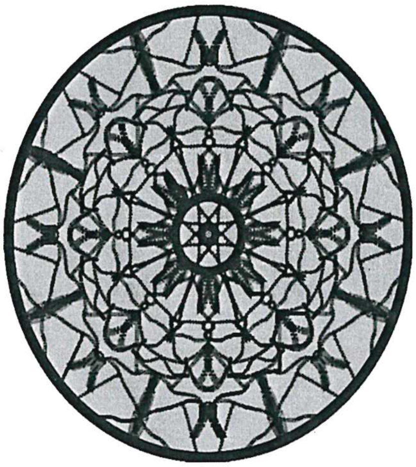
Mark Scheme.



GCSE Mathematics
2019 Final Predicted Paper 2h (Calculator)
1MA1
Higher Tier (1hr 50mins)

Remember: *These questions are just a guide. There are no guarantees that these questions/topics will come up! So, revise all you can before the calculator exams!*

Instructions

- Use **black** ink or ball-point pen.
- Answer **all** questions.
- Answer the questions in the spaces provided
  – there may be more space than you need

- You must show all your working
- **Calculators may be used**
- Diagrams are **NOT** accurately drawn, unless otherwise indicated

Information

- The total mark for this paper is **100**.
- The marks for **each** question are shown in brackets
  – use this as a guide as to how much time to spend on each question.

Advice
- Read each question carefully before you start to answer it.
- Keep an eye on time.
- Try to answer every question.
- Check your answers if you have time at the end.

1. The value of a van depreciates at the rate of 20% per year.
Gary buys a new van for £27 500
After *n* years the value of the van is £11 264

Find the value of *n*.

$$27500 \times 0.8^n = 11264$$

$$0.8^n = \frac{11264}{27500}$$

$$0.8^n = \frac{256}{625}$$

$$0.8^n = \frac{4^4}{5^4}$$

$$\left(\frac{4}{5}\right)^n = \left(\frac{4}{5}\right)^4$$

$$n = 4$$

4 ....(2)

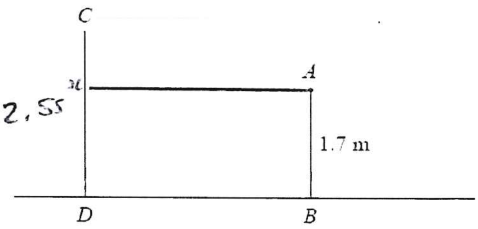
2. The diagram shows two vertical posts, *AB* and *CD*, on horizontal ground.

AB = 1.7 m

CD : *AB* = 1.5 : 1

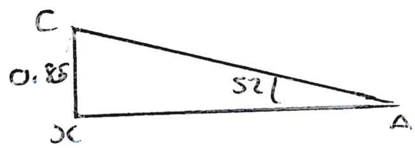
The angle of elevation of *C* from *A* is 52°

Calculate the length of *BD*.



Give your answer correct to 3 significant figures.

$CD = 1.7 \times 1.5$
$= 2.55$



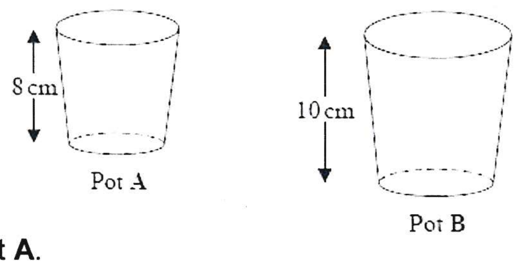
$$\tan 52 = \frac{0.85}{Ax}$$

$$Ax = \frac{0.85}{\tan 52} = 0.6640927825 = BD$$

0.664 ....(4)

3. Here are two pots.

Pot **A** and pot **B** are mathematically similar.



The area of the base of pot **B** is 160 cm².

Work out the area of the base of pot **A**.

$$160 \div \frac{25}{16} = 102.4\text{cm}^2$$

	A		B
L	8	$\xrightarrow{\times \frac{5}{4}}$	10
A		$\xrightarrow{\times \frac{25}{16}}$	160
V			

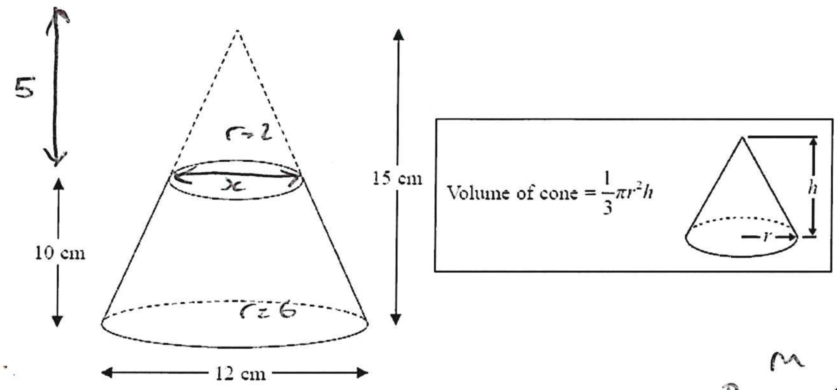
102.4cm^2 ....(2)

4. A frustum is made by removing a small cone from a large cone as shown in the diagram.

$\frac{x}{5} = \frac{12}{15}$

$15x = 60$

$x = 40$



M

P ✓

The frustum is made from glass. The glass has a density of 2.5 g / cm^3

Work out the mass of the frustum.

$\text{Vol } f = \text{Vol}(L) - \text{Vol}(s)$

$= \left(\frac{1}{3}\pi \times 6^2 \times 15\right) - \left(\frac{1}{3}\pi \times 2^2 \times 5\right)$

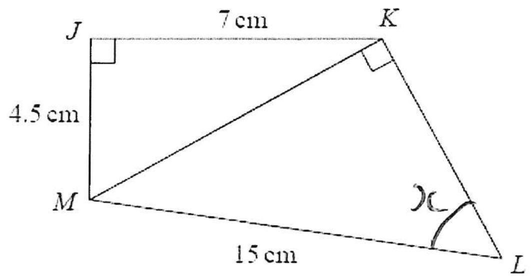
$= \frac{520\pi}{3}$

$\text{mass} = \frac{520\pi}{3} \times 2.5 = 1361.356817$

1361.4g ....(5)

5. The diagram shows a quadrilateral *JKLM*.

Work out the size of angle *KLM*. Give your answer correct to 3 significant figures.



$km = \sqrt{4.5^2 + 7^2}$

$= \sqrt{69.25}$

$\sin(x) = \frac{\sqrt{69.25}}{15}$

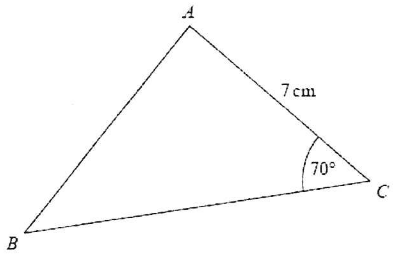
$x = \sin^{-1}\left(\frac{\sqrt{69.25}}{15}\right) = 33.69537249$

$= 33.7°$

……………………………(4)

6. The area of triangle *ABC* is 42 cm^2

Find the length of *AB*.



Give your answer correct to 3 significant figures.

$42 = \frac{1}{2} \times BC \times 7 \sin 70$

$\frac{84}{7\sin 70} = BC = 12.77013327$

$AB = \sqrt{BC^2 + 7^2 - 2 \times BC \times 7 \cos 70}$

$= 12.28532883$

12.3 ……………………(5)

7. On a school trip the ratio of the number of teachers to the number of students is 1 : 15

The ratio of the number of male students to the number of female students is 7 : 5

Work out what percentage of all the people on the trip are female students.
Give your answer correct to the nearest whole number.

$\frac{15}{16} \times \frac{5}{12} = \frac{25}{64}$

$\frac{25}{64} \times 100 = 39.0625\%$

$= 39\%$

39% ……………………(3)

8. (a) Show that the equation $x^3 + 5x - 4 = 0$ has a solution between $x = 0$ and $x = 1$

$0^3 + 5(0) - 4 = -4$

$1^3 + 5(1) - 4 = 2$

Answer moves from negative to positive ∴ there is a solution between the two inputs

..............................(2)

(b) Show that the equation $x^3 + 5x - 4 = 0$ can be arranged to give $x = \dfrac{4}{x^2 + 5}$

$x^3 + 5x = 4$

$x(x^2 + 5) = 4$

$x = \dfrac{4}{x^2 + 5}$

..................................(2)

(c) Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \dfrac{4}{x_n^2 + 5}$ twice, to find an estimate for the solution of
$x^3 + 5x - 4 = 0$

$x_1 = \dfrac{4}{0^2 + 5} = \dfrac{4}{5} = 0.8$

$x_2 = \dfrac{4}{(\frac{4}{5})^2 + 5} = \dfrac{100}{141} = 0.709219858 2$

...............................(3)

9. Asha and Lucy are selling pencils in a school shop.
They sell boxes of pencils and single pencils.

Asha sells 7 boxes of pencils and 22 single pencils.
Lucy sells 5 boxes of pencils and 2 single pencils.
Asha sells twice as many pencils as Lucy.

Work out how many pencils there are in a box.

$7x + 22 = 2(5x + 2)$

$7x + 22 = 10x + 4$

$22 = 3x + 4$

$18 = 3x$

$6 = x$

Check

$42 + 22 = 64$

$30 + 2 = 32$

$32 \times 2 = 64$ ✓

6 ....**(4)**

10. Here is the graph of $y = x^2 - 2x - 4$

(a) Write down estimates for the roots of $x^2 - 2x - 4 = 0$

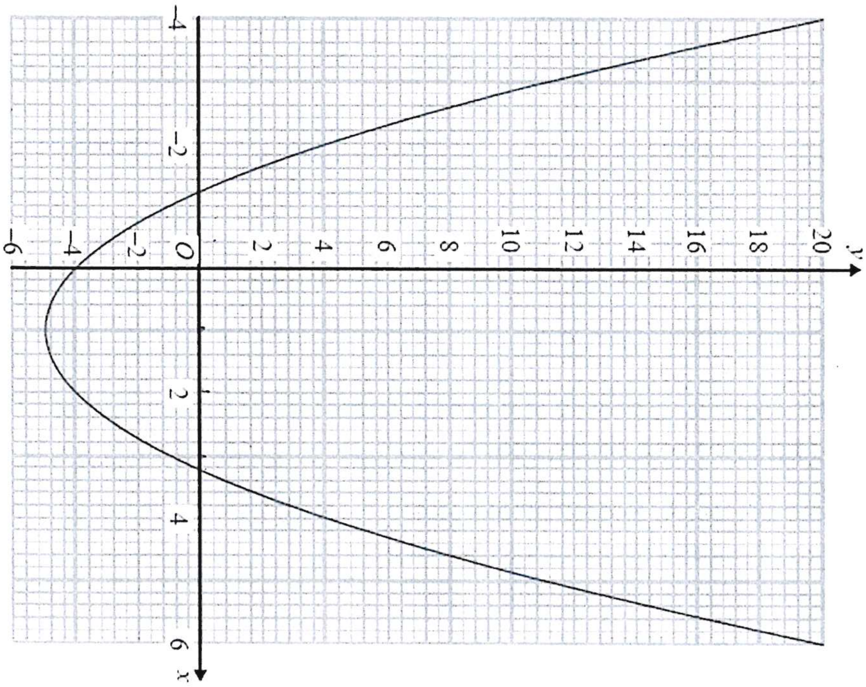
$x = 3.2 \quad x = -1.2$
..............................(2)

(b) Write down the coordinates of the turning point of $y = x^2 - 2x - 4$

$(x-1)^2 - 1^2 - 4$

$(x-1)^2 - 5$

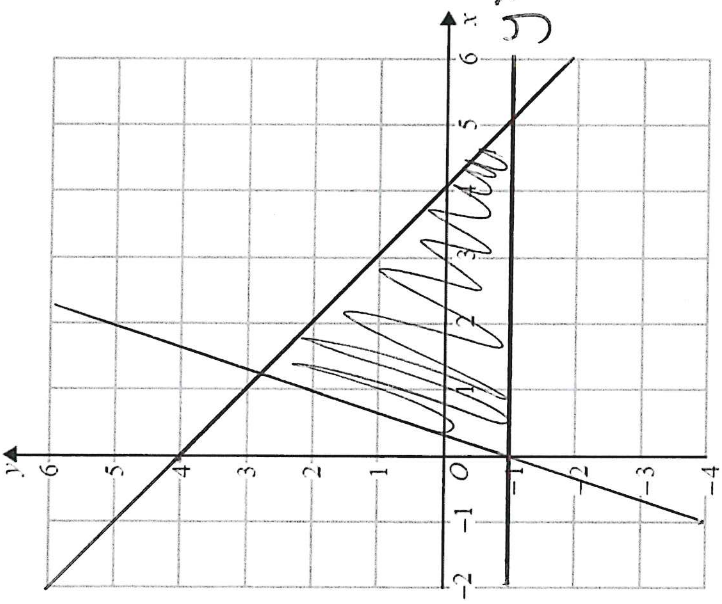
$(+1, -5)$ .... (1)



11. On the grid below show, by shading, the region defined by the inequalities

$y \geqslant -1 \qquad y \leqslant 4 - x \qquad y \leqslant 3x - 1$

Mark this region with the letter R.



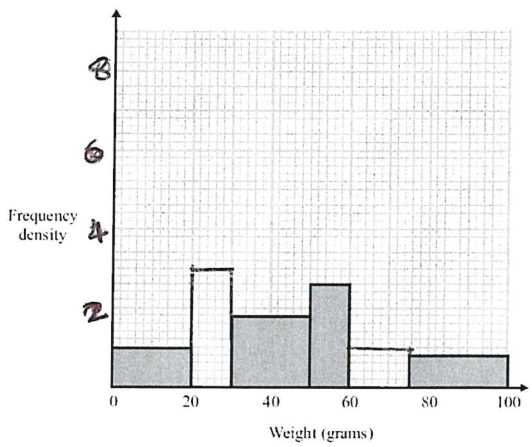
..............................(4)

12. The table shows some information about the weights of oranges.

Weight (*w* grams)	Frequency
$0 < w \leq 20$	20
$20 < w \leq 30$	15
$30 < w \leq 50$	36
$50 < w \leq 60$	13
$60 < w \leq 75$	15
$75 < w \leq 100$	10

(a) Use the histogram to complete the table. ....(2)

(b) Use the table to complete the histogram. ....(2)

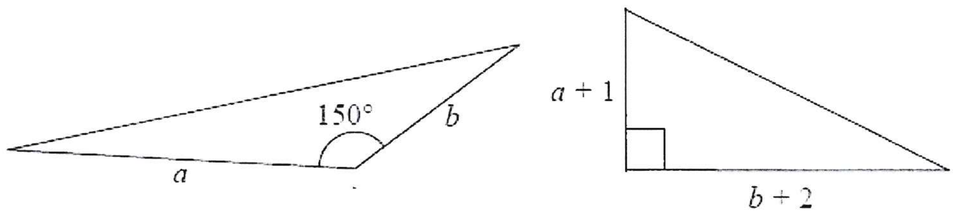


$20 \times 1 = 20$

$20 \times 1.8 = 36$

13. The diagram shows two triangles, **A** and **B**.

Diagram **NOT** accurately drawn



Triangle **A** Triangle **B**

The area of triangle **B** is 3 times the area of triangle **A**.

Given that $b > 4$, find an expression for a in terms of b.

$3\left(\frac{1}{2} \times a \times b \sin 150\right) = \frac{1}{2}(a+1)(b+2)$

$\frac{3}{4}ab = \frac{1}{2}(ab + 2a + b + 2)$

$\frac{6}{4}ab = ab + 2a + b + 2$

$\frac{1}{2}ab = 2a + b + 2$

$ab = 4a + 2b + 2$

$ab - 4a = 2b + 2$

$a(b-4) = 2b + 2$

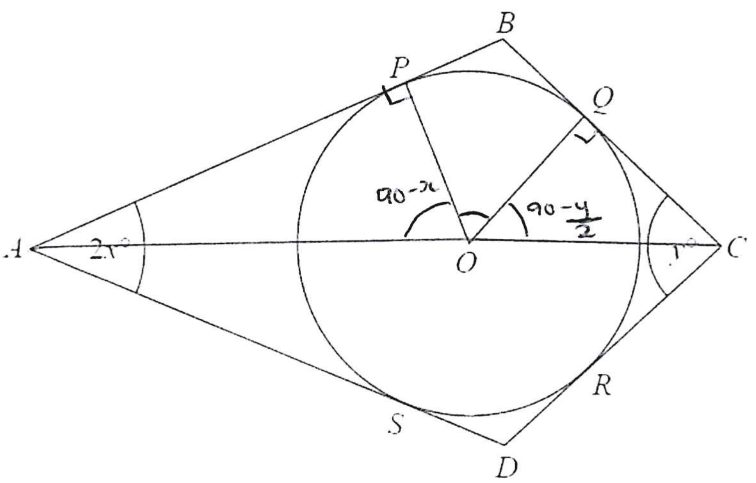
$a = \dfrac{2b+2}{b-4}$

$a = $ $\dfrac{2b+2}{b-4}$

(Total 5 marks)

14.

Diagram **NOT** accurately drawn



P, Q, R and S are points on the circumference of a circle, centre O.
APB, BQC, CRD and DSA are tangents to the circle.
$ABCD$ is a kite.

Angle $PAS = 2x°$
Angle $QCR = y°$

Find an expression in terms of x and y for the size, in degrees, of the angle POQ.
Give your expression in its simplest form.
Give reasons for your answer.

$POQ = 180 - (90 - x) - (90 - \frac{y}{2})$

$= 180 - 90 + x - 90 + \frac{y}{2}$

$= x + \frac{y}{2}$

(Total 5 marks)

15.

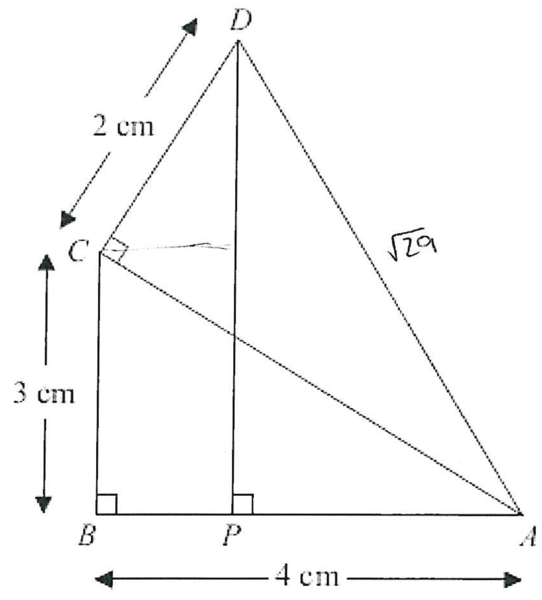


Diagram **NOT** accurately drawn

$AC = \sqrt{3^2 + 4^2} = 5$

$DA = \sqrt{2^2 + 5^2} = \sqrt{29}$

In the diagram,

ABC, *ACD* and *APD* are right-angled triangles.
$AB = 4$ cm.
$BC = 3$ cm.
$CD = 2$ cm.

Work out the length of *DP*.

....

(Total 5 marks)

*16. A is the point with coordinates (1, 3)
B is the point with coordinates (4, −1)
The straight line L goes through both A and B.

Is the line with equation $2y = 3x - 4$ perpendicular to line L?
You must show how you got your answer.

$$\frac{3 - -1}{1 - 4} = \frac{4}{5}$$

$$m_1 \times \frac{4}{5} = -1$$

$$m_1 = -\frac{5}{4}$$

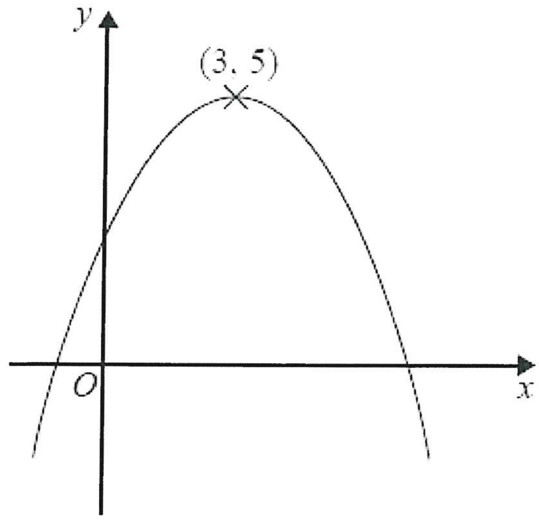
$y = -\frac{5}{4}x + c$ is perpendicular

$2y = 3x - 4$

$y = \frac{3}{2}x - 2$ is not as $-\frac{5}{4} \neq \frac{3}{2}$

(Total 4 marks)

17.



The diagram shows part of the curve with equation $y = \mathrm{f}(x)$.
The coordinates of the maximum point of the curve are (3, 5).

(*a*) Write down the coordinates of the maximum point of the curve with equation

(i) $y = \mathrm{f}(x + 3)$

(0 , 5)

(ii) $y = 2\mathrm{f}(x)$

(3 , 10)

(iii) $y = \mathrm{f}(3x)$

(1 , 5)

(3)

The curve with equation $y = \mathrm{f}(x)$ is transformed to give the curve with equation $y = \mathrm{f}(x) - 4$

(*b*) Describe the transformation.

The Curve shifts 4 units down

(1)

(Total 4 marks)

18. $f(x) = 3x - 2$

$g(x) = \dfrac{10}{x+2}$

(*a*) Express the inverse function f^{-1} in the form $f^{-1}(x) = ...$

$y = 3x - 2$

$y + 2 = 3x$

$\dfrac{y+2}{3} = x$

$f^{-1}(x) = \dfrac{x+2}{3}$

(2)

(*b*) Find gf(x)
Simplify your answer.

$gf(x) = \dfrac{10}{3x-2+2} = \dfrac{10}{3x}$

$gf(x) = \dfrac{10}{3x}$

(2)

(Total 4 marks)

***19.**

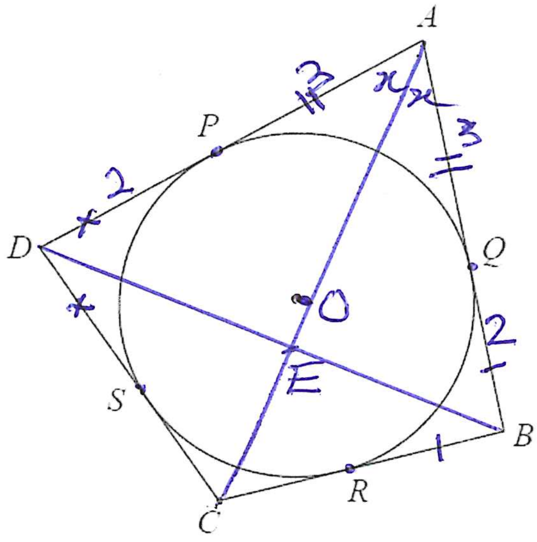


Diagram **NOT** accurately drawn

ABCD is a quadrilateral.
AB, *AD*, *BC* and *CD* are tangents to a circle.
The tangents touch the circle at *Q*, *P*, *R* and *S* respectively.
AC goes through the centre of the circle.

AP : *PD* is in the ratio 3 : 2
AQ : *QB* is in the ratio 3 : 2

Prove that *ABCD* is a kite.

Proof:
In $\triangle ADE$ and $\triangle ABE$
$AP = AQ$ (2 tgts from same point are equal)
$AP = \frac{3}{5}AD$
$AQ = \frac{3}{5}AB$
$\therefore AD = AB$ (since $AP = AQ$)
$D\hat{A}E = B\hat{A}E$ (AO bisects $D\hat{A}B$)
$AE = AE$ (same side of both $\triangle$s: reflexive)
$\therefore \triangle ADE$ is congruent to $\triangle ABE$ (SAS)
$\Rightarrow D\hat{E}A = B\hat{E}A$ (Congruency)
$= 90^\circ$ (since DEB is a str. line)
$\Rightarrow AC \perp BD$ (a property of kite)
Next, $QB = RB$ (2 tangents from same pt equal)
and $QB = PD$ ($AD = AB$); $PD = DS$ (2 tgts equal)
also $SC = RC$ (2 tgts theorem)
$\therefore DC = DS + SC = BR + RC = BC$
$\Rightarrow \triangle BCD$ is isosceles with $B\hat{D}C = D\hat{B}C$ (base $\angle$s of isosc. $\triangle$)
with $A\hat{D}E = A\hat{B}E$ (shown above)
$\Rightarrow A\hat{D}C = A\hat{B}C \Rightarrow ABCD$ is a kite since diagonals $AC \perp BD$, and a pair of opposite angles ($A\hat{D}C = A\hat{B}C$) are equal.

(Total is 5 marks)

***20.** Given that a and b are two consecutive even numbers, prove algebraically that $\left(\frac{a+b}{2}\right)^2$ is always 1 less than $\frac{a^2+b^2}{2}$.

$$\frac{a^2+2ab+b^2}{4}$$

let $a = 2n$

$b = 2n+2$

$$= \frac{4n^2 + 2\times 2n(2n+2) + (2n+2)(2n+2)}{4}$$

$$= \frac{4n^2 + 8n^2 + 8n + 4n^2 + 8n + 4}{4}$$

$$= \frac{16n^2 + 16n + 4}{4} = 4n^2 + 4n + 1$$

$$\frac{(2n)^2 + (2n+2)^2}{2} = \frac{4n^2 + 4n^2 + 8n + 4}{2}$$

$$= \frac{8n^2 + 8n + 4}{2}$$

$$= 4n^2 + 4n + 2$$

$$4n^2 + 4n + 2 - (4n^2 + 4n + 1) = 1$$

QED

(Total 5 marks)

21. Clive wants to estimate the number of bees in a beehive.
Clive catches 50 bees from the beehive.

He marks each bee with a dye.
He then lets the bees go.

The next day, Clive catches 40 bees from the beehive.
8 of these bees have been marked with the dye.

(i) Work out an estimate for the number of bees in the beehive.

$$\frac{50}{x} = \frac{8}{40}$$

$$2000 = 8x$$

$$250 = x$$

250 bees

(ii) Write down any assumptions you have made.

That the bees are caught at the same rate

(Total 4 marks)

***22.** Prove algebraically that the difference between the squares of any two consecutive integers is equal to the sum of these two integers.

$(2n+1)^2 - (2n)^2 = 4n^2 + 4n + 1 - 4n^2 = 4n + 1$

$2n+1 + 2n = 4n+1$ QED

(Total 4 marks)

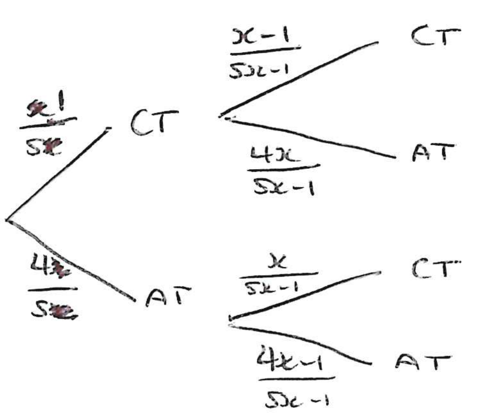
23. Hannah has an empty box.
She puts some crème tangerines and some nice apple tarts into the box.

The ratio of the number of crème tangerines to the number of nice apple tarts is 1 : 4

Hannah's sister Holly takes at random 2 items from the box.

The probability that she takes 2 crème tangerines is $\frac{6}{155}$.

How many crème tangerines did Hannah put into the box?



$\frac{1}{5} \times \frac{x-1}{5x-1} = \frac{6}{155}$

$\frac{x-1}{25x-5} = \frac{6}{155}$

$155x - 155 = 150x - 30$

$5x - 155 = -30$

$5x = 125$

$x = 25$

....

(Total 4 marks)

***24.** Zara is the manager of a shop.
The table gives information about the expenses the shop had last year.

Expense	**Wages**	**Rent**	**Goods**	**Other expenses**	Total
Amount	£92 000	£10 800	£72 000	£7000	181800 ~~[illegible]~~
New	98900	8400		3500	181800

This year

the wages will increase by 7.5%,
the rent will be 7/9 of the rent last year,
the other expenses will halve.

Zara wants to increase the amount of money she spends on goods.
She also wants the total expenses the shop has this year to be the same as last year.

Can Zara increase the amount of money she spends on goods?

$92000 \times 1.075 = 98900$

$10800 \times \frac{7}{9} = 8400$ +

$\frac{7000}{2} = 3500$ +

110800

$181800 - 110800 = 71000$ $\therefore$ No.

(Total 4 marks)

25. Solve the simultaneous equations:

$$x^2 + y^2 = 18$$
$$y = 5 - x$$

Give your answer correct to 3 significant figures.

$x^2 + (5-x)^2 = 18$

$x^2 + 25 - 10x + x^2 = 18$

$2x^2 - 10x + 7 = 0$

$x = \frac{10 \pm \sqrt{10^2 - (4 \times 2 \times 7)}}{4} = \frac{10 \pm \sqrt{44}}{4}$

$x = 4.16$ $\qquad$ $x = 0.842$

$y = 5 - 4.16 = 0.84$ $\qquad$ $y = 4.16$

(Total 5 marks)

26. **Tony designs a game.**
It costs £1.20 to play the game.
The probability of winning the game is $\frac{3}{10}$
The prize for each win is £2.50
150 people play the game.
Work out an estimate of the profit that Tony should expect to make.

$150 \times \frac{3}{10} = 45$ winners

$150 \times 1.2 = 180$

$45 \times 2.5 = 112.5$ −

£67.50

£ 67.50

(Total 4 marks)

27.

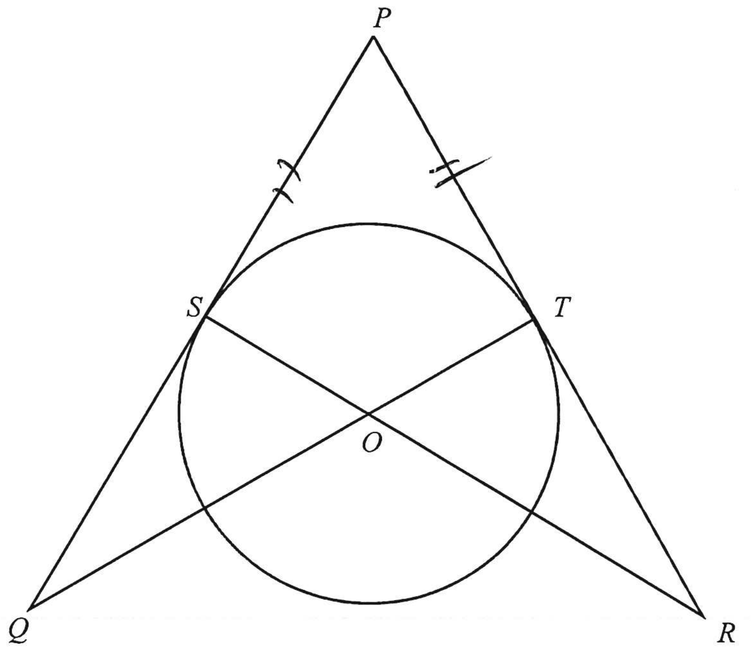


Diagram **NOT** accurately drawn

S and *T* are points on a circle, centre *O*.
PSQ and *PTR* are tangents to the circle.
SOR and *TOQ* are straight lines.

(a) Prove that triangle *PQT* and triangle *PRS* are congruent.

$P\hat{T}Q = P\hat{S}R = 90°$ (tangent $\perp$ radius)

$T\hat{P}Q = S\hat{P}R$ (same angle)

$\overline{PT} = \overline{PS}$ (2 tangents to a circle from a point outside the circle are equal)

(3)

$\therefore \triangle PQT$ is congruent to $\triangle PRS$ (ASA ie 2 angles and included side of both triangles are correspondingly equal).

Asif says that triangle *STQ* and triangle *STR* have equal areas.

(b) Explain why Asif is correct.

Two triangles congruent $\therefore$ Equal in all respect ie $PT = PS$ and $QT = RS$ in particular

$\therefore$ Area of $\triangle PQT = \frac{1}{2} QT \times PT = \frac{1}{2} RS \times PS$
$= \text{Area } \triangle PSR$

(2)

(Total 5 marks)

28. The diagram shows a cylinder inside a cone on a horizontal base.

The cone and the cylinder have the same vertical axis.
The base of the cylinder lies on the base of the cone.

The circumference of the top face of the cylinder touches the curved surface of the cone.

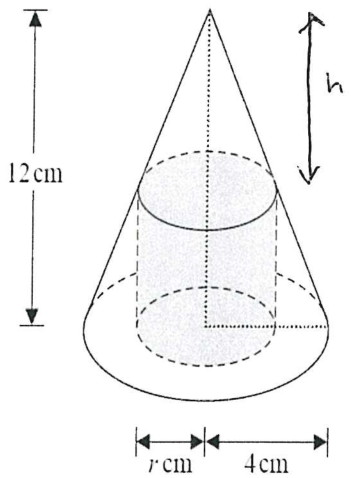
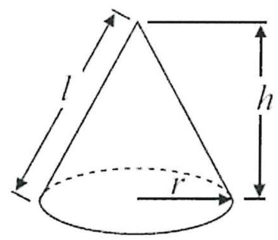


Diagram **NOT** accurately drawn

Volume of cone $\frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$



The height of the cone is 12 cm and the radius of the base of the cone is 4 cm.

(*a*) Work out the curved surface area of the cone.
Give your answer correct to 3 significant figures.

$l = \sqrt{12^2 + 4^2}$

$= \sqrt{144 + 16}$

$= \sqrt{160}$

$= 4\sqrt{10}$

$\pi \times 4 \times 4\sqrt{10} = 16\sqrt{10}\,\pi$

$= 158.9534123$

$= 159$

..........159.......... cm^2

(3)

The cylinder has radius r cm and volume V cm^3

(*b*) Show that $V = 12\pi r^2 - 3\pi r^3$

$\frac{4}{12} = \frac{r}{h}$

$\frac{4h}{12} = r$

$4h = 12r$

$h = 3r$

$V = \pi \times r^2 \times \text{height}$

$= \pi \times r^2 \times (12 - 3r)$

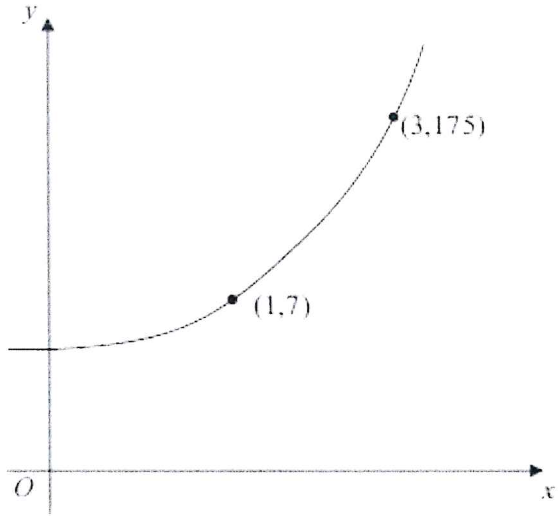
$= \cancel{4\pi \times r^2}$

$= 12\pi r^2 - 3\pi r^3$

(3)

(Total 6 marks)

29.



The sketch shows a curve with equation

$y = ka^x$

where k and a are constants, and $a > 0$

The curve passes through the points (1, 7) and (3, 175).

Calculate the value of k and the value of a.

$175 = ka^3$

$7 = ka$ $\div$

$25 = a^2$

$5 = a$

$7 = 5k$

$1.4 = k$

$k =$ 1.4

$a =$ 5

(Total 3 marks)

30.

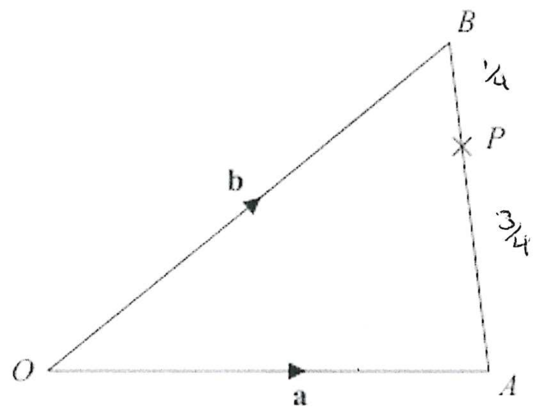


Diagram **NOT** accurately drawn

OAB is a triangle.

$\overrightarrow{OA} = \mathbf{a}$
$\overrightarrow{OB} = \mathbf{b}$

(a) Find $\overrightarrow{AB}$ in terms of **a** and **b**.

$\overrightarrow{AB} = -a+b$

$-a+b$

(1)

P is the point on *AB* such that *AP* : *PB* = 3 : 1

(b) Find $\overrightarrow{OP}$ in terms of **a** and **b**.
Give your answer in its simplest form.

$OP = \overrightarrow{OA} + \overrightarrow{AP}$

$= a + \frac{3}{4} AB$

$= a + \frac{3}{4}(-a+b)$

$= a - \frac{3}{4}a + \frac{3}{4}b$

$= \frac{1}{4}a + \frac{3}{4}b$

$= \frac{1}{4}(a+3b)$

$\frac{1}{4}(a+3b)$

(3)

(Total 4 marks)

31. Here are the first 4 terms of a quadratic sequence.

7 18 33 52

Find an expression, in terms of n, for the nth term of the sequence. $2n^2 + 5n$

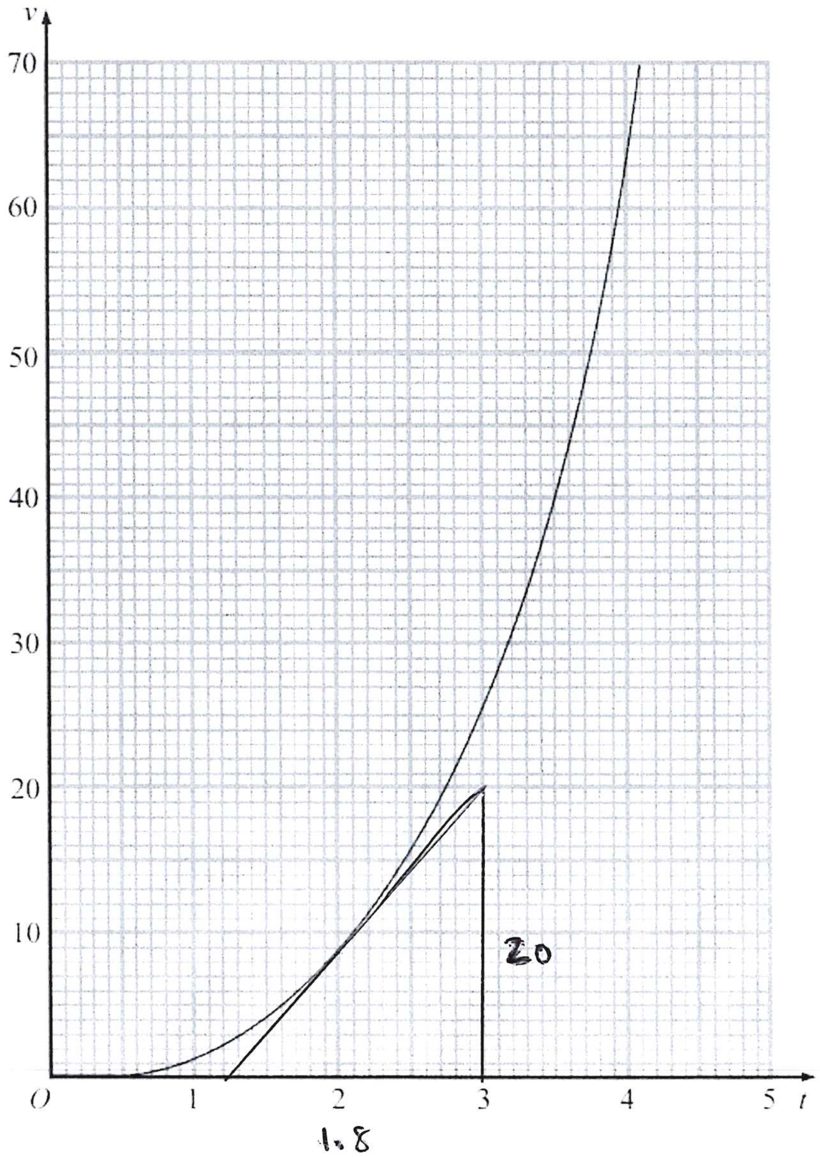
$9a + 3b + c = 33$
$4a + 2b + c = 18$
$5a + b = 15$
$3a + b = 11$
$2a = 4$
$a = 2$

$4a + 2b + c = 18$
$a + b + c = 7$
$3a + b = 11$
$3(2) + b = 11$
$6 + b = 11$
$b = 5$

$2 + 5 + c = 7$
$7 + c = 7$
$c = 0$

(Total 3 marks)

32. The graph shows the velocity, v metres per second, of a rocket at time t seconds.



Find an estimate for the rate of change of the velocity of the rocket at $t = 2$

$\frac{20}{1.8} = 11.\dot{1}\ ms^{-2}$

11.1 .... m/s^2

(Total 3 marks)

33. The line **L** is a tangent to the circle $x^2 + y^2 = 45$ at the point $(-3, 6)$.

The line **L** crosses the x-axis at the point P.

Work out the coordinates of P.

gradient of normal = $\frac{6-0}{-3-0} = -2$

gradient of perpendicular = $\frac{1}{2}$

$y = \frac{1}{2}x + c$

$6 = -\frac{3}{2} + c$

$\frac{12}{2} + \frac{3}{2} = c$

$\frac{15}{2} = c$

$y = \frac{1}{2}x + \frac{15}{2}$

$0 = \frac{1}{2}x + \frac{15}{2}$

$0 = \frac{1}{2}(x+15)$

$0 = x + 15$

$x = -15$

$(-15, 0)$ ....

(Total 4 marks)