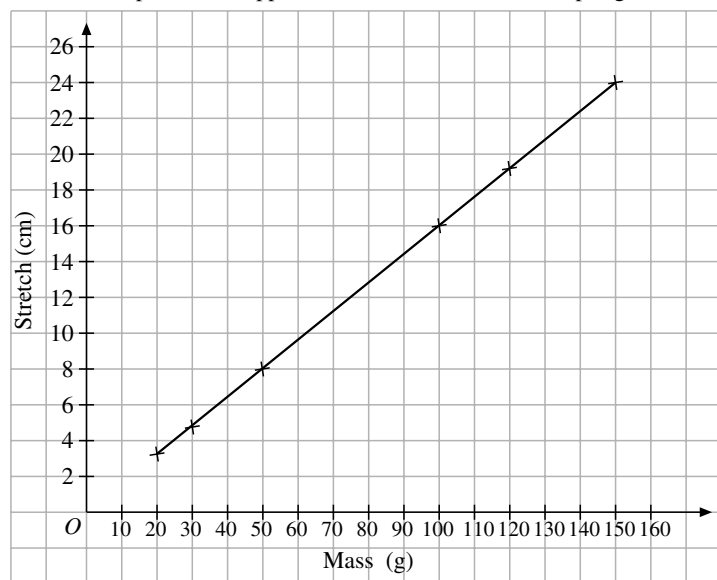


**2.** $\dfrac{8}{50} = 0.16$ (or other equivalent calculation)

**3.** The gradient would be less than 0.16 because the change in the $y$ value would be less for the same change in the $x$ value on the current spring.

# DISCOVERING MATHEMATICS

**3B**

**Victor Chow**

**UK Consultant: Robert Wilne**

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# PROBLEMS IN REAL-WORLD CONTEXTS

## A.    Utility Bill

Here is a simplified version of a household utility bill for a three-month period.

Billing period:  1 April 2019 to 30 June 2019

| Energy used | Rate | Charge |
|---|---|---|
| **Electricity**<br>    380 kWh<br>    Standing charge | 16p per kWh<br>25p per day | £60.80<br>£22.75 |
| **Gas**<br>    1600 kWh<br>    Standing charge | 3.75p per kWh<br>28p per day | £60.00<br>£25.48 |
| **Total electricity and gas used** | | |
| VAT | 5% | £8.45 |
| **Total charge including VAT** | | |

> **REMARK**
> The kilowatt-hour (kWh) is a unit of energy.

1.  Look at the row in the table for 'Electricity'. Show how the two charges for electricity are calculated.

2.  Work out the charge for the total electricity and gas used.

3.  Show how the charge of £8.45 is calculated.

4.  Work out the total charge including VAT.

5.  On 1 July, the electricity rate changed to 18p per kWh and the gas rate changed to 4p per kWh. If the amount of energy used and the standing charges were unchanged, find

    **(a)**  the total charge including VAT for the three-month period from 1 July 2019 to 30 September 2019,

    **(b)**  the percentage increase in the total charge including VAT for the second billing period compared with the first billing period, to three significant figures.

# B.    Population Age Pyramid

The population age pyramid is an example of a statistical diagram showing the distribution of the population across different age groups. This population age pyramid shows the distribution of the population of England by age group and gender in the year 2017.



1. State the modal age group for

    (a)   females,

    (b)   males.

2. (a)   Estimate the female population for the 15–19 age group.

    (b)   Estimate the male population for the 45–49 age group.

3. Estimate the ratio of males to females in the age group

    (a)   5–9,

    (b)   75–79.

4. Using the graph provided, can you conclude that females live longer than males? Explain your answer.

5. (a)   In which age group would a person fall if they were

        (i)    90 years old,

        (ii)   6 months old,

        (iii)  9 years and 11 months old?

    (b)   Describe the age group '5–9' using inequality signs.

    (c)   A journalist wants to include this population age pyramid in a newspaper article. Suggest why the journalist might not want to describe the age groups using inequality signs.

# C.  Wind Turbines

One form of renewable energy comes from wind farms. Wind turbines are used to generate electricity without emitting carbon dioxide. Each wind turbine consists of a tower and a rotor with three turbine blades. When the wind blows, the blades rotate and the rotation generates electricity.

The table shows the rotor diameter ($D$ metres) and the nominal power generated ($P$ megawatts) of some wind turbines. The nominal power is the peak power generated under standard test conditions.

| Rotor diameter, $D$ m | 114 | 132 | 170 |
|---|---|---|---|
| Nominal power, $P$ MW | 2.6 | 3.5 | 5.8 |

1.  If the rotor diameter is 114 m, what is the area swept by a blade in one revolution? Give your answer to three significant figures.

2.  It is known that $P$ is directly proportional either to $D$ or to $D^2$. Based on the data, which is true? Show your working.

3.  Another wind turbine has a rotor diameter of 193 m. Using your result from Question **2**, what is its nominal power? Give your answer to one decimal place.

# D.  Wooden Fences



Figure 1          Figure 2                              Figure 3

Figure 1 shows a wooden plank used to make fences. Figure 2 and Figure 3 show two fences constructed using these identical planks. Each plank has a horizontal side of $x$ cm and two vertical sides of $y$ cm. Each slanting edge of the plank is 5 cm long.

1.  Express, in terms of $x$ and $y$, the perimeter of the fence in

    **(a)**  Figure 2,          **(b)**  Figure 3.

2.  The perimeter of the fence in Figure 2 is 192 cm. The perimeter of the fence in Figure 3 is 256 cm. Find the values of $x$ and $y$.

3.  Find the angle between the two slanting edges of a plank. Give your answer to one decimal place.

4.  Find the area of the fence plank in Figure 1. Give your answer to three significant figures.

# E.   How are Paper Sizes Related?

Did you know that paper sizes are standardised and that the different paper sizes are related proportionally? The A4-sized paper that is commonly used is one of the paper sizes in the A series of paper sizes according to the International Standard (ISO216) for paper size.

The diagram shows some paper sizes (not drawn to scale) in the A series joined together to form a rectangle.



1.  The ratio of the length to the width of a piece of A4-sized paper is $\sqrt{2}:1$. Given that its width is 210 mm, what is its length? Give your answer to the nearest mm.

2.  As shown in the diagram, two pieces of paper of the same size can be joined to form the next larger paper size in the A series. For example, two pieces of A4-sized paper form a piece of A3-sized paper.

    (a)  Show that the ratio of the length to the width of a piece of A3-sized paper is also $\sqrt{2}:1$.

    (b)  Are all the paper sizes in the A series similar to one another? Explain your answer.

# F.    Cable Cars

The Heights of Abraham Cable Car in the Peak District consists of 12 cabins
and an aerial cableway. It starts from the base station in the Derwent Valley
and ends at the top station on the summit of Masson Hill.



Here are some technical specifications:

**(a)** Length of cable between the base station and the top station: 568 m

**(b)** Altitude of the top station: approximately 339 m above sea level

**(c)** Altitude of the base station: approximately 170 m above sea level

**(d)** Average speed of the cable car: between 1.5 m/s and 2.0 m/s

The system can be modelled using the diagram shown.
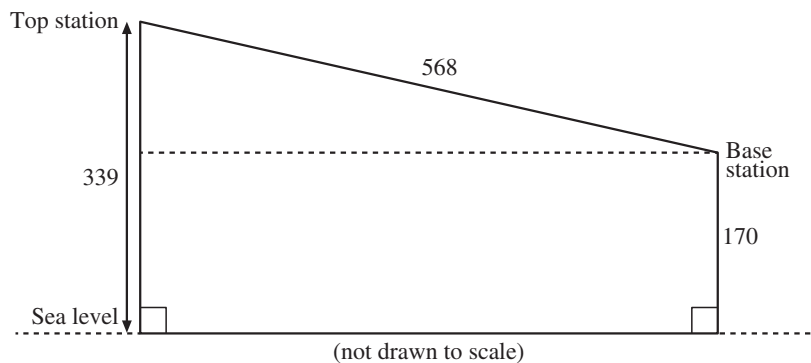


(not drawn to scale)

1. Estimate the horizontal distance between the base station and the top station,
   to three significant figures. Show your working and state any assumptions made.

2. Estimate the gradient of the cable between the base station and the top station,
   to three significant figures.

3. Estimate the angle that the cable makes with the horizontal. Give your answer
   to one decimal place.

4. **(a)** Estimate the time taken to travel from the base station to the top station, to
   the nearest minute. Show your working and state any assumptions made.

   **(b)** Give one possible factor that would affect the actual travelling time in part (**a**).

# G.   Premier League Statistics

The table shows some Premier League club statistics for 2018/19.

| Club | Number of goals | Number of wins | Number of losses |
|---|---|---|---|
| Manchester City | 95 | 32 | 4 |
| Liverpool | 89 | 30 | 1 |
| Arsenal | 73 | 21 | 10 |
| Tottenham Hotspur | 67 | 23 | 13 |
| Manchester United | 65 | 19 | 10 |
| Chelsea | 63 | 21 | 8 |
| AFC Bournemouth | 56 | 13 | 19 |
| Everton | 54 | 15 | 14 |
| Watford | 52 | 14 | 16 |
| West Ham United | 52 | 15 | 16 |

1. Find the mean, median, mode and range of the number of goals for these clubs.

2. Which measure of central tendency is the best representation for the average number of goals scored by these teams? Explain your answer.

3. One of the top scorers, Mohamed Salah, scored 22 goals for Liverpool.

   (a) If you watch a video of one of Liverpool's goals selected at random, what is the probability that the scorer is not Mohamed Salah?

   (b) What percentage of Liverpool's goals were scored by Mohamed Salah? Give your answer to three significant figures.

4. (a) Draw a scatter graph of the number of losses against the number of wins for these teams.

   (b) What can you say about the relationship between the number of wins and the number of losses?

# Problems in Real-world Contexts

## A. Utility Bill

Here is a simplified version of a household utility bill for a three-month period.

Billing period: 1 April 2019 to 30 June 2019

| Energy used | Rate | Charge |
|---|---|---|
| **Electricity** <br>     380 kWh <br>     Standing charge | 16p per kWh <br> 25p per day | £60.80 <br> £22.75 |
| **Gas** <br>     1600 kWh <br>     Standing charge | 3.75p per kWh <br> 28p per day | £60.00 <br> £25.48 |
| **Total electricity and gas used** | | |
| VAT | 5% | £8.45 |
| **Total charge including VAT** | | |

**GCSE Grade 2⁺**

**1.** Look at the row in the table for 'Electricity'. Show how the two charges for electricity are calculated.

**GCSE Grade 2**

**2.** Work out the charge for the total electricity and gas used.

**GCSE Grade 3⁻**

**3.** Show how the charge of £8.45 is calculated.

**GCSE Grade 2**

**4.** Work out the total charge including VAT.

**GCSE Grade 4⁻**

**5.** On 1 July, the electricity rate changed to 18p per kWh and the gas rate changed to 4p per kWh. If the amount of energy used and the standing charges were unchanged, find

   **(a)**   the total charge including VAT for the three-month period from 1 July 2019 to 30 September 2019,

   **(b)**   the percentage increase in the total charge including VAT for the second billing period compared with the first billing period, to three significant figures.

***Solution***

1. First electricity charge:
   $380 \times £0.16 = £60.80$

   Second electricity charge:
   Number of days from 1 April 2019 to 30 June 2019 = 91
   $91 \times £0.25 = £22.75$

2. Total charge for electricity and gas used
   = £60.80 + £22.75 + £60.00 + £25.48
   = £169.03

3. 100% $\rightarrow$ 169.03
   1% $\rightarrow$ 169.03 ÷ 100 = 1.6903
   5% $\rightarrow$ 1.6903 × 5 = 8.4515

   The 5% VAT is £8.45.

4. Total charge including VAT = £169.03 + £8.45
                                = £177.48

5. **(a)** Number of days from 1 July 2019 to 30 September 2019 = 92

   | Energy used | Rate | Charge |
   |---|---|---|
   | **Electricity** | | |
   |     380 kWh | 18p per kWh | $380 \times £0.18 = £68.40$ |
   |     Standing charge | 25p per day | $92 \times £0.25 = £23.00$ |
   | **Gas** | | |
   |     1600 kWh | 4p per kWh | $1600 \times £0.04 = £64.00$ |
   |     Standing charge | 28 per day | $92 \times £0.28 = £25.76$ |
   | **Total electricity and gas used** | | £181.16 |
   | VAT | 5% | £9.06 |
   | **Total charge including VAT** | | £190.22 |

   The total charge is £190.22.

   **(b)** Amount of increase = £190.22 – £177.48
                              = £12.74

   Percentage increase = $\dfrac{12.74}{177.48} \times 100\%$
                       = 7.18% (to 3 sf)

## B. Population Age Pyramid

The population age pyramid is an example of a statistical diagram showing the distribution of population across different age groups. This population age pyramid shows the distribution of the population of England by age group and gender in the year 2017.
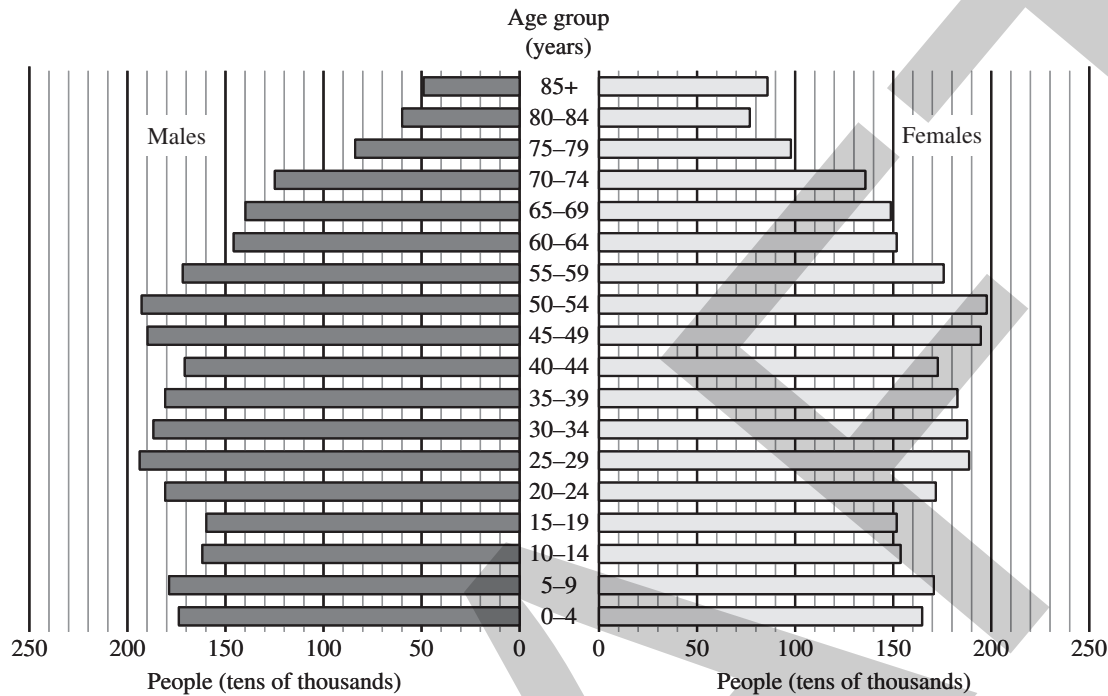


**GCSE Grade 4⁻**

**1.** State the modal age group for

(a) females,

(b) males.

**GCSE Grade 4⁻**

**2.** (a) Estimate the female population for the 15–19 age group.

(b) Estimate the male population for the 45–49 age group.

**GCSE Grade 4**

**3.** Estimate the ratio of males to females in the age group

(a) 5–9,

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**4.** Using the graph provided, can you conclude that females live longer than males? Explain your answer.

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**5.** (a) In which age group would a person fall if they were

(i) 90 years old,

(ii) 6 months old,

(iii) 9 years and 11 months old?

(b) Describe the age group '5–9' using inequality signs.

(c) A journalist wants to include this population age pyramid in a newspaper article. Suggest why the journalist might not want to describe the age groups using inequality signs.

### Solution

1. (a) The modal age group for females is 50–54.
   (b) The modal age group for males is 25–29.

2. (a) The female population for the 15–19 age group is approximately 1 520 000.
   (b) The male population for the 45–49 age group is approximately 1 900 000.

3. Ratio of males to females in the age group
   (a) Ratio of males to females in the age group 5–9
   $$\approx 1\,800\,000 : 1\,700\,000$$
   $$= 18 : 17$$
   (b) Ratio of males to females in the age group 75–79
   $$\approx 850\,000 : 1\,000\,000$$
   $$= 85 : 100$$
   $$= 17 : 20$$

4. Yes, you can conclude that females live longer than males. For each of the higher age groups 75–79, 80–84 and 85+, the population of females is higher than the population of males.

5. (a) (i) A person who is 90 years old would fall in the age group 85+.
   (ii) A person who is 6 months old would fall in the age group 0–4.
   (iii) A person who is 9 years and 11 months old would fall in the age group 5–9.
   (b) $5 \leq x < 10$
   (c) Describing the age groups using inequality signs may be considered more precise but may be less easily and quickly understood by the general public.

## C. Wind Turbines

One form of renewable energy comes from wind farms. Wind turbines are used to generate electricity without emitting carbon dioxide. Each wind turbine consists of a tower and a rotor with three turbine blades. When the wind blows, the blades rotate and the rotation generates electricity.

The table shows the rotor diameter ($D$ in metres) and the nominal power generated ($P$ megawatts) by some wind turbines.

| Rotor diameter, $D$ m | 114 | 132 | 170 |
|---|---|---|---|
| Nominal power, $P$ MW | 2.6 | 3.5 | 5.8 |

**GCSE Grade 4**
1. If the rotor diameter is 114 m, what is the area swept by a blade in one revolution? Give your answer to three significant figures.

**GCSE Grade 5+**
2. It is known that $P$ is directly proportional either to $D$ or to $D^2$. Based on the data, which is true? Show your working.

**GCSE Grade 6**
3. Another wind turbine has a rotor diameter of 193 m. Using your result from Question 2, what is its nominal power? Give your answer to one decimal place.

### Solution

**1.** Rotor diameter = 114 m

Rotor radius = 57 m

Area swept by blade in one revolution = $\pi r^2$

$$= \pi \times 57^2$$
$$= 10\,207.034\ldots$$
$$= 10\,200\,\text{m}^2 \quad \text{(to 3 sf)}$$

**2.**

| $D$ | 114 | 132 | 170 |
|---|---|---|---|
| $D^2$ | 12 996 | 17 424 | 28 900 |
| $P$ | 2.6 | 3.5 | 5.8 |
| $\dfrac{P}{D}$ | 0.0228... | 0.0265... | 0.0341... |
| $\dfrac{P}{D^2}$ | 0.000 200... | 0.000 200... | 0.000 200... |

$P$ is directly proportional to $D^2$.

**3.** $P = 0.0002D^2$

If $D = 193$, $P = 0.0002 \times 193^2$
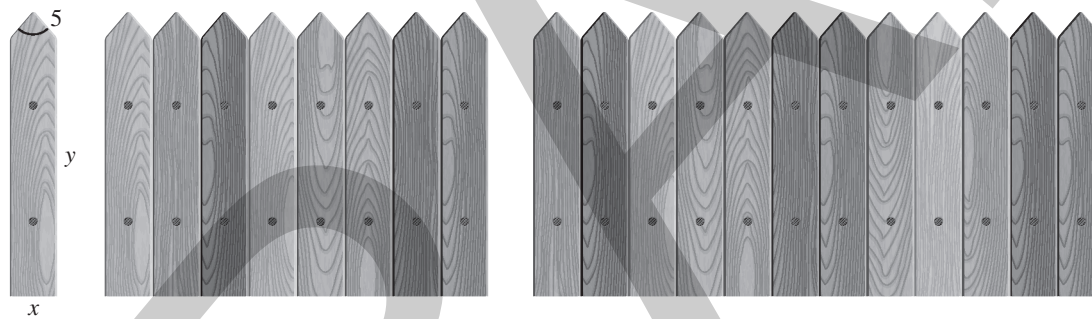$$= 7.4\,\text{MW (to 1 dp)}$$

## D. Wooden Fences



Figure 1     Figure 2        Figure 3

Figure 1 shows a wooden plank used to make fences. Figure 2 and Figure 3 show two fences constructed using these identical planks. Each plank has a horizontal side of $x$ cm and two vertical sides of $y$ cm. Each slanting edge of the plank is 5 cm long.

**GCSE Grade 3**
**1.** Express, in terms of $x$ and $y$, the perimeter of the fence in
 **(a)** Figure 2,
 **(b)** Figure 3.

**GCSE Grade 5⁺**
**2.** The perimeter of the fence in Figure 2 is 192 cm. The perimeter of the fence in Figure 3 is 256 cm. Find the values of $x$ and $y$.

**GCSE Grade 6**
**3.** Find the angle between the two slanting edges of a plank. Give your answer to one decimal place.

**GCSE Grade 5⁺**
**4.** Find the area of the fence plank in Figure 1. Give your answer to three significant figures.

***Solution***

1.  **(a)**  Length of base = $x \times 8$
    $\qquad\qquad\qquad\quad = 8x$ cm
    Length of two sides = $y + y$
    $\qquad\qquad\qquad\qquad = 2y$ cm
    Length of slanting edges = $5 \times 16$
    $\qquad\qquad\qquad\qquad\qquad = 80$ cm
    Perimeter of fence in Figure 2 = $(8x + 2y + 80)$ cm

    **(b)**  Length of base = $x \times 12$
    $\qquad\qquad\qquad\quad = 12x$ cm
    Length of two sides = $y + y$
    $\qquad\qquad\qquad\qquad = 2y$ cm
    Length of slanting edges = $5 \times 24$
    $\qquad\qquad\qquad\qquad\qquad = 120$ cm
    Perimeter of fence in Figure 2 = $(12x + 2y + 120)$ cm

2.  Figure 2:
    $8x + 2y + 80 = 192$
    $\qquad 8x + 2y = 112$
    $\qquad\quad 4x + y = 56 \qquad\qquad$ [1]

    Figure 3:
    $12x + 2y + 120 = 256$
    $\qquad\quad 12x + 2y = 136$
    $\qquad\qquad 6x + y = 68 \qquad\qquad$ [2]

    Solve simultaneous equations [1] and [2].
    [2] – [1]: $2x = 12$
    $\qquad\qquad\quad x = 6$
    Substitute $x = 6$ into [1].
    $4(6) + y = 56$
    $\quad 24 + y = 56$
    $\qquad\qquad y = 32$

3.
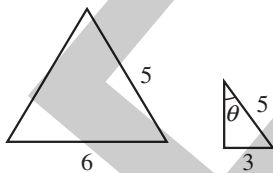
    

    $\sin \theta = \dfrac{3}{5}$
    $\qquad \theta = 36.869\ldots°$
    Angle between two slanting edges = $2 \times 36.869\ldots°$
    $\qquad\qquad\qquad\qquad\qquad\qquad = 73.7°$ (to 1 dp)

4.  Vertical height of triangular section = $\sqrt{5^2 - 3^2}$
    $\qquad\qquad\qquad\qquad\qquad\qquad\qquad = \sqrt{16}$
    $\qquad\qquad\qquad\qquad\qquad\qquad\qquad = 4$ cm

    Area of triangular section = $\dfrac{1}{2} \times 6 \times 4$
    $\qquad\qquad\qquad\qquad\qquad\qquad = 12$ cm$^2$

$$\text{Area of rectangular section} = 6 \times 32$$
$$= 192\,\text{cm}^2$$

$$\text{Total area of the fence plank} = 12 + 192$$
$$= 204\,\text{cm}^2$$

## E.   How are Paper Sizes Related?

Did you know that paper sizes are standardised and that the different paper sizes are related proportionally? The A4-sized paper that is commonly used is one of the paper sizes in the A series of paper sizes according to the International Standard (ISO216) for paper size.

The diagram shows some paper sizes (not drawn to scale) in the A series joined together to form a rectangle.
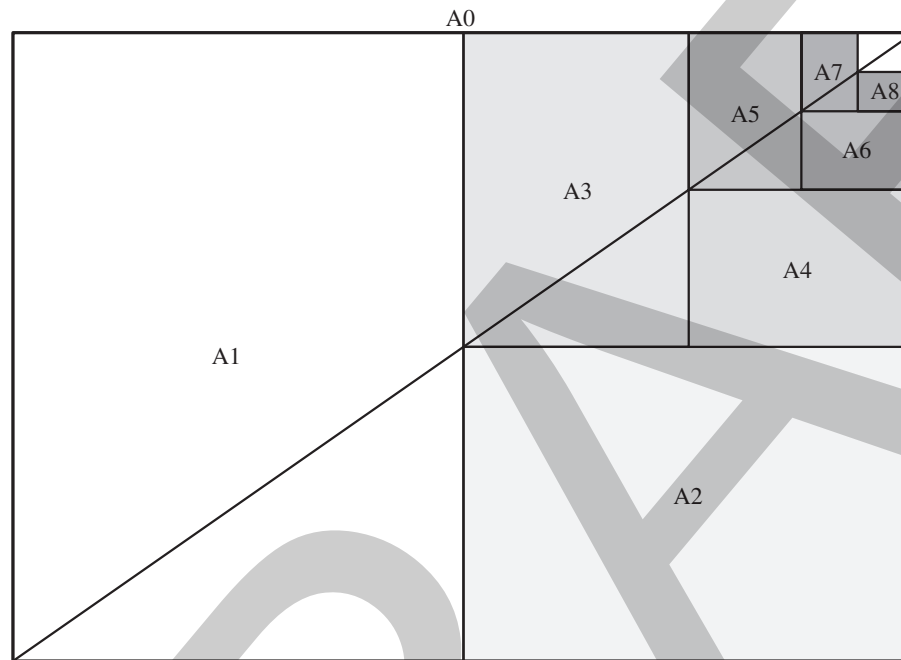


**GCSE Grade 5**

**1.**   The ratio of the length to the width of a piece of A4-sized paper is $\sqrt{2}:1$. Given that its width is 210 mm, what is its length? Give your answer to the nearest mm.

**GCSE Grade 6**

**2.**   As shown in the diagram, two pieces of paper of the same size can be joined to form the next larger paper size in the A series. For example, two pieces of A4-sized paper form a piece of A3-sized paper.

   **(a)**   Show that the ratio of the length to the width of a piece of A3-sized paper is also $\sqrt{2}:1$.

   **(b)**   Are all the paper sizes in the A series similar to one another? Explain your answer.

### Solution

**1.**   Length of an A4-sized paper $= 210 \times \sqrt{2}$
$$= 297\,\text{mm} \quad \text{(to the nearest mm)}$$

**2.**   **(a)**   Let $w$ cm be the width of an A4-sized paper.

Length of an A3-sized paper $= 2 \times$ width of an A4-sized paper
$$= 2w \text{ cm}$$

Width of an A3-sized paper $=$ length of an A4-sized paper
$$= \sqrt{2}w \text{ cm}$$

The ratio of the length to the width of an A3-sized paper $= 2w : \sqrt{2}w$
$$= \sqrt{2} : 1$$

**(b)** In general, for each paper size in the A-series,

$$\text{length} : \text{width} = \sqrt{2} : 1.$$

All the paper sizes in the A-series are similar to one another since all the corresponding angles are equal and all the corresponding sides are proportional.

## F.  Cable Cars

The Heights of Abraham Cable Car in the Peak District consists of 12 cabins and an aerial cableway. It starts from the base station in the Derwent Valley and ends at the top station on the summit of Masson Hill.



Here are some technical specifications:

**(a)**  Length of cable between the base station and the top station: 568 m
**(b)**  Altitude of the top station: approximately 339 m above sea level
**(c)**  Altitude of the base station: approximately 170 m above sea level
**(d)**  Average speed of the cable car: between 1.5 m/s and 2.0 m/s

The system can be modelled using the diagram below.



(not drawn to scale)
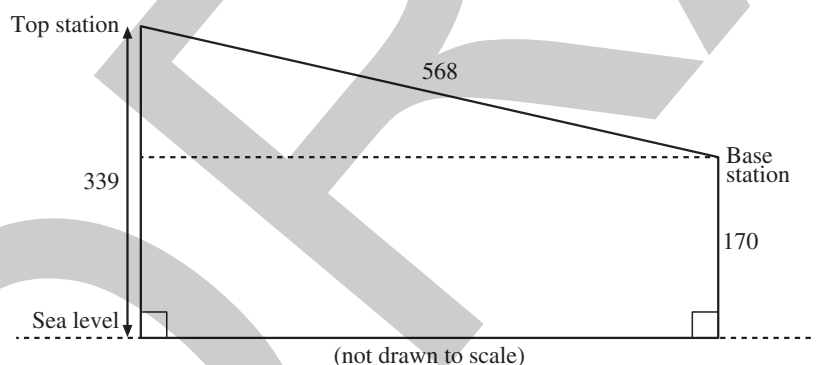
**GCSE Grade 5**

**1.**  Estimate the horizontal distance between the base station and the top station, to three significant figures. Show your working and state any assumptions made.

**GCSE Grade 4⁺**

**2.**  Estimate the gradient of the cable between the base station and the top station, to three significant figures.

**GCSE Grade 6⁻**

**3.**  Estimate the angle that the cable makes with the horizontal. Give your answer to one decimal place.

**4.** **(a)** Estimate the time taken to travel from the base station to the top station, to the nearest minute. Show your working and state any assumptions made.

   **(b)** Give one possible factor that would affect the actual travelling time in part (**a**).

### Solution

**1.** Difference in height between top station and base station = 339 – 170
$$= 169\,\text{m}$$

Assume that the cable between the top station and the base station is straight.

Horizontal distance between the two stations $= \sqrt{568^2 - 169^2}$
$$= \sqrt{294\,063}$$
$$= 542\,\text{m}\quad\text{(to 3 sf)}$$

**2.** Horizontal distance between top station and base station $= \sqrt{294\,063}$ m.
Vertical rise from base station to top station = 169 m.
Assume the cable is a straight line.

The gradient of the cable $= \dfrac{169}{\sqrt{294\,063}}$
$$= 0.312\qquad\text{(to 3 sf)}$$

**3.** Let the angle that the cable makes with the horizontal be $\theta$.
$$\sin\theta = \frac{169}{568}$$
$$\theta = 17.3096...$$
$$= 17.3°\qquad\text{(to 1 dp)}$$

**4.** **(a)** Total distance travelled from base station to top station = 568 m.
Assume that the cable car does not stop between the base station and the top station, and take the average speed to be $\dfrac{1.5+2}{2} = 1.75\,\text{m/s}$.

Travel time from base station to top station $= \dfrac{568}{1.75}$ seconds
$$= \frac{568}{1.75\times 60}\ \text{min}$$
$$= 5\,\text{min}\quad\text{(to the nearest minute)}$$

**Note:** You may also take the speed of the cable car to be 1.5 m/s or 2 m/s to perform the estimation.

   **(b)** Some possible factors that would affect the actual travelling time are:
   - time that the cable car stays at each station
   - the transition between towers and stations
   - weather conditions
   - the loading of the whole cable car system

## G.  Premier League Statistics

The table shows some Premier League club statistics for 2018/19.

| Club | Number of goals | Number of wins | Number of losses |
|---|---|---|---|
| Manchester City | 95 | 32 | 4 |
| Liverpool | 89 | 30 | 1 |
| Arsenal | 73 | 21 | 10 |
| Tottenham Hotspur | 67 | 23 | 13 |
| Manchester United | 65 | 19 | 10 |
| Chelsea | 63 | 21 | 8 |
| AFC Bournemouth | 56 | 13 | 19 |
| Everton | 54 | 15 | 14 |
| Watford | 52 | 14 | 16 |
| West Ham United | 52 | 15 | 16 |

**GCSE Grade 3**

**1.** Find the mean, median, mode and range of the number of goals for these clubs.

**GCSE Grade 4⁻**

**2.** Which measure of central tendency is the best representation for the average number of goals scored by these teams? Explain your answer.

**GCSE Grade 4⁻**

**3.** One of the top scorers, Mohamed Salah, scored 22 goals for Liverpool.
   **(a)** If you watch a video of one of Liverpool's goals selected at random, what is the probability that the scorer is not Mohamed Salah?
   **(b)** What percentage of Liverpool's goals were scored by Mohamed Salah? Give your answer to three significant figures.

**GCSE Grade 4**

**4.** **(a)** Draw a scatter graph of the number of losses against the number of wins for these teams.
   **(b)** What can you say about the relationship between the number of wins and the number of losses?

### Solution

**1.** Total number of goals
= 95 + 89 + 73 + 67 + 65 + 63 + 56 + 54 + 52 + 52
= 666

Mean = $\frac{666}{10}$ = 66.6

Arrange the numbers of goals in ascending order as follows:

52    52    54    56    63    65    67    73    89    95

↑

middle position

Median = $\frac{1}{2} \times (63 + 65)$
= 64

Mode = 52

Range = 95 – 52
= 43

**2.** The mean is the best representation for the average number of goals as the distribution is fairly symmetrical and the mean includes all of the data in the calculation.

**3.** **(a)** Number of Liverpool goals = 89
Number of Liverpool goals scored by Mohamed Salah = 22
Number of Liverpool goals **not** scored by Mohamed Salah = 67

P(scorer of Liverpool goal is not Mohamed Salah) = $\frac{67}{89}$

**(b)** Percentage of Liverpool goals scored by Mohamed Salah = $\frac{22}{89} \times 100\%$

$= 24.7\%$ (to 3 sf)

**4.** **(a)**

Number of losses against number of wins for
Premier League teams 2018/19
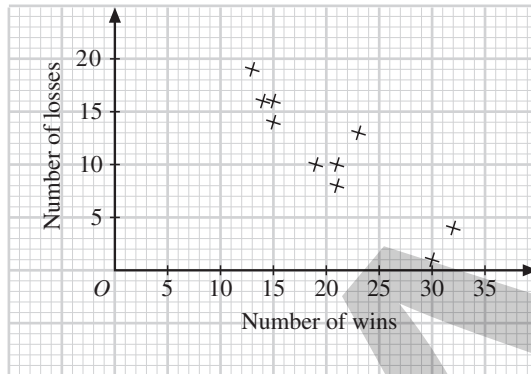


**(b)** There is a negative correlation between the number of wins and the number of losses.