Centre No.	1	0	6	5	8	Paper Reference							Surname	Initial(s)
Candidate No.	5	0	8	7		6	6	6	5	/	0	1	FORWARD	LF

Signature

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced

Friday 25 January 2013 – Afternoon

Time: 1 hour 30 minutes

HA011261554

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	7
2	8
3	9
4	8
5	11
6	11
7	11
8	9
Total	74

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

Printer's Log. No.
P41486A
W850/R6665/57570 5/5/5/5/6/
P 4 1 4 8 6 A 0 1 2 8

Turn over

PEARSON

596775/1

Leave blank

1. The curve C has equation

$$y = (2x - 3)^5$$

The point P lies on C and has coordinates $(w, -32)$.

$x \quad y$

Find

(a) the value of w,

(2)

1 Q01aM
1 Q01aA

(b) the equation of the tangent to C at the point P in the form $y = mx + c$, where m and c are constants.

(5)

1 Q01bM1
1 Q01bA1
1 Q01bM2
1 Q01bM3
1 Q01bA2

a) $-32 = (2x - 3)^5$

$\sqrt[5]{-32} = 2x - 3$

$\dfrac{\sqrt[5]{-32} + 3}{2} = \cancel{2x}\ x = \frac{1}{2}$

$\dfrac{-2 + 3}{2} = \frac{1}{2} = w$

b) $P = (\frac{1}{2}, -32)$

$\dfrac{dy}{dx} = 5 \times (2x - 3)^4 \times 2 = 10(2x - 3)^4$

when $x = \frac{1}{2}$

$10(1 - 3)^4 = 160$ gradient

$y = 160x + c$

$(y - y_1) = 160(x - x_1)$

$y + 32 = 160(x - \frac{1}{2})$

$y = 160x - 80 - 32$

$y = 160x - 112$

Leave blank

Question 1 continued

(Total 7 marks)

Q1 | 7

P 4 1 4 8 6 A 0 3 2 8

Turn over

596775/3

Leave blank

2. $$g(x) = e^{x-1} + x - 6$$

(a) Show that the equation $g(x) = 0$ can be written as

$$x = \ln(6 - x) + 1, \qquad x < 6$$

(2)

1 Q02aM
1 Q02aA

The root of $g(x) = 0$ is α.

The iterative formula

$$x_{n+1} = \ln(6 - x_n) + 1, \qquad x_0 = 2$$

is used to find an approximate value for α.

(b) Calculate the values of x_1, x_2 and x_3 to 4 decimal places.

(3)

1 Q02bM
1 Q02bA1
1 Q02bA2

(c) By choosing a suitable interval, show that $\alpha = 2.307$ correct to 3 decimal places.

(3)

1 Q02cM1
1 Q02cM2
1 Q02cA

a) $g(x) = e^{x-1} + x - 6$

$0 = e^{x-1} + x - 6$

$e^{x-1} = 6 - x$

$x - 1 = \ln(6 - x)$

$x = \ln(6-x) + 1 \qquad x < 6$

b) $x_1 = \ln(6-2) + 1 = 2.386294361 \approx 2.3863$

$x_2 = \ln(6 - x_1) + 1 = 2.284733739 \approx 2.2847$

$x_3 = \ln(6 - x_2) + 1 = 2.312450348 \approx 2.3125$

c) $\alpha = 2.307$

upper bound $= 2.3075$

lower bound $= 2.3065$

$g(2.3075) = e^{2.3075-1} + 2.3075 - 6 = 4.41985077 \times 10^{-3}$

$g(2.3065) = e^{2.3065-1} + 2.3065 - 6 = -2.752219294 \times 10^{-4}$

~~There is a sign change of g(x) between x = 2.3075 and x = 2.3065 ... g(x) = 0 ... between these two values and ... 2.307 ...~~

Leave blank

Question 2 continued

~~places~~

There is a sign change of $g(x)$ between $x=2.3075$ and $x=2.3065$ therefore $g(x)=0$ when $2.3065 < x < 2.3075$ therefore $x=2.307$ to three decimal places when $g(x)=0$. Any number greater than 2.3065 but less than 2.3075 rounds to 2.307 (3 decimal places).

Q2 8

(Total 8 marks)

Turn over

596775/5

P 4 1 4 8 6 A 0 5 2 8

Leave blank

3.

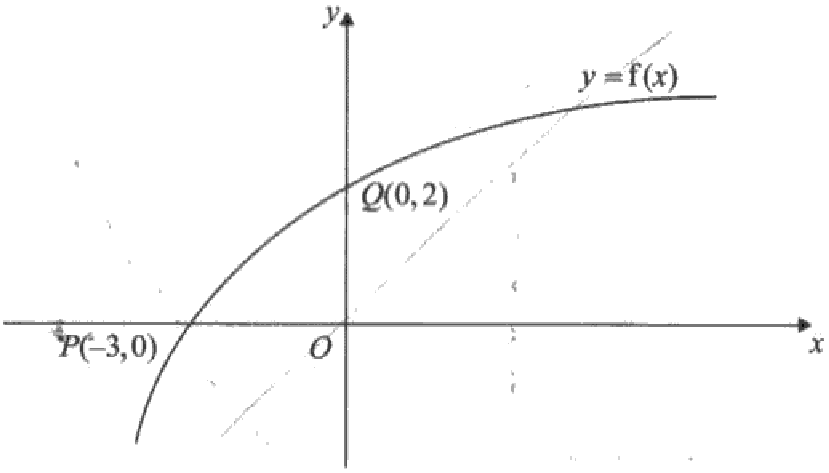


Figure 1

Figure 1 shows part of the curve with equation $y = f(x)$, $x \in \mathbb{R}$.

The curve passes through the points $Q(0,2)$ and $P(-3,0)$ as shown.

(a) Find the value of $ff(-3)$.

(2) 1 Q03aM

1 Q03aA

On separate diagrams, sketch the curve with equation

(b) $y = f^{-1}(x)$, 1 Q03bB1

(2) 1 Q03bB2

(c) $y = f(|x|) - 2$, 1 Q03cB1

(2) 1 Q03cB2

(d) $y = 2f\left(\frac{1}{2}x\right)$. 1 Q03dB1

(3) 1 Q03dB2

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes. 1 Q03dB3

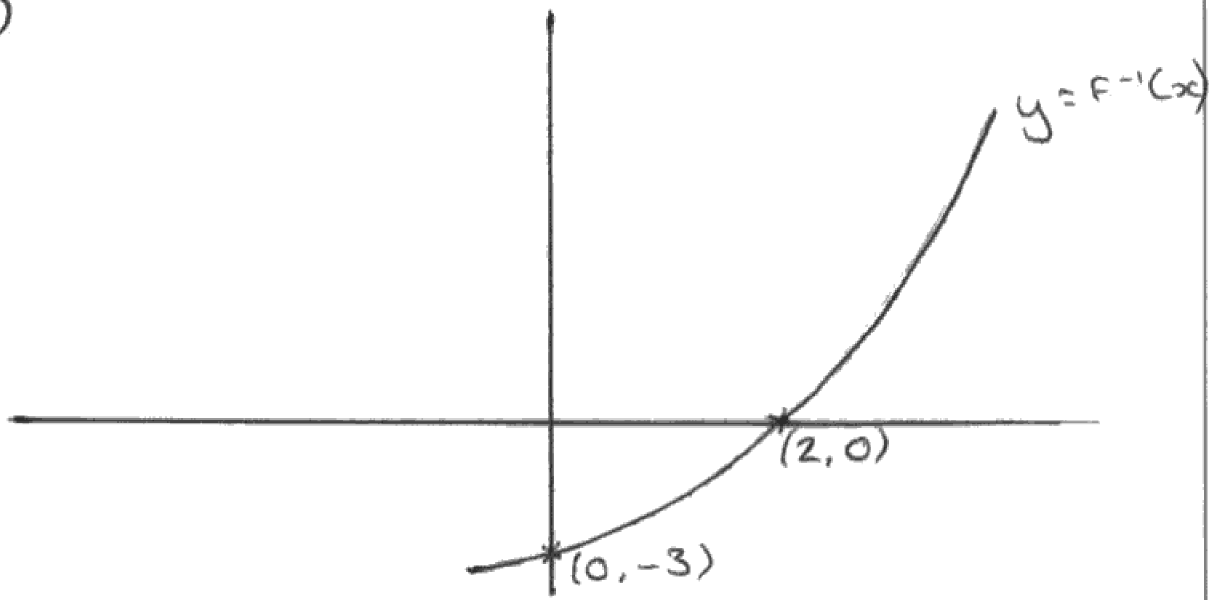
a) $ff(-3) = f(0) = 2$

b)

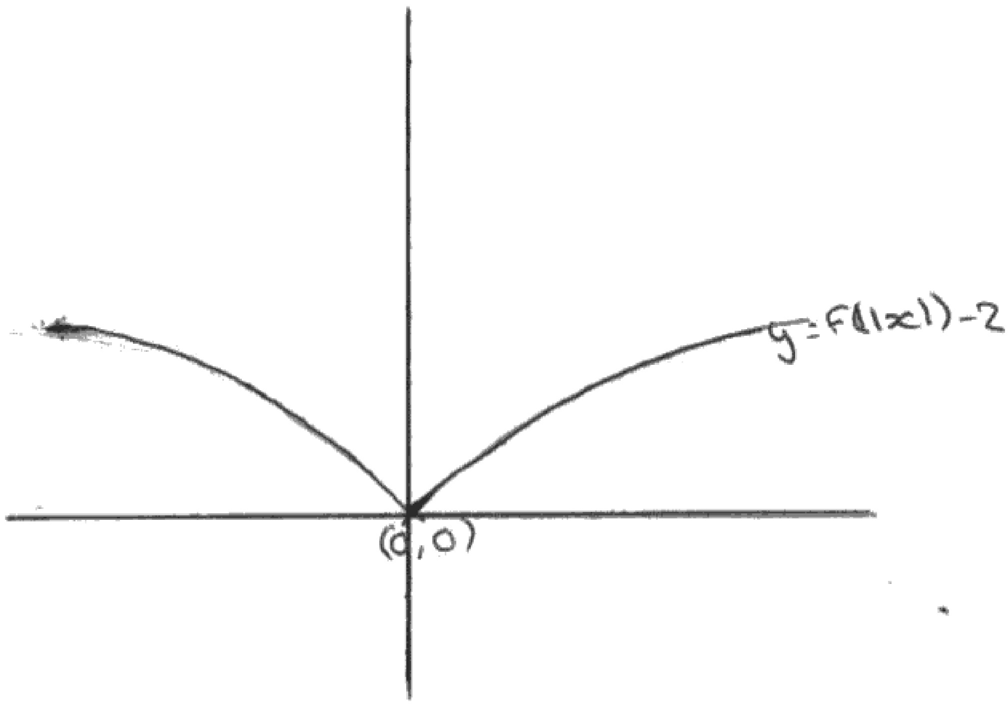
Leave blank

Question 3 continued

b)

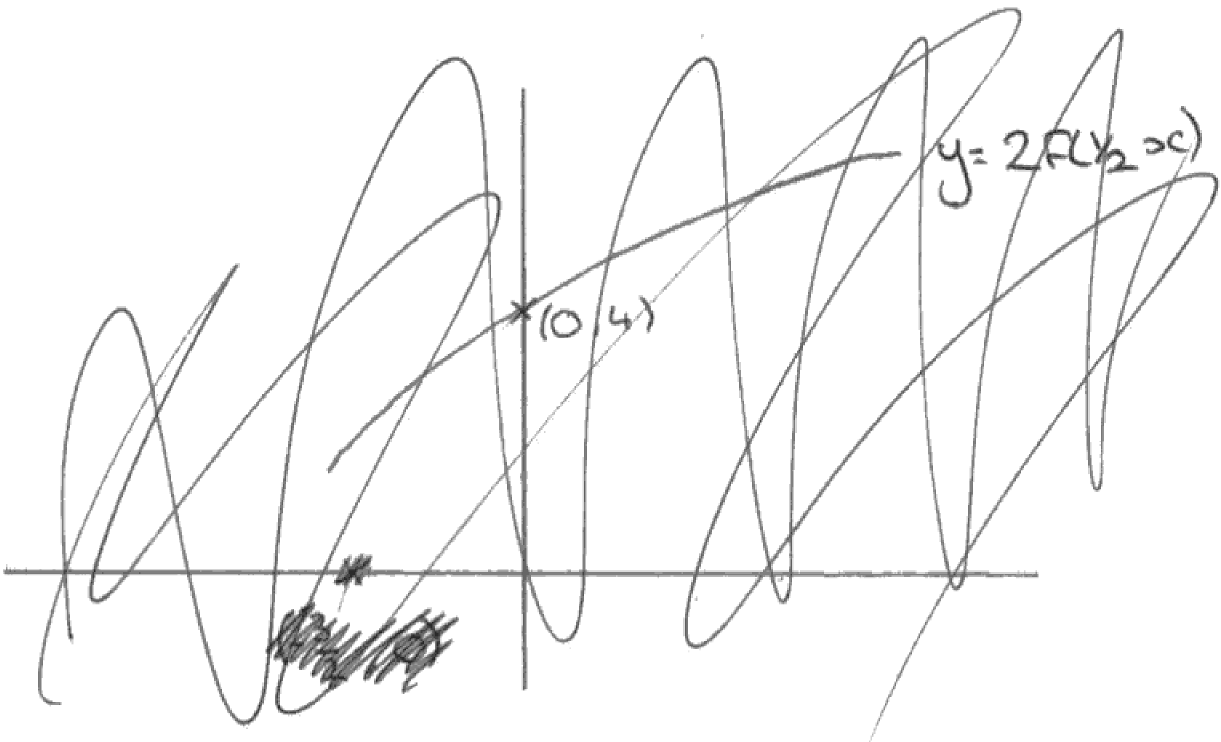


c)

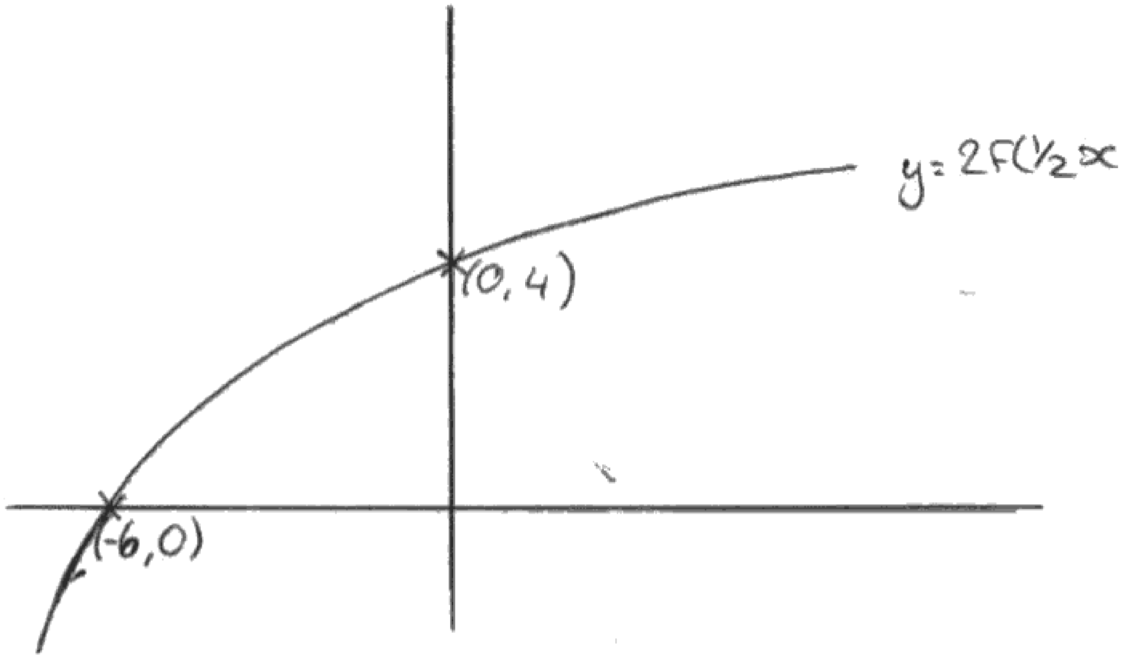


$y = 2f(\tfrac{1}{2}x)$

Question 3 continued



c)



Leave blank

Leave blank

Question 3 continued

Q3

(Total 9 marks) 9

P 4 1 4 8 6 A 0 9 2 8

Turn over

596775/9

Leave blank

4. (a) Express $6\cos\theta + 8\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 decimal places.

(4)

1 Q04aM1
1 Q04aA1
1 Q04aM2
1 Q04aA2

(b) $$p(\theta) = \frac{4}{12 + 6\cos\theta + 8\sin\theta}, \quad 0 \leqslant \theta \leqslant 2\pi$$

Calculate

(i) the maximum value of $p(\theta)$,

(ii) the value of θ at which the maximum occurs.

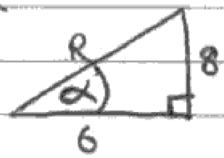
(4)

1 Q04biM
1 Q04biA
1 Q4biiM
1 Q4biiA

a) $R\cos(\theta - \alpha) = R\cos\theta\cos\alpha + R\sin\alpha\sin\theta$

$R\cos\alpha = 6$ $\quad$ $R\sin\alpha = 8$

$\cos\alpha = 6/R$ $\quad$ $\sin\alpha = 8/R$



$R = \sqrt{8^2 + 6^2} = 10$

$\alpha = \tan^{-1}\left(\frac{8}{6}\right) = 0.927295218 \approx 0.927$

$6\cos\theta + 8\sin\theta = 10\cos(\theta - 0.927)$

b) $p(\theta) = \dfrac{4}{12 + 6\cos\theta + 8\sin\theta}$

i) $p(\theta) = \dfrac{4}{12 + 10\cos(\theta - 0.927)}$

$= \dfrac{2}{6 + 5\cos(\theta - 0.927)}$

for maximum value $\quad p(\theta) =$

Question 4 continued

~~maximum value of $6 + 10\cos(\theta - 0.927)$~~

~~$0 = 6 + 10\cos(\theta - 0.927)$~~

~~$\cos(\theta - 0.927) = \frac{-6}{10}$~~

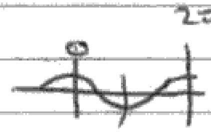
~~$p(\theta) = \infty$ there is no maximum value~~

~~ii) $\cos(\theta - 0.927) = \frac{-6}{10}$~~

~~$\theta - 0.927 = 2.21429\ldots,\ 4.06888872$~~

i) $p(\theta) = \frac{2}{6-5} = 2$ maximum value

ii) $\cos(\theta - 0.927) = -1$



$\theta - 0.927 = \pi$

$\theta = \pi + 0.927\ldots$

$= 4.068887672 = 4.069$

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Q4

(Total 8 marks)

8

P 4 1 4 8 6 A 0 1 3 2 8

596775/13

5. (i) Differentiate with respect to x

(a) $y = x^3 \ln 2x$

(b) $y = (x + \sin 2x)^3$

(6)

Given that $x = \cot y$,

(ii) show that $\frac{dy}{dx} = \frac{-1}{1+x^2}$

(5)

Leave blank

1 Q05iaM
1 Q5iaA1
1 Q5iaA2
1 Q05ibB
1 Q05ibM
1 Q05ibA
1 Q5iiM1
1 Q5iiA1
1 Q5iiM2
1 Q5iiM3
1 Q5iiA2

i) a) $y = x^3 \ln 2x$

$\frac{dy}{dx} = 3x^2 \ln 2x + \frac{x^3 \times 2}{2x}$

$= 3x^2 \ln x + x^2$

$x^2(3\ln x + 1)$

b) $y = (x + \sin 2x)^3$

$\frac{dy}{dx} = 3(x+\sin 2x)^2 \times (1 + 2\cos 2x)$

$= 3(1 + 2\cos 2x)(x + \sin 2x)^2$

ii) $x = \cot y$

$\cos^2 x + \sin^2 x = 1$

$\cot^2 x + 1 = \operatorname{cosec}^2 x$

$\frac{dx}{dy} = -\operatorname{cosec}^2 y$

$= -(1 + \cot^2 y)$

$x = \cot y$

$x^2 = \cot^2 y$

$= -1 - x^2$

$\frac{dy}{dx} = \frac{1}{-1 - x^2} = \frac{-1}{1+x^2}$

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Q5

11

(Total 11 marks)

Leave blank

6\. (i) Without using a calculator, find the exact value of

$$(\sin 22.5° + \cos 22.5°)^2$$

You must show each stage of your working.

(5)

(ii) (a) Show that $\cos 2\theta + \sin \theta = 1$ may be written in the form

$$k \sin^2 \theta - \sin \theta = 0, \text{ stating the value of } k.$$

(2)

(b) Hence solve, for $0 \leqslant \theta < 360°$, the equation

$$\cos 2\theta + \sin \theta = 1$$

(4)

1 Q06iM1
1 Q06iB
1 Q06iM2
1 Q06iA1
1 Q06iA2
1 Q6iiaM
1 Q6iiaA
1 Q6iibM
1 6iibA1
1 Q6iibB
1 6iibA2

6i) $(\sin 22.5 + \cos 22.5)^2 =$

$\sin^2 22.5 + \cos^2 22.5 + 2\sin 22.5\cos 22.5 =$
$(\sin^2 x + \cos^2 x = 1)$

$1 + 2\sin 22.5\cos 22.5$
$(2\sin A\cos A = \sin 2A)$

$1 + \sin(2\times 22.5) = 1 + \sin 45 = 1 + \frac{\sqrt{2}}{2} = \frac{2+\sqrt{2}}{2}$

ii) a) $\cos 2\theta + \sin\theta = 1$
$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$

$1 - 2\sin^2\theta + \sin\theta = 1$
$-2\sin^2\theta + \sin\theta = 0$
$2\sin^2\theta - \sin\theta = 0$

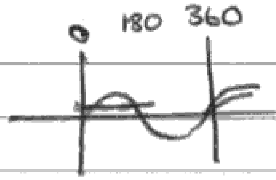
$k = 2$

Leave blank

Question 6 continued

b) $2\sin^2\theta - \sin\theta = 0$

$\sin\theta(2\sin\theta - 1) = 0$



$\sin\theta = 0$ $\quad\quad$ $\sin\theta = \frac{1}{2}$

$\theta = 0, 180$ $\quad\quad$ $\theta = 30, 150$

$\theta = 0, 30, 150, 180$

Turn over

596775/19

P 4 1 4 8 6 A 0 1 9 2 8

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 11 marks)

11

Leave blank

7. $$h(x) = \frac{2}{x+2} + \frac{4}{x^2+5} - \frac{18}{(x^2+5)(x+2)}, \qquad x \geqslant 0$$

(a) Show that $h(x) = \dfrac{2x}{x^2+5}$ **(4)**

1 Q07aM1
1 Q07aA1
1 Q07aM2
1 Q07aA2

(b) Hence, or otherwise, find $h'(x)$ in its simplest form. **(3)**

1 Q07bM
1 Q07bA1
1 Q07bA2

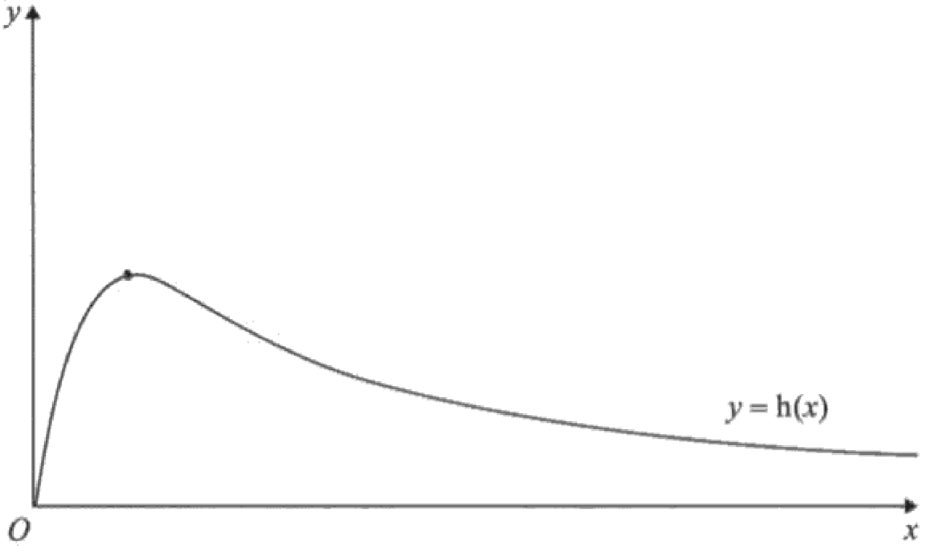


Figure 2

Figure 2 shows a graph of the curve with equation $y = h(x)$.

(c) Calculate the range of $h(x)$. **(5)**

1 Q07cM1
1 Q07cA1
1 Q07cM2
1 Q07cA2
0 Q07cA3

a) $h(x) = \dfrac{2}{x+2} + \dfrac{4}{x^2+5} - \dfrac{18}{(x^2+5)(x+2)}$

$h(x) = \dfrac{2(x^2+5) + 4(x+2) - 18}{(x^2+5)(x+2)}$

$= \dfrac{2x^2 + 10 + 4x + 8 - 18}{(x^2+5)(x+2)}$

$= \dfrac{2x^2 + 4x}{(x^2+5)(x+2)} = \dfrac{2x(x+2)}{(x^2+5)(x+2)}$

$= \dfrac{2x}{x^2+5}$

Leave blank

Question 7 continued

b) $h(x) = 2x(x^2+5)^{-1}$

$h'(x) = 2(x^2+5)^{-1} - 2x \times 2x(x^2+5)^{-2}$

$= \dfrac{2(x^2+5) - 4x^2}{(x^2+5)^2}$

$= \dfrac{2x^2+10-4x^2}{(x^2+5)^2} = \dfrac{-2x^2+10}{(x^2+5)^2} = \dfrac{10-2x^2}{(x^2+5)^2}$

$= \dfrac{2(5-x^2)}{(x^2+5)^2}$

c) $y \geq 0$

$0 = \dfrac{10-2x^2}{(x^2+5)^2}$ $\quad 10-2x^2=0$

$2x^2 = 10$

$x^2 = 5$

$x = \sqrt{5} \simeq 2.236067977$

for stationary point on graph (i.e maximum)

$h(\sqrt{5}) = \dfrac{2\sqrt{5}}{5+5} = \dfrac{\sqrt{5}}{5} \simeq 0.4472135955$

$0 < h(x) \leq \dfrac{\sqrt{5}}{5} \simeq 0.472135955$

596775/23

P 4 1 4 8 6 A 0 2 3 2 8

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Q7

11

(Total 12 marks)

Leave blank

8. The value of Bob's car can be calculated from the formula

$$V = 17000e^{-0.25t} + 2000e^{-0.5t} + 500$$

where V is the value of the car in pounds (£) and t is the age in $\boxed{\text{years}}$

(a) Find the value of the car when $t = 0$

(1)

(b) Calculate the exact value of t when $V = 9500$

(4)

(c) Find the rate at which the value of the car is decreasing at the instant when $t = 8$. Give your answer in pounds per year to the nearest pound.

(4)

1 Q08aB
1 Q08bM1
1 Q08bM2
1 Q08bA1
1 Q08bA2
1 Q08cM1
1 Q08cA1
1 Q08cM2
1 Q08cA2

$V = (17000 \times e^0) + (2000 \times e^0) + 500$

a) $V = 17000 + 2000 + 500$

$= £19500$

b) $9500 = 17000e^{-0.25t} + 2000e^{-0.5t} + 500$

$9000 = 17000e^{-0.25t} + 2000e^{-0.5t}$

$9 = 17e^{-0.25t} + 2e^{-0.5t}$

$9 = 17e^{-0.25t} + 2(e^{-0.25t})^2$

let $e^{-0.25t} = u$

$9 = 17u + 2u^2$

$0 = 2u^2 + 17u - 9$

$0 = (2u - 1)(u + 9)$

$u = -9$ ✗

$u = \frac{1}{2}$

$e^{-0.25t} = \frac{1}{2}$

$-0.25t = \ln \frac{1}{2}$

$t = -4\ln \frac{1}{2}$

596775/26

P 4 1 4 8 6 A 0 2 6 2 8

Question 8 continued

c) $\cancel{\frac{dV}{dt} = -0.25 \times 17000 e^{0.25t}}$

$\frac{dV}{dt} = (-0.25 \times 17000)e^{-0.25t} - (0.5 \times 2000)e^{-0.5t}$

$= -4250 e^{-0.25t} - 1000 e^{-0.5t}$

$t = 8$

$\frac{dV}{dt} = -4250 e^{-(0.25 \times 8)} - 1000 e^{-(0.5 \times 8)}$

$= -4250 e^{-2} - 1000 e^{-4}$

$= -575.1749538 - 18.3156389 =$

$-593.4905927 \approx$

$-£593$ per year

Leave blank

Leave blank

Question 8 continued

Q8

9

(Total 9 marks)

TOTAL FOR PAPER: 75 MARKS

END