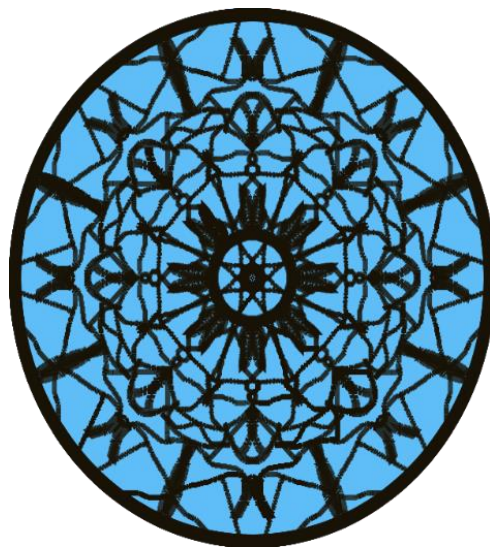# Maths Set 1 Christmas Pack

Name: …………………………….

Teacher: ……………

1. Cosine Rule
2. Sine Rule
3. Iteration
4. Transformation (Enlargement)
5. Transforming trig graphs
6. Quadratic/Cubic graphs
7. Area of non- right-angle triangle
8. Bounds/Limits
9. Histogram
10. Proof (Geometry)
11. Compound measures (Pressure, Speed distance and time, Density, volume and mass)
12. Equation of a circle
13. Loci/construction/Bearings
14. LCM/HCF
15. Volume of a cone/sphere
16. Frequency Polygons
17. Laws of Indices
18. Simultaneous Equations
19. Use of Calculator/reciprocal
20. Compound
21. Simultaneous Equations (non-linear)
22. Congruent Triangles
23. Conditional Probability
24. Circle Theorems Proofs
25. Recurring Decimals to Fractions
26. Quadratic inequalities/graphs
27. Quadratic sequence nth term
28. Quadratic Formula
29. Venn Diagrams
30. Capture Recapture
31. Estimated Mean
32. Reverse Percentages
33. Similar shapes
34. Scatter graphs
35. Comparing Box Plot
36. Shaded Area
37. Similar Shapes
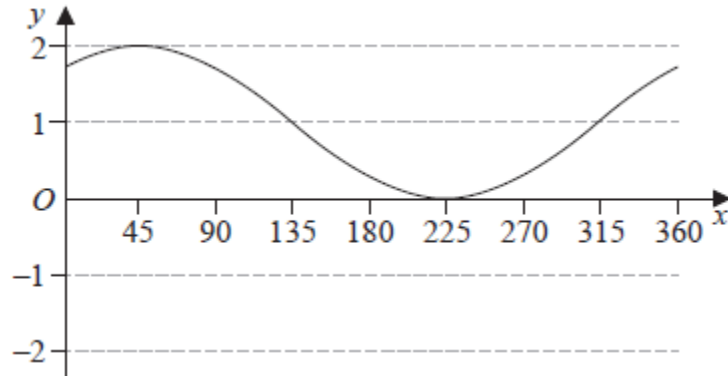38. Polygon/Angles
39. surds

1 Here is a sketch of the graph of a trigonometric function for $0 \leq x \leq 360$
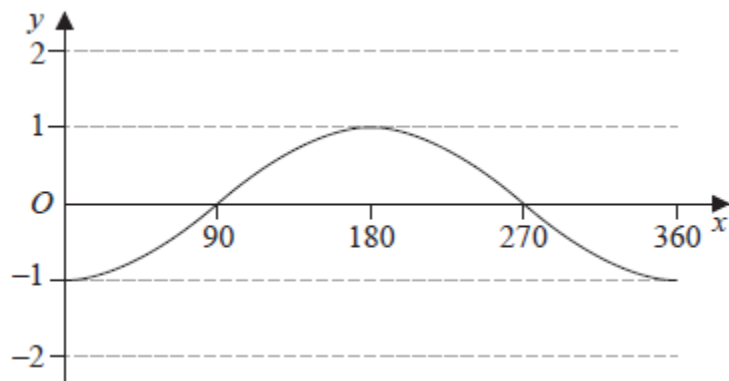
(a) Write down a possible equation of the graph.

....

(1)

Here is a sketch of the graph of $y = \cos (x - p)° + r$ for $0 \leq x \leq 360$

(b) Find the value of p and the value of r.

$p = $ ....

$r = $ ....

(2)

(Total is 3 marks)

2   Solve algebraically the simultaneous equations

$$y = 2x^2 - 3x - 10$$

$$2x - y = -2$$

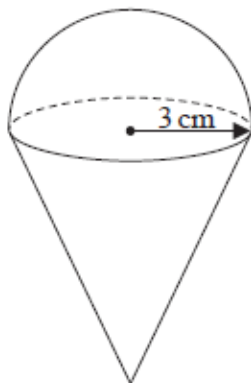...
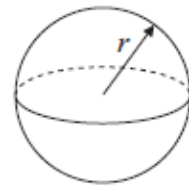
(Total 5 marks)

3   The diagram shows a solid made by joining a solid hemisphere to a solid circular cone.

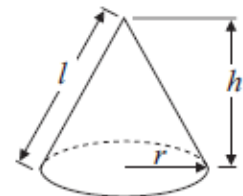The centre of the plane face of the cone coincides with the centre of the plane face of the hemisphere.

Surface area of sphere = $4\pi r^2$

Volume of sphere = $\frac{4}{3}\pi r^3$

Curved surface area of cone = $\pi r l$

Volume of cone = $\frac{1}{3}\pi r^2 h$

The radius of the hemisphere is 3 cm.
The radius of the base of the cone is 3 cm.

The volume of the solid is 30 π cm^3

Work out the total surface area of the solid.
Give your answer as a multiple of π.

....cm^2

(Total 5 marks)

4 The table gives information about the times that 100 people took to travel to work.

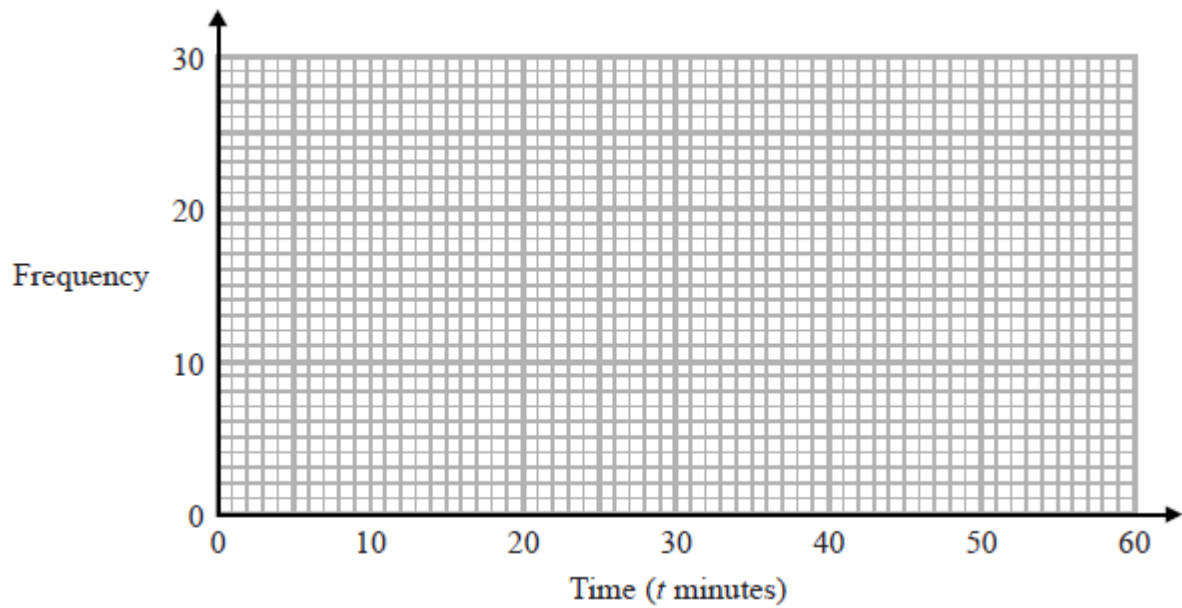| Time (t minutes)	Frequency
$0 < t \leq 10$	10
$10 < t \leq 20$	26
$20 < t \leq 30$	23
$30 < t \leq 40$	19
$40 < t \leq 50$	14
$50 < t \leq 60$	8

(a) Find the class interval that contains the median.

....

(1)

(b) Draw a frequency polygon for the information in the table.

(2)

(Total 3 marks)

5 (a) Complete the table of values for $y = x^3 - 3x + 1$

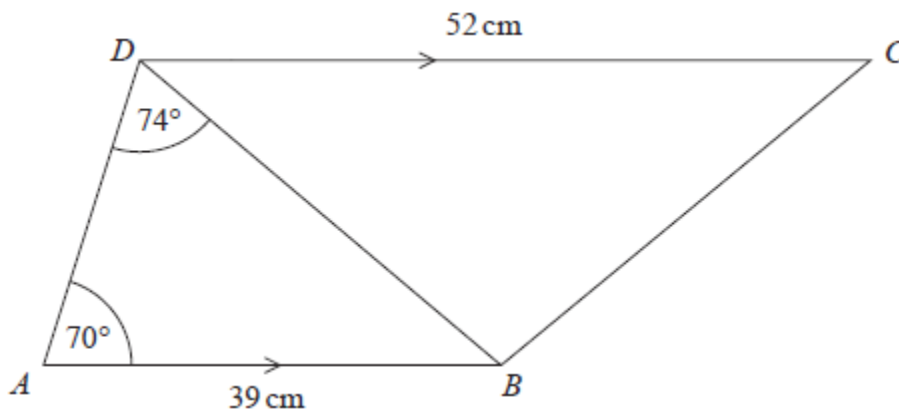| x	-2	-1	0	1	2
y					

(2)

(b) On the grid below, draw the graph of $y = x^3 - 3x + 1$ for values of x from -2 to 2
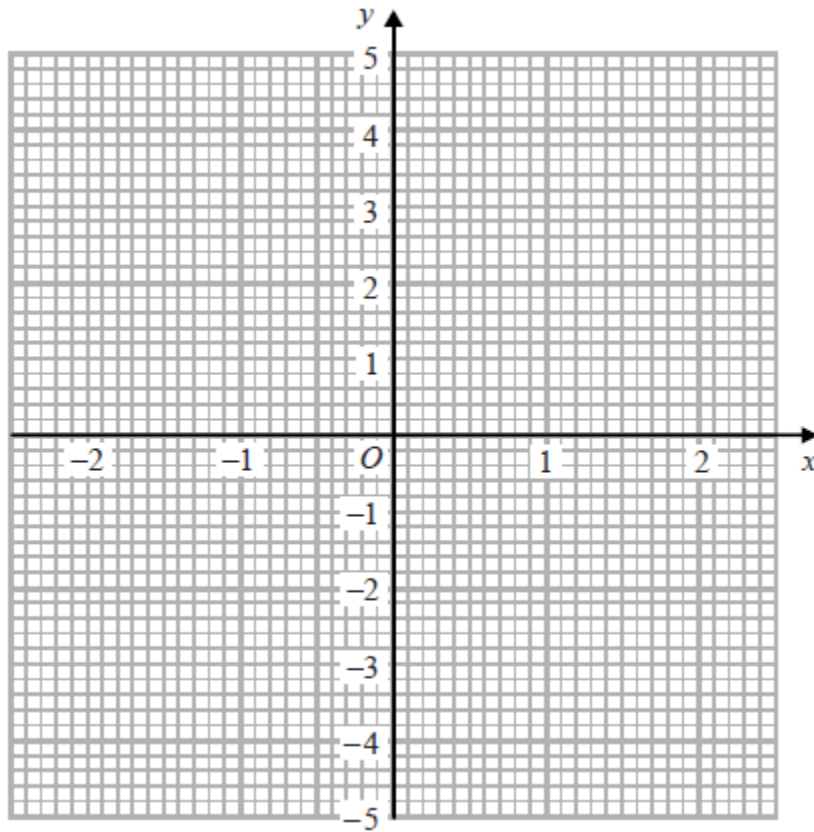
(2)

(Total 4 marks)

6   Here is trapezium *ABCD*.

AB and *DC* are parallel.

Work out the area of triangle *BCD*.
Give your answer correct to 3 significant figures.

....cm^2

(Total 5 marks)

7

C

105°

7 cm   10 cm

A   B
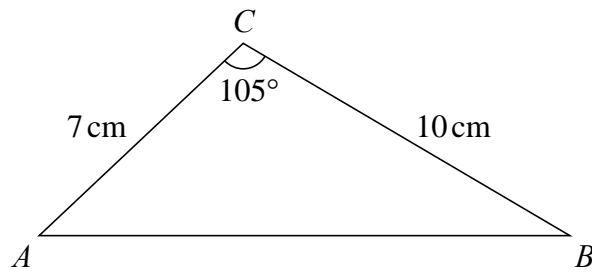
Diagram **NOT**
accurately drawn

(*a*) Work out the area of triangle *ABC*.
Give your answer correct to 3 significant figures.

.... cm^2

(**2**)

(*b*) Work out the size of angle *BAC*.
Give your answer correct to 1 decimal place.

8 Here is a quadrilateral *PQRS*.

Diagram **NOT** accurately drawn

12 cm

P

Q

120°

9 cm

8 cm

27°

S

R
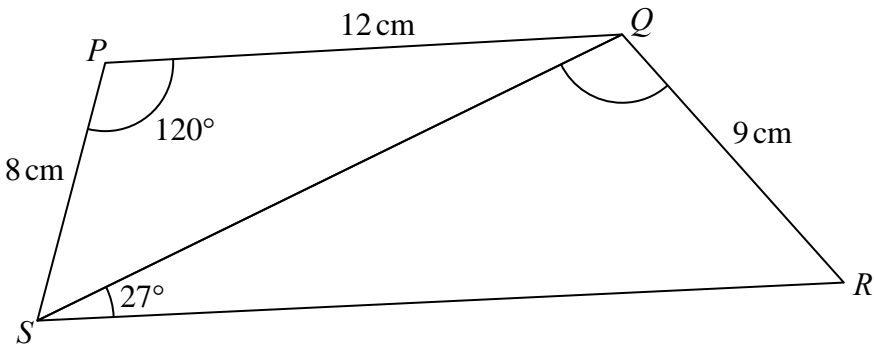
Angle *SRQ* is acute.

Work out the size of angle *SQR*.
Give your answer correct to 1 decimal place.

.... °

(**Total 6 marks**)

9 Marta breeds dogs.
32 dogs give birth to puppies.
The table shows information about the number of puppies born to each dog.

Number of puppies	Frequency
1 − 3	5
4 − 6	12
7 − 9	10
10 − 12	4
13 − 15	1

(*a*) Write down the modal class.

....

(**1**)

(*b*) Work out an estimate for the mean number of puppies born to each dog.

....

(**4**)

(**Total 5 marks**)

10 The histogram shows some information about the salaries of a sample of people.
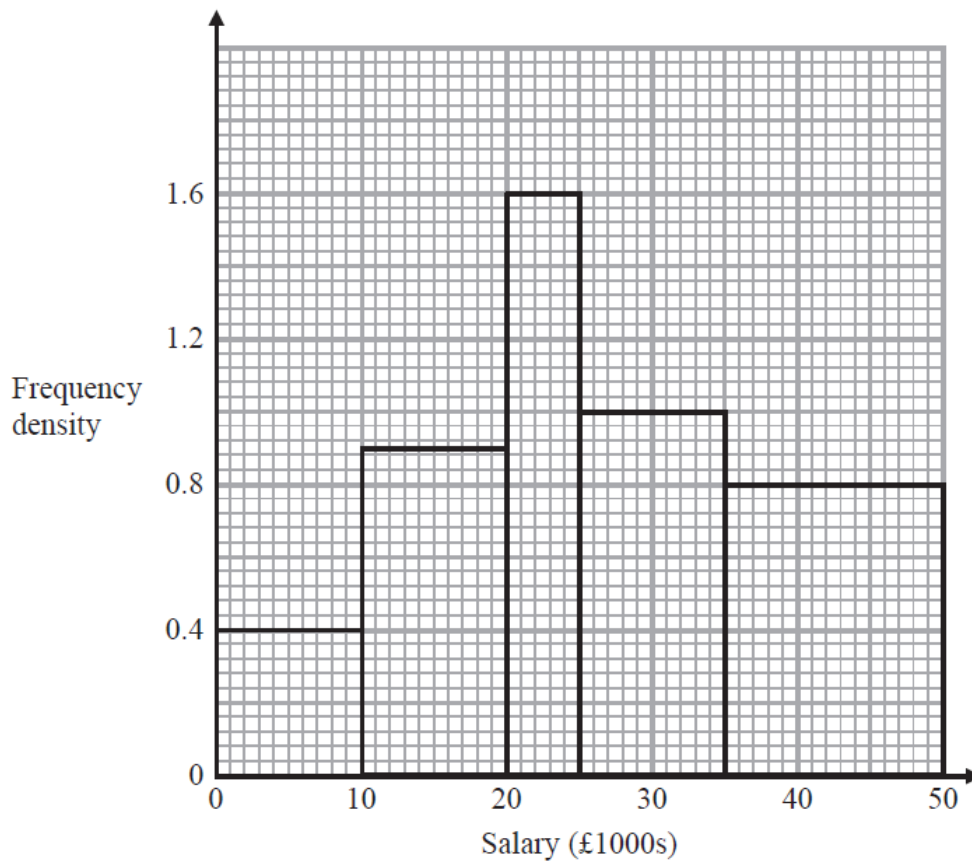
Salary (£1000s)

(a) Use the histogram to complete the frequency table.

Salary (p) in £1000s	Frequency
$0 < p \le 10$	4
$10 < p \le 20$	
$20 < p \le 25$	
$25 < p \le 35$	
$35 < p \le 50$	

(2)

(b) Work out the proportion of people in the sample who have a salary greater than £45 000.

....

(*c*) Find an estimate for the median salary.

£...
(2)

(Total 6 marks)

11. Here is a scale drawing of an office.
The scale is 1 cm to 2 metres.

A photocopier is going to be put in the office.
The photocopier has to be closer to *C* than it is to *D*.
The photocopier also has to be less than 8 metres from B.

Show, by shading, the region where the photocopier can be put.

(Total 3 marks)

12. The points *A*, *B* and *C* lie in order on a straight line.

The coordinates of *A* are (2, 5)

The coordinates of *B* are (4, *p*)
The coordinates of *C* are (*q*, 17)

Given that $AC = 3AB$, find the values of *p* and *q*.

p = ...

q = ...

(Total 3 marks)

13. Martin and Janet are in an orienteering race.

Martin runs from checkpoint *A* to checkpoint *B*, on a bearing of 075°
Janet is going to run from checkpoint *B* to checkpoint *A*.

Work out the bearing of *A* from *B*.

....°

(Total 2 marks)

16. Liquid A has a density of 0.6 g/cm³.

Liquid B has a density of 1.6 g/cm³.

120 g of liquid A and 128 g of liquid B are mixed to make liquid C.

Work out the density of liquid C.

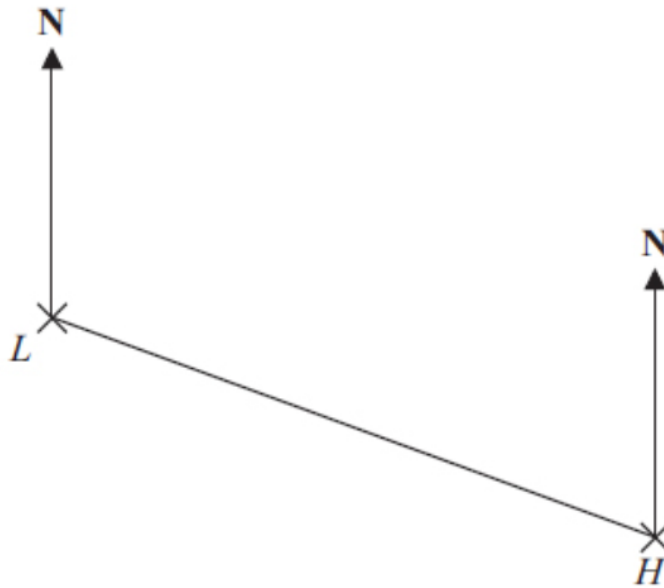.. g/cm^3

(Total 4 marks)

14   Solve the equation $2x^2 + 5x - 10 = 0$
Give your solutions correct to 2 decimal places.

....

(Total 3 marks)

15.   The diagram shows the position of a lighthouse *L* and a harbour *H*.

The scale of the diagram is 1 cm represents 5 km.

(a) Work out the real distance between *L* and *H*.

…………………………….. km

(1)

(b) Measure the bearing of *H* from *L*.

…………………………….. °

(1)

A boat *B* is 20 km from *H* on a bearing of 040°.

(c) On the diagram, mark the position of boat *B* with a cross (×).
Label it *B*.

(2)

(Total 4 marks)

16. $A = 2^2 \times 3 \times 5^2$

$B = 2^3 \times 5$

(a) Find the Highest Common Factor (HCF) of *A* and *B*.

....
(1)

(b) Find the Lowest Common Multiple (LCM) of *A* and *B*.

....
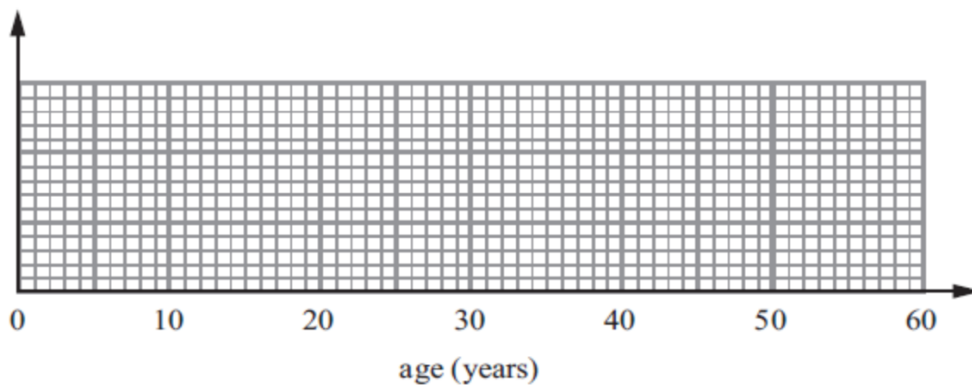**(2)**

(Total 3 marks)

17. Here are the ages, in years, of 15 women at West Ribble Tennis Club.

   16,   18,   18,   20,   25,   25,   27,   28,   30,   35,   38,   42,   45,   46,   50
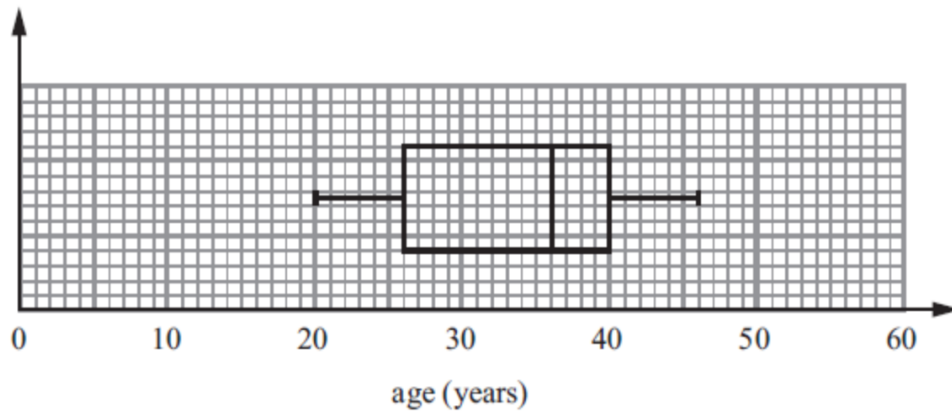
(a) On the grid, draw a box plot for this information.

(3)

The box plot below shows the distribution of the ages of the men at West Ribble Tennis Club.
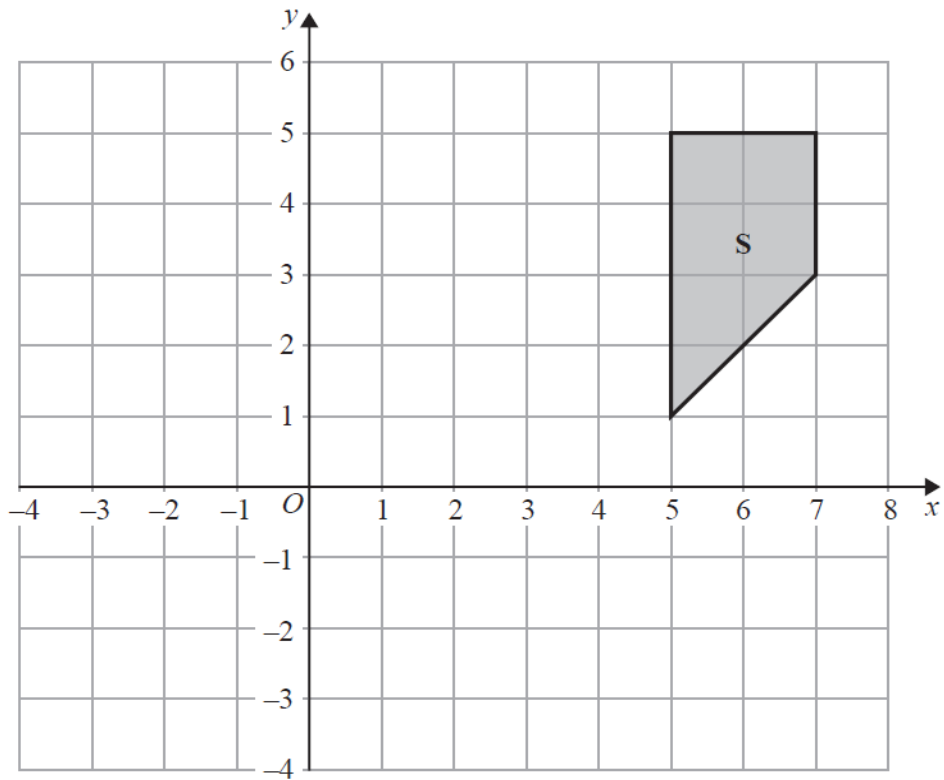
age (years)

(b) Use the box plots to compare the distributions of the ages of these women and the distributions of the ages of these men.

(2)

(Total 5 marks)

18.

Enlarge shape **S** with scale factor $\dfrac{1}{2}$ and centre (1, 3).
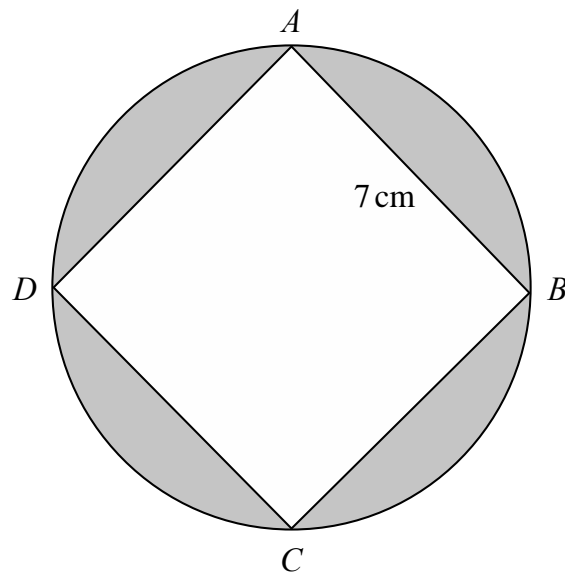
(Total 2 marks)

19.

Diagram **NOT** accurately drawn

A, *B*, *C* and *D* are points on a circle.
ABCD is a square of side 7 cm.

Work out the total area of the shaded regions.
Give your answer correct to the nearest whole number.

.... cm^2

(Total 5 marks)

20. (a) Show that the equation $x^3 + 4x = 1$ has a solution between $x = 0$ and $x = 1$

(2)

(b) Show that the equation $x^3 + 4x = 1$ can be arranged to give $x = \dfrac{1}{4} - \dfrac{x^3}{4}$
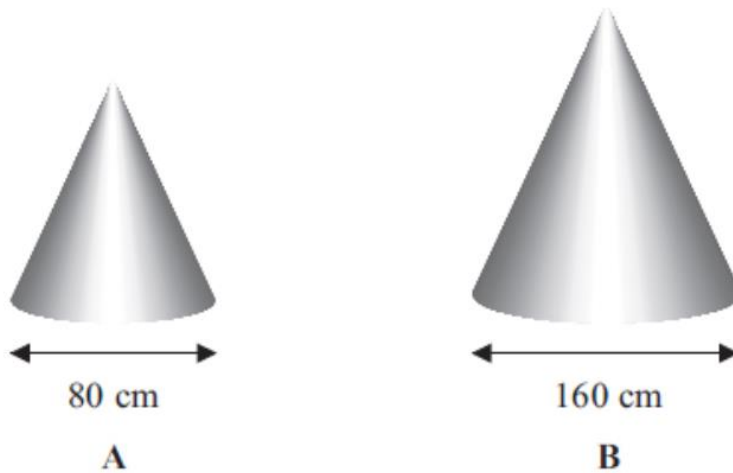
(1)

(c) Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \dfrac{1}{4} - \dfrac{x_n^{\,3}}{4}$ twice, to find an estimate for the solution of $x^3 + 4x = 1$

....
(3)

(Total 6 marks)

21. Ali has two solid cones made from the same type of metal.

Diagram **NOT** accurately drawn

A   **B**

The two solid cones are mathematically similar.
The base of cone **A** is a circle with diameter 80 cm.
The base of cone **B** is a circle with diameter 160 cm.
Ali uses 80 m*l* of paint to paint cone **A**.
Ali is going to paint cone **B**.

(a) Work out how much paint, in m*l*, he will need.

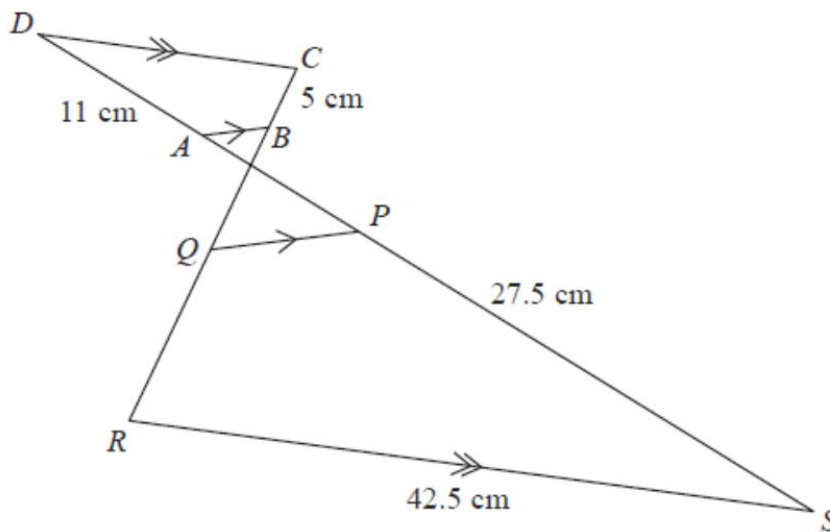.……………………………… m*l*
(2)

The volume of cone **A** is 171 700 cm^3.

(b) Work out the volume of cone **B**.

…………………………………… cm^3

(3)

(Total 5 marks)

22. In the diagram, *DAPS* and *CBQR* are straight lines.
AB is parallel to *QP* and *DC* is parallel to *RS*.
AD = 11 cm, *BC* = 5 cm, *PS* = 27.5 cm and *RS* = 42.5 cm.

Quadrilateral *ABCD* is similar to quadrilateral *PQRS*.

(a) Work out the length of *RQ*.

.... cm

(2)

(b) Work out the length of *CD*.

.... cm

(2)

(Total 4 marks)

23. A field is in the shape of a rectangle.
The width of the field is 28 metres, measured to the nearest metre.

(*a*) Work out the upper bound of the width of the field.

.... metres

(1)

The length of the field is 145 metres, measured to the nearest 5 metres.

(*b*) Work out the upper bound for the perimeter of the field.

.... metres

(3)

(Total 4 marks)

24. The diagram shows a regular pentagon *ABCDE*.
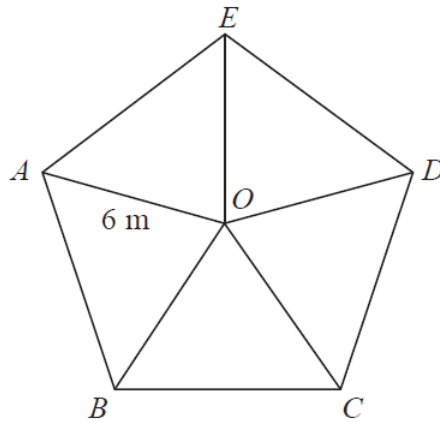
The pentagon is divided into 5 isosceles triangles.
$OA = OB = OC = OD = OE = 6$ m

Work out the area of the pentagon.
Give your answer correct to 1 decimal place.

.... m²

(Total 4 marks)

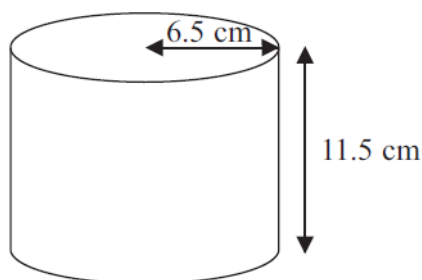**25.** The diagram shows a large tin of pet food in the shape of a cylinder.

Diagram **NOT** accurately drawn

The large tin has a radius of 6.5 cm and a height of 11.5 cm.

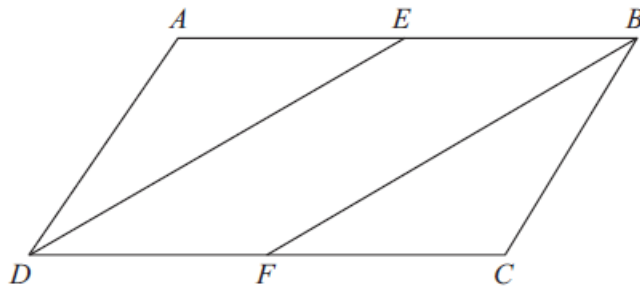A pet food company wants to make a new size of tin.

The new tin will have a radius of 5.8 cm.
It will have the same volume as the large tin.

23 @cchristian

Calculate the height of the new tin.
Give your answer correct to one decimal place.

.... cm

(Total 3 marks)

26.

ABCD is a parallelogram.
E is the midpoint of *AB*.
F is the midpoint of *DC*.

(a) Prove that triangle *AED* is congruent to triangle *CFB*.

(3)

(b) Hence, prove that *DE = FB*

27. Each student in a group of 32 students was asked the following question.

"Do you have a desktop computer (*D*), a laptop (*L*) or a tablet (*T*)?"

Their answers showed that

> 19 students have a desktop computer
> 17 students have a laptop
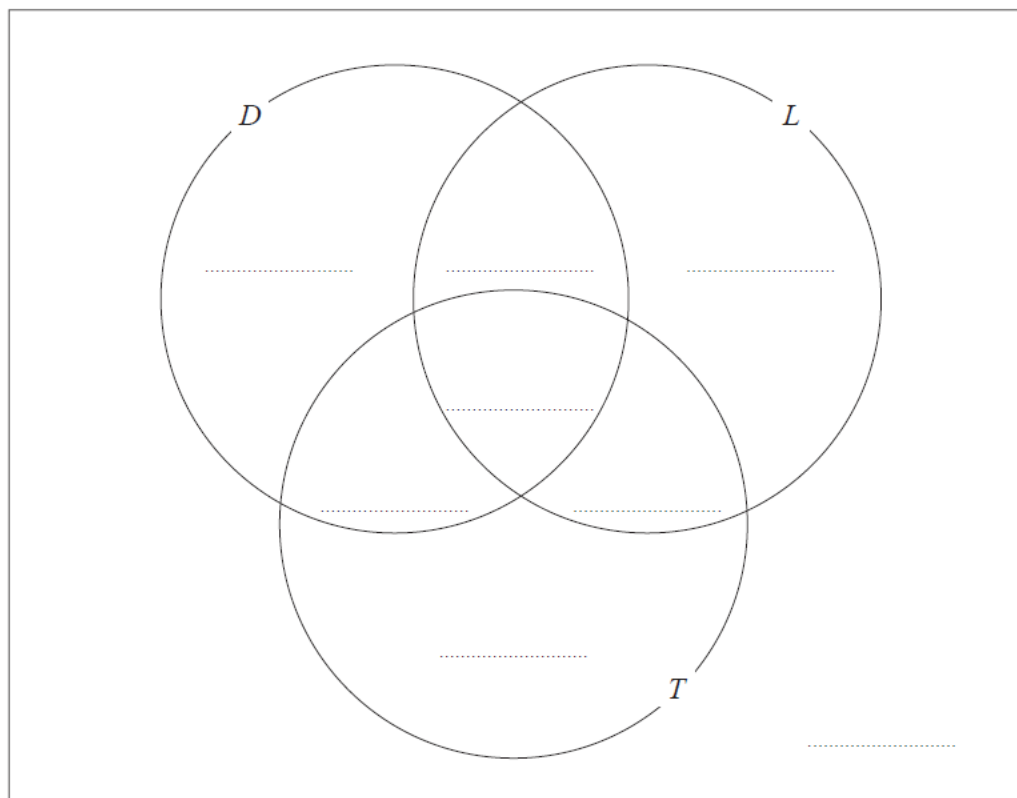> 16 students have a tablet
> 9 students have both a desktop computer and a laptop
> 11 students have both a desktop computer and a tablet
> 7 students have both a laptop and a tablet
> 5 students have all three.

(*a*) Using this information, complete the Venn diagram to show the number of students in each appropriate subset.
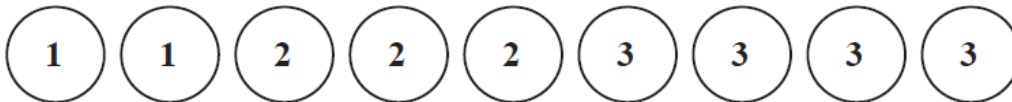
One of the students with both a desktop computer and a laptop is chosen at random.

(*b*) Find the probability that this student also has a tablet.

....

(1)

(Total 4 marks)

28. There are 9 counters in a bag.
There is a number on each counter.

Kal takes at random 3 counters from the bag.

He adds together the numbers on the 3 counters to get his Total.

Work out the probability that his Total is 6.

....

(Total 5 marks)

29. Toby invested £4500 for 2 years in a savings account.

He was paid 4% per annum compound interest.

(a) How much did Toby have in his savings account after 2 years?

£ ....

(3)

Jaspir invested £2400 for n years in a savings account.

He was paid 7.5% per annum compound interest.

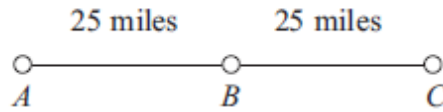At the end of the n years he had £3445.51 in the savings account.

(b) Work out the value of n.

....

(2)

(Total 5 marks)

30.

25 miles   25 miles

A   B   C

A, *B* and *C* are 3 service stations on a motorway.

AB = 25 miles

BC = 25 miles

Aysha drives along the motorway from *A* to *C*.

Aysha drives at an average speed of 50 mph from *A* to *B*.

She drives at an average speed of 60 mph from *B* to *C*.

Work out the difference in the time Aysha takes to drive from *A* to *B* and the time Aysha takes to drive from *B* to *C*.

Give your answer in minutes.

.... minutes

(Total 3 marks)

31. A rugby team played six games.
The mean score for the six games is 14.5

The rugby team played one more game.
The mean score for all seven games is 16

Work out the number of points the team scored in the seventh game.

.... points

(Total 2 marks)

32.   The scatter graph shows some information about 10 cars.
It shows the time, in seconds, it takes each car to go from 0 mph to 60 mph.
For each car, it also shows the maximum speed, in mph.
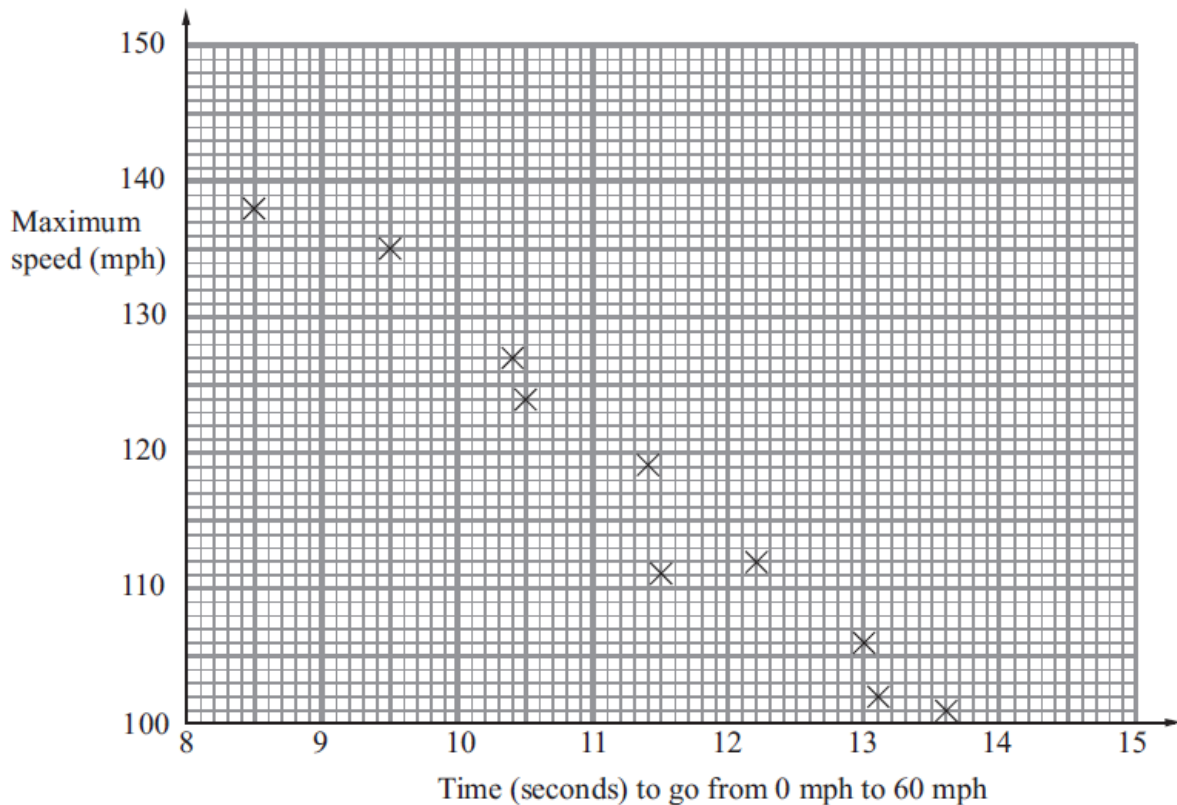
(a)   What type of correlation does this scatter graph show?

....

(1)

The time a car takes to go from 0 mph to 60 mph is 11 seconds.

(b) Estimate the maximum speed for this car.

.... mph

(2)

(Total 3 marks)

33. Jarek uses the formula

$$\text{Area} = \frac{1}{2}ab\sin C$$

to work out the area of a triangle.

For this triangle,

$a = 7.8$ cm correct to the nearest mm.
$b = 5.2$ cm correct to the nearest mm.
$C = 63°$ correct to the nearest degree.

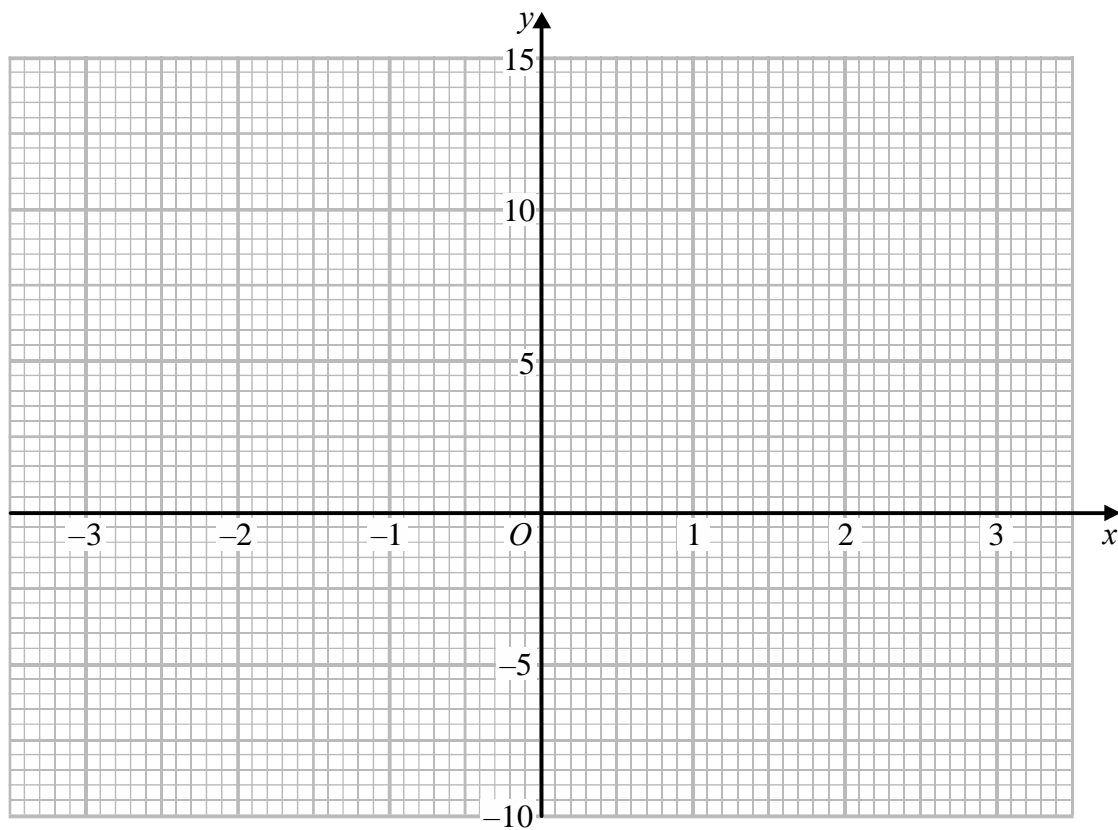Calculate the lower bound for the area of the triangle.

.... cm^2

(Total 3 marks)

34 (*a*) Complete this table of values for $y = x^2 + x - 4$

x	–3	–2	–1	0	1	2	3
y		–2	–4		–2		

(2)

(*b*) On the grid, draw the graph of $y = x^2 + x - 4$ for values of x from –3 to 3

(2)

(*c*) Use the graph to estimate a solution to $x^2 + x - 4 = 0$

....

(1)

(Total 5 marks

35
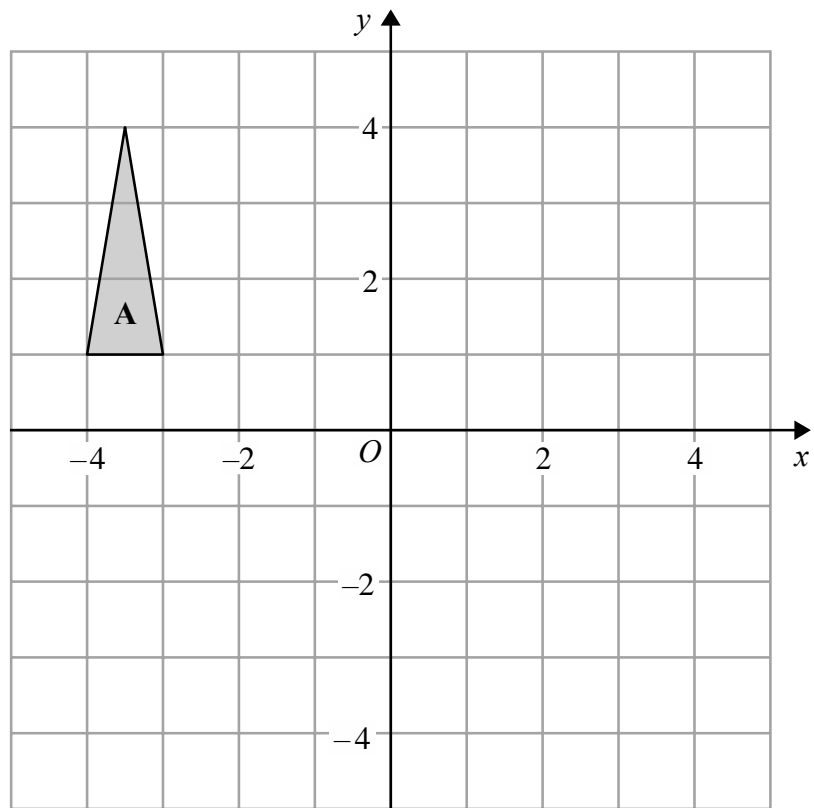
Triangle A is transformed by the combined transformation of a rotation of 180° about the

point (−2, 0) followed by a translation with vector $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$

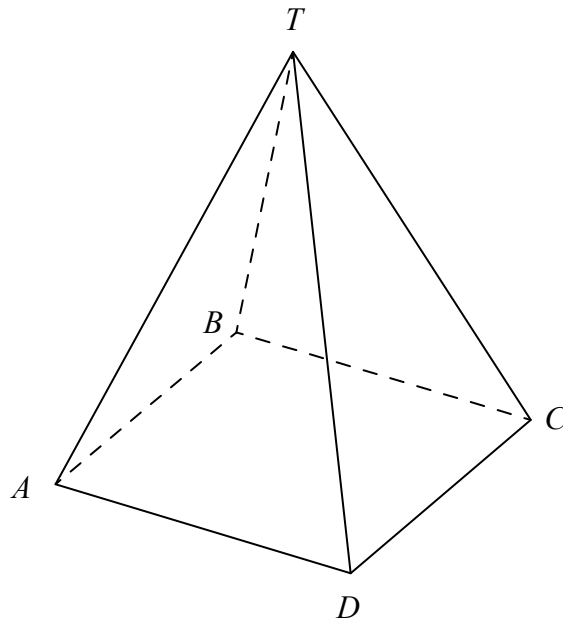**One point on triangle A is invariant under the combined transformation.**

Find the coordinates of this point.

(........................... , ...)

(Total 2 marks)

36 Here is a pyramid with a square base *ABCD*.

AB = 5 m

The vertex *T* is 12 m vertically above the midpoint of *AC*.

Calculate the size of angle *TAC*.

....°

(Total for Question 12 is 4 marks)

37   *BEG* is a triangle.
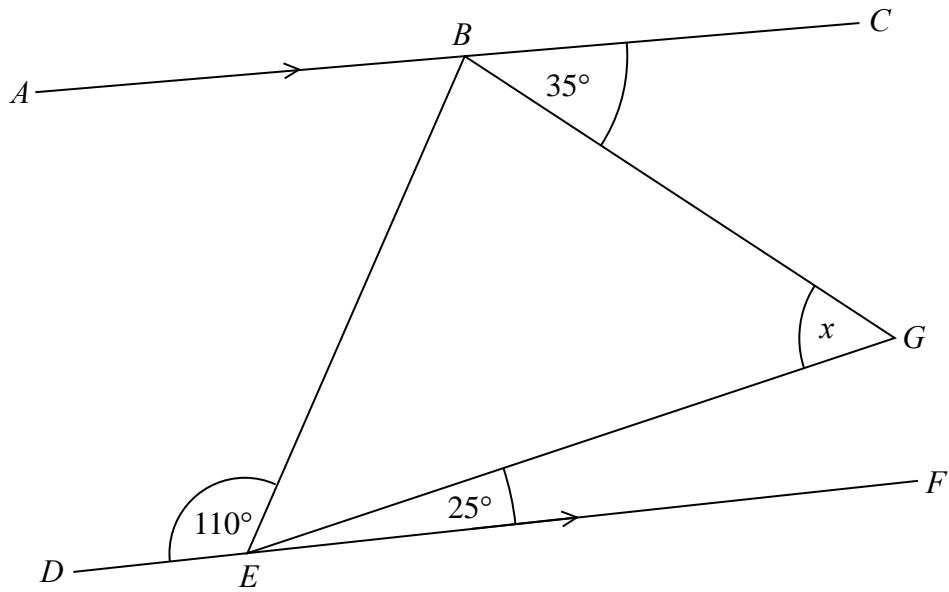
ABC and *DEF* are parallel lines.

Work out the size of angle *x*.
Give a reason for each stage of your working.

....°

(Total 4 marks)

38 Northern Bank has two types of account.
Both accounts pay compound interest.

| **Cash savings account** |
| Interest |
| 2.5% per annum |

| **Shares account** |
| Interest |
| 3.5% per annum |

Ali invests £2000 in the cash savings account.
Ben invests £1600 in the shares account.

(*a*) Work out who will get the most interest by the end of 3 years.
You must show all your working.

(4)

In the 3rd year the rate of interest for the shares account is changed to 4% per annum.

(*b*) Does this affect who will get the most interest by the end of 3 years?
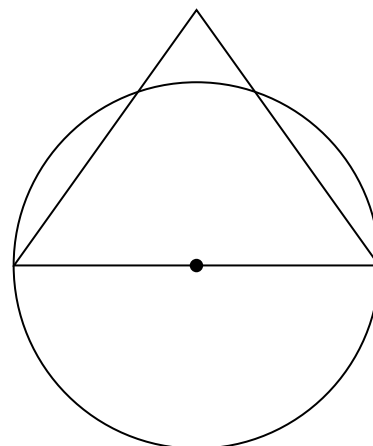Give a reason for your answer.

....

....

....
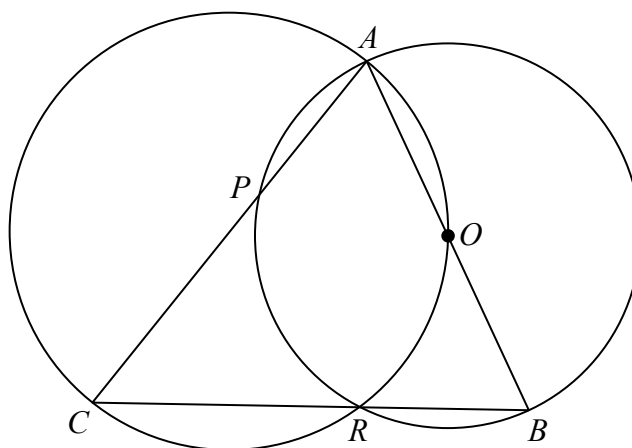
(1)

(Total 5 marks)

39 The diagram shows a circle and an equilateral triangle.

One side of the equilateral triangle is a diameter of the circle.
The circle has a circumference of 44 cm.

Work out the area of the triangle.
Give your answer correct to 3 significant figures.

....cm^2

(Total 3 marks)

40

A, *B*, *R* and *P* are four points on a circle with centre *O*.
A, *O*, *R* and *C* are four points on a different circle.
The two circles intersect at the points *A* and *R*.

CPA, *CRB* and *AOB* are straight lines.

Prove that angle *CAB* = angle *ABC*.

41 In a sale, the price of a TV is reduced by 25%.

A week later, the sale price of the TV is reduced by 15%.
The price of the TV is now £293.25.

What was the price of the TV before the sale?

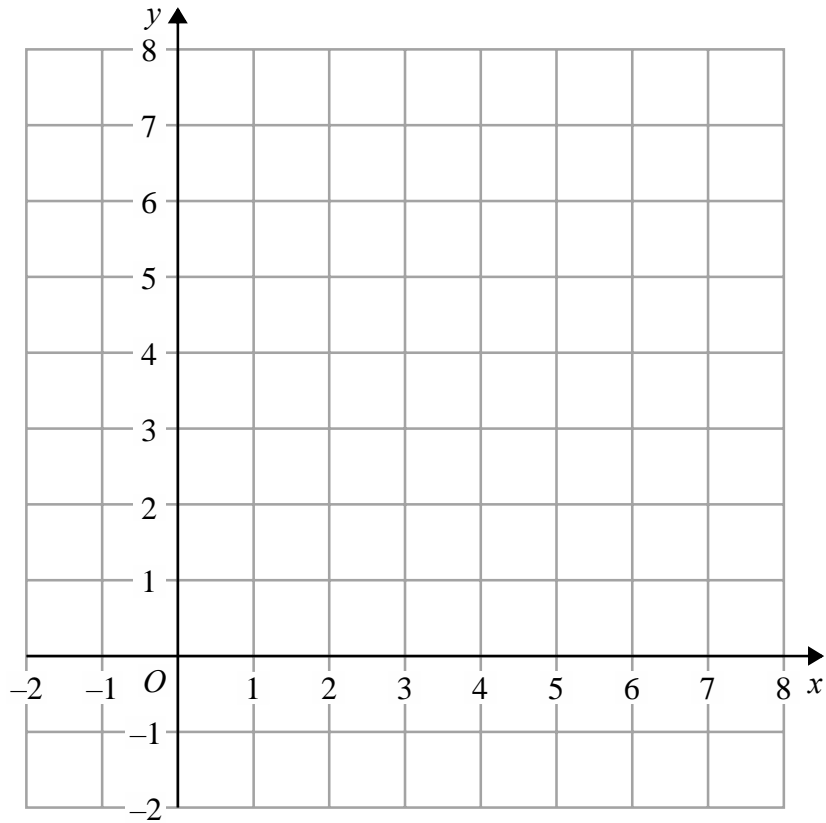£ ....

42 Prove algebraically that the recurring decimal $0.45\dot{7}$ can be written as $\dfrac{151}{330}$

43 On the grid show, by shading, the region defined by the inequalities
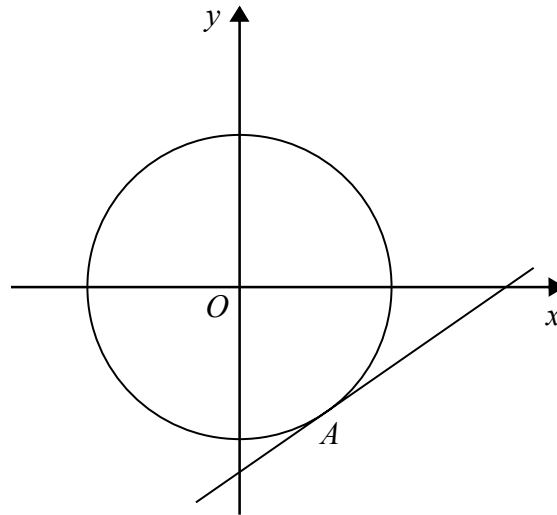
$$x < 4 \qquad 2x + y > 6 \qquad y > \frac{1}{3}x$$

Label the region **R**.

(Total 3 marks)

44 The diagram shows the circle with equation $x^2 + y^2 = 261$

A tangent to the circle is drawn at point A with coordinates $(p, -15)$, where $p > 0$

Find an equation of the tangent at A.

....

(Total 5 marks)

45   (*a*) Find the reciprocal of 5.

....
(1)

(*b*) Use your calculator to work out $\sqrt[3]{5\tan 60° + 1}$
Write down all the figures on your calculator display.

....
(2)

(Total 3 marks)

46   Solve the simultaneous equations

$$x + y = 15$$
$$7x - 5y = 3$$

Show clear algebraic working.

$x = $ ....

$y = $ ....

(**Total for Question 7 is 3 marks**)

47 Simplify fully $\left(\dfrac{256x^{20}}{y^8} \right)^{-\frac{1}{4}}$

....

(**2**)

(**Total 2 marks**)

48 A box is put on a table.

The face of the box in contact with the table is in the shape of a rectangle, 2 m by 1.25 m.
The pressure on the table due to the box is 42 newtons/m^2

Work out the force exerted by the box on the table.

$$\boxed{\text{pressure} = \dfrac{\text{force}}{\text{area}}}$$

.... newtons

(**Total 3 marks**)

49 Solve the inequality $4x^2 - 5x - 6 > 0$

....
(**Total 4 marks**)

50. A farmer wants to estimate the number of rabbits on his farm.

On Monday he catches 120 rabbits.
He puts a tag on each rabbit.
He then lets the rabbits run away.

On Tuesday the farmer catches 70 rabbits.
15 of these rabbits have a tag on them.

Work out an estimate for the total number of rabbits on the farm.
You must write down any assumptions you have made.

....

51 The nth term of a sequence is given by $an^2 + bn$ where a and b are integers.

The 2nd term of the sequence is –2.
The 4th term of the sequence is 12.

(a) Find the 6th term of the sequence.

....

(4)

Here are the first five terms of a different quadratic sequence.

$$0 \qquad 2 \qquad 6 \qquad 12 \qquad 20$$

(b) Find an expression, in terms of n, for the nth term of this sequence.

....
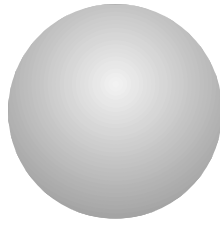
(2)

(Total 6 marks)

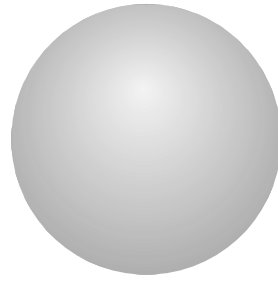**52** Here are three spheres.
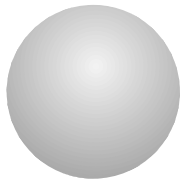
P   Q   R

The volume of sphere **Q** is 150% more than the volume of sphere **P**.
The volume of sphere **R** is 50% more than the volume of sphere **Q**.

Find the volume of sphere **P** as a fraction of the volume of sphere **R**.

....

(Total for Question 12 is 3 marks)

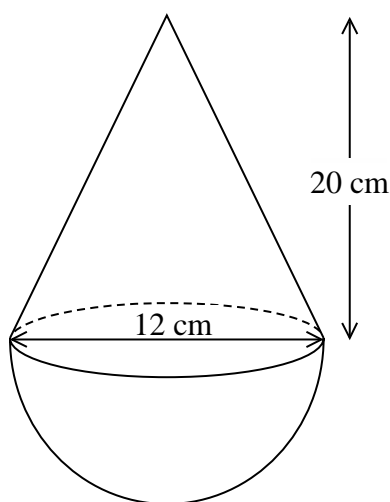**53** n is a whole number. Prove that $n^2 + (n + 1)^2$ is always an odd number.

Total for Question 13 is 2 marks)
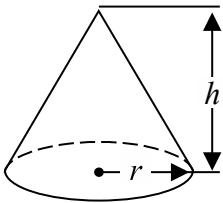
54 Find the exact value of tan 60° × sin 30°
Give your answer in its simplest form.

....

(Total for Question 14 is 2 marks)

_____
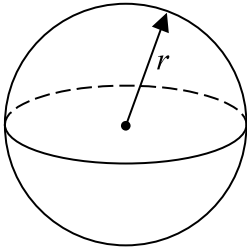
55 The diagram shows a solid shape.
The shape is a cone on top of a hemisphere.

The height of the cone is 20 cm.
The base of the cone has a diameter of 12 cm.
The hemisphere has a diameter of 12 cm.

The total volume of the shape is $k\,\pi$ cm^3, where k is an integer.

Work out the value of k.

$k = $ ....

56 Given that

$$x^2 : (-2x + 12) = 1 : 2$$

find the possible values of x.

....

57 (*a*) Express $\sqrt{2} + \sqrt{18}$ in the form $a\sqrt{2}$ where a is an integer.

58 Given that $x^2 - 8x + 1 = (x - a)^2 - b$ for all values of x,

(i) find the value of a and the value of b.

$a = $ ....

$b = $ ....

(2)

(ii) Hence write down the coordinates of the turning point on the graph of $y = x^2 - 8x + 1$

$$(\text{............................} , \text{............................})$$
(1)

(Total for Question 19 is 3 marks)

___

58 The functions f and g are such that

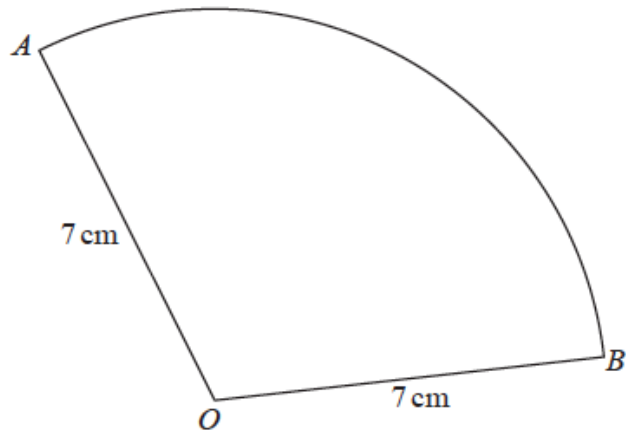$$f(x) = 3x - 2 \quad \text{and} \quad g(x) = x^2 + 2$$

(*a*)   Find $f^{-1}(x)$

$$f^{-1}(x) = \text{..}$$
(2)

Given that $fg(x) = 2gf(x)$,

(*b*)   show that $15x^2 - 24x + 8 = 0$

(5)

60   *OAB* is a sector of a circle with centre *O* and radius 7 cm.

The area of the sector is 30 cm^2

Calculate the perimeter of the sector.
Give your answer correct to 3 significant figures.

.... cm

(Total for Question 12 is 4 marks)

_____

61   Show that $6 + \left[(x+4) \div \dfrac{x^2 + 2x - 8}{x-1} \right]$ simplifies to $\dfrac{ax - b}{cx - d}$ where a, b, c and d are integers.
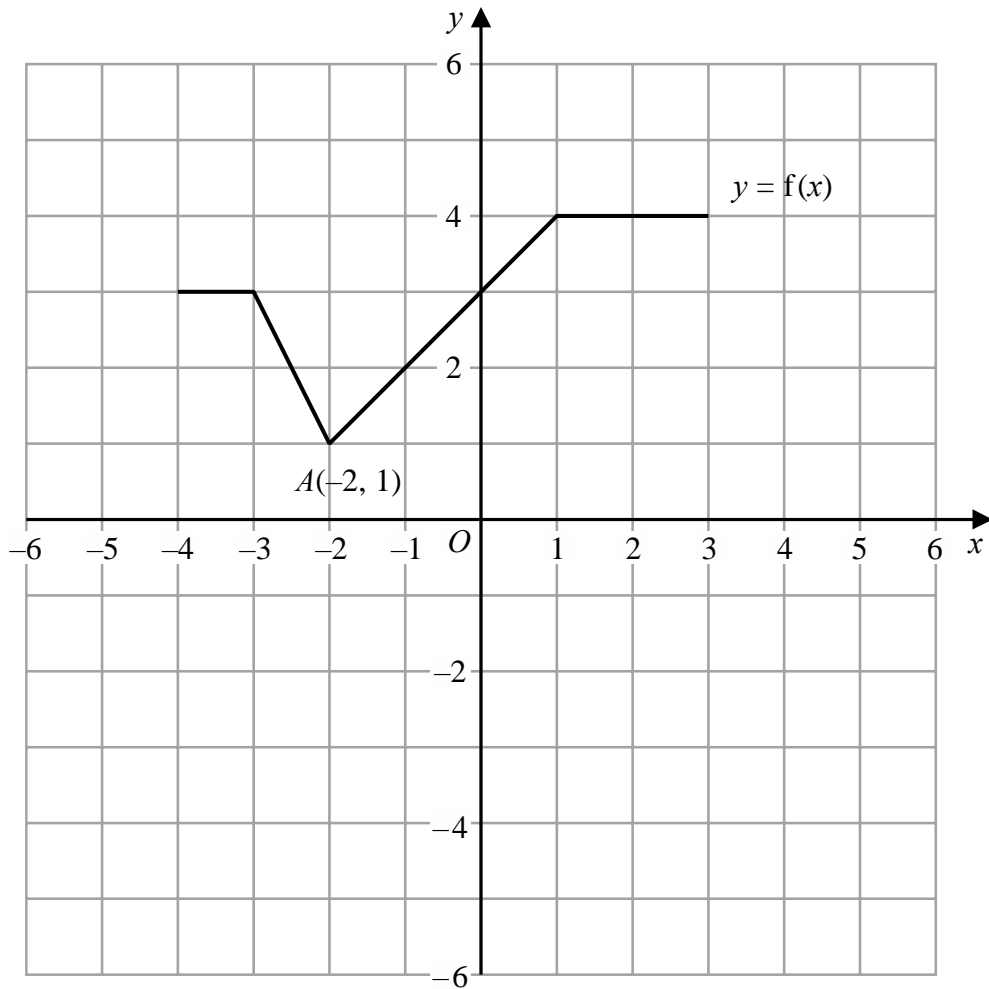
(Total for Question 13 is 4 marks)

62 Given that $9^{-\frac{1}{2}} = 27^{\frac{1}{4}} \div 3^{x+1}$

find the exact value of x.

= ....

(Total for Question 19 is 3 marks)

_____

63 The graph of $y = f(x)$ is shown on the grid.

$A(-2, 1)$

$y = f(x)$

(a) On the grid, draw the graph with equation $y = f(x + 1) - 3$

(2)

Point $A(-2, 1)$ lies on the graph of $y = f(x)$.

When the graph of $y = f(x)$ is transformed to the graph with equation $y = f(-x)$, point A is mapped to point B.

(*b*) Write down the coordinates of point B.

(............................ , ...)

(1)

(Total for Question 20 is 3 marks)
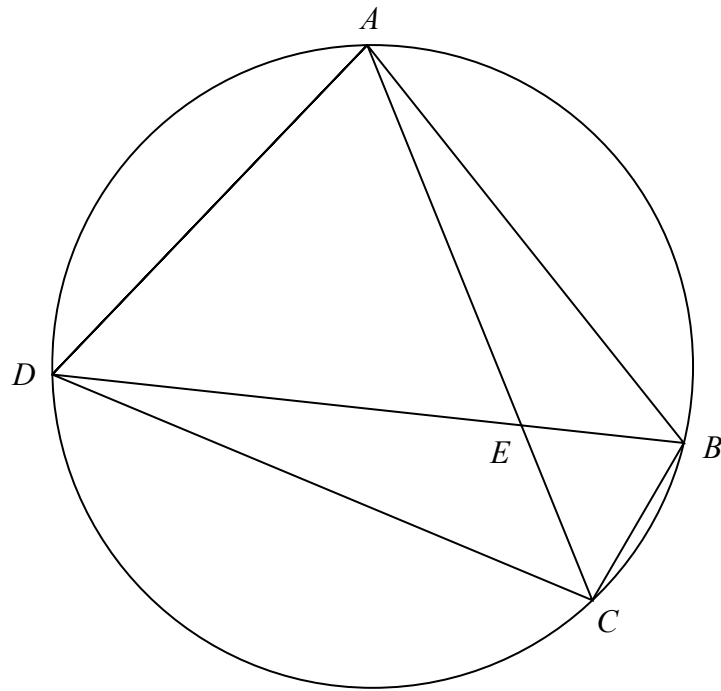
_____

64 Sketch the graph of

$$y = 2x^2 - 8x - 5$$

showing the coordinates of the turning point and the exact coordinates of any intercepts with the coordinate axes.

(Total for Question 21 is 5 marks)

_____

65 *A*, *B*, *C* and *D* are four points on a circle.

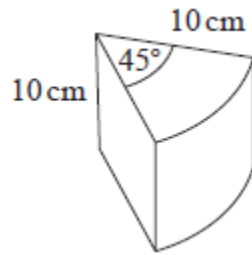*AEC* and *DEB* are straight lines.

Triangle *AED* is an equilateral triangle.

Prove that triangle *ABC* is congruent to triangle *DCB*.
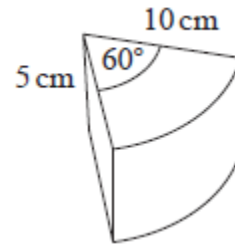
(Total for Question 22 is 4 marks)

66 Here are two solid prisms, prism **A** and prism **B**.

prism **A**   prism **B**

The cross section of prism **A** is a sector, with angle 45°, of a circle of radius 10 cm.
The prism has a depth of 10 cm and a mass of 40π grams.

The cross section of prism **B** is a sector, with angle 60°, of a circle of radius 10 cm.
The prism has a depth of 5 cm and a mass of 50π grams.

Express the difference in the densities of the two prisms as a percentage of the density of prism **A**.

.... %

(Total for Question 19 is 5 marks)

67 Show that $\dfrac{12 + \sqrt{128}}{1 - \sqrt{2}}$ can be written in the form $a + b\sqrt{2}$, where a and b are integers.
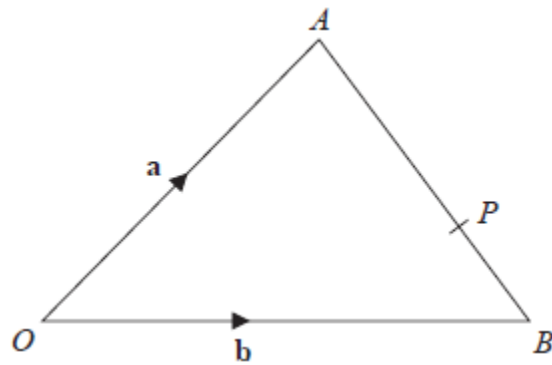
68
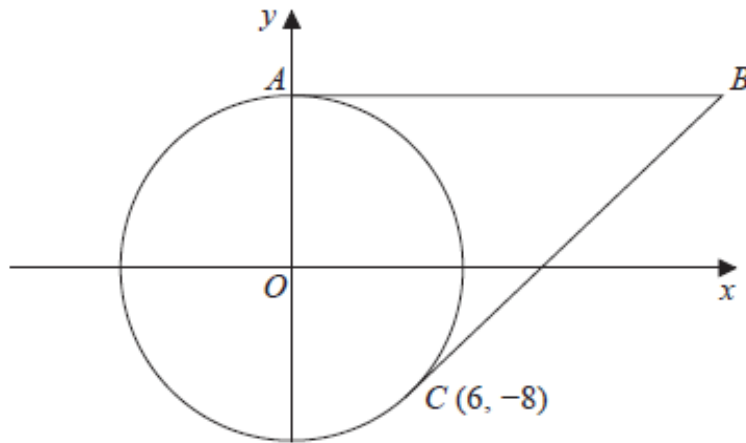
The diagram shows the circle with equation $x^2 + y^2 = 100$
The unit of length on both axes is one centimetre.

The circle intersects the positive y-axis at the point A.
The point C on the circle has coordinates $(6, -8)$
The straight lines AB and CB are tangents to the circle.

Find the area of quadrilateral $ABCO$.

.... cm^2

(Total for Question 21 is 4 marks)

69 *OAB* is a triangle.

$\overrightarrow{OA} = \mathbf{a}$

$\overrightarrow{OB} = \mathbf{b}$

P is the point on *AB* such that *AP* : *PB* = 3 : 2

Find $\overrightarrow{OP}$ in terms of **a** and **b**.
Give your answer in its simplest form.

....

(Total for Question 69 is 3 marks)

70   *OXYZ* is a parallelogram.
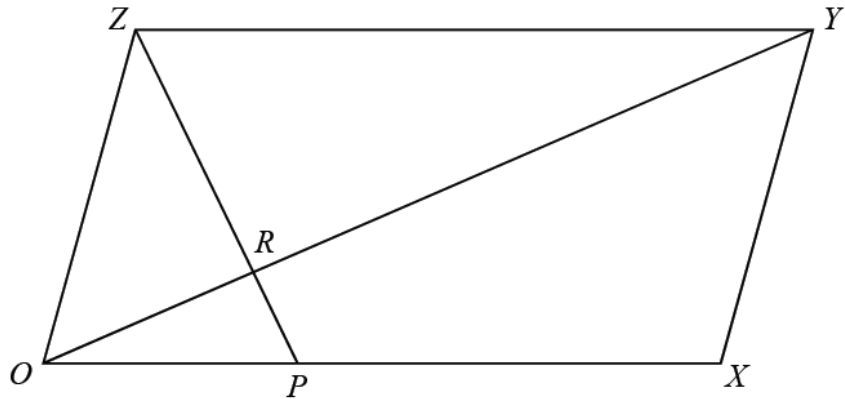
$\overrightarrow{OX}$ = **a**

$\overrightarrow{OY}$ = **b**
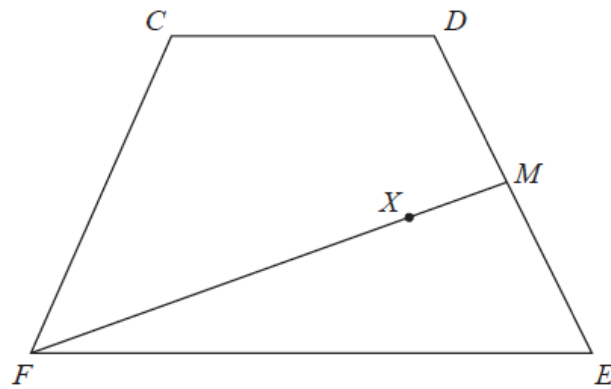
P is the point on *OX* such that *OP* : *PX* = 1 : 2

R is the point on *OY* such that *OR* : *RY* = 1 : 3

Work out, in its simplest form, the ratio *ZP* : *ZR*

You must show all your working.

....

(Total for Question 70 is 5 marks)

71 *CDEF* is a quadrilateral.

$$\overrightarrow{CD} = \mathbf{a}, \qquad \overrightarrow{DE} = \mathbf{b} \qquad \text{and} \qquad \overrightarrow{FC} = \mathbf{a} - \mathbf{b}.$$

(*a*)　Express $\overrightarrow{FE}$ in terms of **a** and/or **b**.
　　Give your answer in its simplest form.

....
(2)

M is the midpoint of *DE*.
X is the point on *FM* such that *FX* : *XM* = *n* : 1
CXE is a straight line.

(*b*)　Work out the value of *n*.

n = ...
(4)

(Total for Question 71 is 6 marks)