Number – Place value and Number lines

EZY MATHS

The position of each digit is important as it carries a specific value.

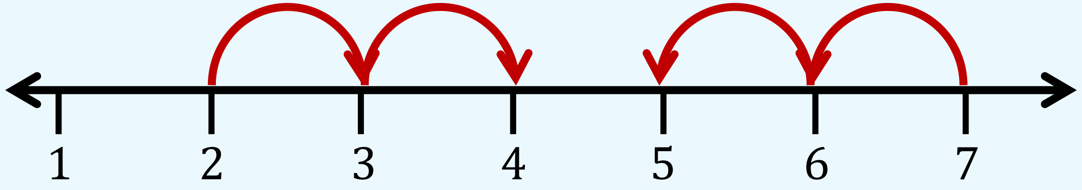
Line numbers up according to place value when calculating

Each place to the left is 10 × larger

654321.1234

- Hundred Thousands
- Ten Thousands
- Thousands
- Hundreds
- Tens
- Units
- Tenths
- Hundredths
- Thousandths
- Ten Thousandths

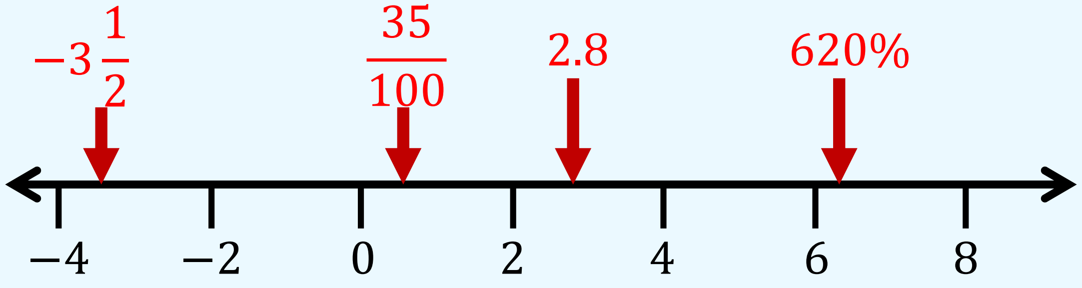
Number lines can be used to count up or down



Markings are the same distance apart each time

An increase means moving to the right, a decrease means moving to the left.

Useful for ordering numbers of different forms



Number – Rounding

EZY MATHS

ROUND DOWN 0,1,2,3,4, **Rules of rounding** 5,6,7,8,9, ROUND UP

Place Value	Decimal Places	Significant Figures
Thousands Hundreds Tens Units	Count Right from the Decimal Point	Count Right from first non-zero Digit
	1 2 3 4	1 2 3 4 5 6
14672	12.5298	325484
To the nearest ten	To 1 decimal place	To 1 significant figure
14670	12.5	300000
To the nearest hundred	To 2 decimal places	To 2 significant figures
14700	12.53	330000
To the nearest thousand	To 3 decimal places	To 3 significant figures
15000	12.530	325000

Be careful when rounding up a nine (A double rounding will occur)

Number – Addition and Subtraction

EZY MATHS

Addition | Subtraction

Chunking method

Add/Subtract according to place value

$436 + 58$

$$\boxed{\begin{array}{r} 430 + 50 = 480 \\ 6 + 8 = 14 \end{array}}$$

$436 + 58 = 494$

$563 - 237$

$$\boxed{\begin{array}{r} 563 - 200 = 363 \\ 363 - 30 = 333 \\ 333 - 7 = 326 \end{array}}$$

$563 - 237 = 326$

Column method

Line up numbers according to place value

$436 + 58$

$$\begin{array}{r} 436 \\ +\ 058 \\ \hline 494 \\ \hline \cancel{1} \end{array}$$

Carry over if column is greater than 10

$563 - 237$

$$\begin{array}{r} {}^{5\,1} \\ 5\cancel{6}3 \\ -\ 237 \\ \hline 326 \\ \hline \end{array}$$

Borrow from next column if too small

Addition and subtraction of positive and negative numbers

Filling the bath analogy

$+ \quad + \quad = \uparrow$

$- \quad - \quad = \uparrow$

$+ \quad - \quad = \downarrow$

$- \quad + \quad = \downarrow$

Number – Multiplication and Division

EZY MATHS

Multiplication

Column method

523×76

Line up numbers according to place value

$$\begin{array}{r} 523 \\ \times\ \ 76 \\ \hline 3138 \\ +\ 36610 \\ \hline 39748 \end{array}$$

Multiply by units first then tens etc.
Don't forget to add zeros
Add using column addition

Box/Grid method

523×76

Separate numbers according to place value

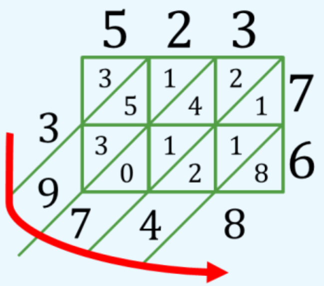
	500	20	3
70	35000	1400	210
6	3000	120	18

Numbers first then zeros
Double check calculations
Add using column addition

Chinese

523×76

Tens in top half
Units in bottom half



Starting from the right
Add the diagonals.
Carry over onto the next diagonal if you need to

Division

Bus shelter method

$958 \div 8$

$$\begin{array}{r|l} & 1\ 1\ 9.7\ 5 \\ \hline 8 & 9\ {}^{1}5\ {}^{7}8.{}^{6}0\ {}^{4}0\ 0 \end{array}$$

Know times tables
Remainders carried over
Add zeros when numbers run out.

Double decker division

$2016 \div 12$

$$\begin{array}{r|l} & 0\ 1\ 6\ 8 \\ \hline 6 & \cancel{1}\ {}^{1}0\ {}^{4}0\ {}^{4}8 \\ \hline 2 & 2\ 0\ {}^{1}\cancel{1}\ 6 \end{array}$$

Split into factors
Divide by one factor
Divide answer by second factor

Negative numbers

$+ \times/\div + = \textit{Positive answer}$

$- \times/\div - = \textit{Positive answer}$

Signs the same = Positive answer

$+ \times/\div - = \textit{Negative answer}$

$- \times/\div + = \textit{Negative answer}$

Signs different = Negative answer

Number – BIDMAS

EZY MATHS

Brackets
Indices
Division
Multiplication
Addition
Subtraction

ORDER

Follow the order of BIDMAS to calculate

If you can do nothing for a given step, move on to the next

Example 1

$$6 + 3 \times 4$$

Multiplication first

$$= 6 + \boxed{3 \times 4}$$

Addition second

$$= \boxed{6 + 12}$$

$$= \underline{\underline{18}}$$

Example 2

$$3 + 8 \times (4 + 6) \div 5 - 2$$

Brackets first

$$= 3 + 8 \times \boxed{(4 + 6)} \div 5 - 2$$

Multiplication and Division second

$$= 3 + \boxed{8 \times 10 \div 5} - 2$$

Addition and Subtraction last

$$= \boxed{3 + 16 - 2}$$

$$= \underline{\underline{17}}$$

Example 3

$$(9 - 3 \times 2)^2 \div (10 \div 5)$$

$$= \boxed{(9 - 3 \times 2)}^2 \div \boxed{(10 \div 5)}$$

$$= \boxed{(3)^2} \div (2)$$

$$= \boxed{9 \div (2)}$$

$$= \underline{\underline{4.5}}$$

Follow the order of operations within brackets

Number – Prime Numbers, Factors, Multiples

EZY MATHS

Prime numbers

A Number is Prime if it has exactly 2 factors: 1 and itself

No other number can divide into it exactly

1 is not a prime number

2 is the only even prime number

Prime numbers up to 50

$2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47$

Really useful for Prime Factor Decomposition, HCF and LCM

Factors

The factors of a number are the numbers which divide into it exactly

No remainder when divided by a factor

Factor Pairs

1 is a factor of all whole numbers

Factors of 24

1×24

2×12

3×8

4×6

$1, 2, 3, 4, 6, 8, 12, 24$

Multiples

A number that features in the times table of another number

The product of two integers will produce a multiple

Close link to factors

Times table knowledge important

Multiples of 9

$9, 18, 27, 36, 45, 54, 63, 72, 81, 90 \ldots$

Multiples of 7 between 30 and 60

$\cancel{7}, \cancel{14}, \cancel{21}, \cancel{28}, \boxed{35, 42, 49, 56}, \cancel{63}, \cancel{70}$

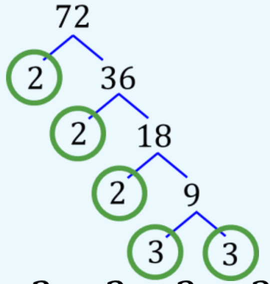
Number – Prime Factor Decomposition, HCF, LCM

EZY MATHS

Prime Factor Decomposition

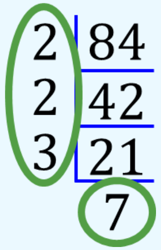
Break numbers down into prime factors

Tree method and ladder method



$72 = 2 \times 2 \times 2 \times 3 \times 3$

$72 = 2^3 \times 3^2$



$84 = 2 \times 2 \times 3 \times 7$

$84 = 2^2 \times 3 \times 7$

Divide by a prime

Multiply Primes

Write in index form

HCF

Highest common factor

The largest number that divides into two or more numbers

Use long format of Prime Factor Decomposition

$HCF\ of\ 48\ and\ 120$

$48 = 2 \times 2 \times 2 \times 2 \times 3$
$180 = 2 \times 2 \times 3 \times 3 \times 5$

$2 \times 2 \times 3 = \underline{12}$

$HCF\ of\ 84\ and\ 980$

$84 = 2 \times 2 \times 3 \times 7$
$980 = 2 \times 2 \times 5 \times 7 \times 7$

$2 \times 2 \times 7 = \underline{28}$

Prime factor decomposition

Identify shared factors

Multiply values

LCM

Lowest common multiple

The smallest number that occurs in the times table of two or more numbers.

$LCM\ of\ 6\ and\ 45$

$6 = 2 \times 3$
$45 = 3 \times 3 \times 5$

$2 \times 3 \times 3 \times 5 = \underline{90}$

$LCM\ of\ 48\ and\ 120$

$48 = 2^4 \times 3$
$180 = 2^2 \times 3^2 \times 5$

$2^4 \times 3^2 \times 5 = \underline{720}$

Multiply together all prime factors apart from duplicates

In index form: Multiply Highest Power of each prime

Number – Powers and Roots

EZY MATHS

Positive powers

Numbers multiplied by themselves

Index notation used to write easily

$$4 \times 4 \times 4 = \underbrace{4}_{Base}^{\overbrace{3}^{Power}}$$

$$8^4 = 8 \times 8 \times 8 \times 8$$

$$\left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2} = \frac{2 \times 2}{3 \times 3} = \frac{4}{9}$$

Anything to the power of 1 is just itself

$5^1 = 5 \qquad 28^1 = 28$

Know square numbers and cube numbers

Negative powers

Negative powers are fractions

5^{-2}	$^1/_{25}$
5^{-1}	$^1/_5$
5^0	1
5^1	5
5^2	$5 \times 5 = 25$
5^3	$5 \times 5 \times 5 = 125$

÷ 5
÷ 5
÷ 5
÷ 5
÷ 5

A negative power means "Take the reciprocal and make the power positive"

$$\left(\frac{4}{5}\right)^{-2} = \left(\frac{5}{4}\right)^2 = \frac{5^2}{4^2} = \frac{25}{16}$$

Find Reciprocal

Apply Positive Power

Apply top and bottom

$x^{-n} \Rightarrow {}^1/_{x^n}$

Roots

Roots are the inverse operations to powers

$$7 \xrightarrow{2} 49 \qquad 49 \xrightarrow{\sqrt{}} 7$$

$\sqrt[3]{Cube\ root}$

$\sqrt[4]{Fourth\ root}$

If you know square numbers and cube numbers,
you can find their roots

Fractional powers

Numerator and denominator are important

2-Stage Powers:

$$x^{\frac{m}{n}} = \left(\sqrt[n]{x}\right)^m$$

Root by denominator first

Then power of numerator

$$16^{\frac{3}{2}} = \left(\sqrt[2]{16}\right)^3 = 64$$

Negative Fractional Powers: Apply reciprocal first!

$$\left(\frac{27}{64}\right)^{-\frac{2}{3}} = \left(\frac{64}{27}\right)^{\frac{2}{3}} = \left(\frac{4}{3}\right)^2$$

$$= \frac{16}{9}$$

Algebra – Notation and Collecting like terms

EZY MATHS

Algebraic notation

Algebra is the language we use to communicate mathematical information

Letters used to represent values are known as variables.

Notation creates shortcuts

$a \times b$ becomes ab

coefficient

$x + x + x + x$ becomes $\boxed{4}x$

$y \times y$ becomes y^2

$$\underbrace{6xy - 5\frac{a}{b} + 21x^4}_{\text{Expression}}$$

$$\underbrace{6xy}\ \underbrace{- 5\frac{a}{b}}\ \underbrace{+ 21x^4}$$

Terms

Same rules of BIDMAS applies to Algebra

Collecting like terms

Collecting like terms enables us to simplify expressions making them easier to use

Terms that contain the exact same variable can be classed as 'like' terms and be simplified

Watch out for the sign before each term

$$\boxed{5x}\boxed{+6y}\boxed{-2x}\boxed{-5y} = 3x + y$$

$$\boxed{5xy}\boxed{+3x}\boxed{-2xy}\boxed{+4y} = 3xy + 3x + 4y$$

$$\boxed{2x^2}\boxed{+3x}\boxed{+5x^2}\boxed{-5x} = 7x^2 - 2x$$

Identify like terms

Use coefficients to collect like terms

First step in many problems involving Algebra

Algebra – Formulae

EZY MATHS

Introduction

Explains how to calculate the value of a variable

"The price of a taxi fare in Manchester depends on the distance driven. Each fare is charged a flat fee of £2 and then £3 for each mile driven."

$$C = 2 + 3M$$

For any given trip, can easily work out the cost of a taxi

Area of circle formula

$$\underbrace{A}_{Subject} = \pi r^2$$

Substitution

Replace letters in the formula with numbers you are given

"The perimeter of a square is 4 times the length of its sides"

$$P = 4l$$

What is perimeter of a square with side length 5cm?

$l = 5$

$$P = 4(5)$$

$$P = 20cm$$

Identify the formula and the values to substitute in.

Substitute values in using brackets

Carry out calculation remembering BIDMAS

Changing the subject

Often it is useful to re-arrange a formula to make a different variable the subject

Make l the subject of the formula

$$P = 4l \Rightarrow \frac{P}{4} = l$$

$\div 4 \quad \div 4$

Use inverse operations

$$y = \frac{18t - 3}{p}$$

Make t the subject

$\times p \quad +3 \quad \div 18$

$$t = \frac{py + 3}{18}$$

Sometimes a variable will appear more than once in a formula

Make x the subject of the formula:

$$a = 5x + xy \rightarrow a = x(5 + y)$$

Factorise first

$$\frac{a}{5 + y} = x$$

Algebra – Laws of indices

EZY MATHS

Basic Laws of Indices

Special indices to consider

$x^1 = x$	Anything to the power 1 = itself
$x^0 = 1$	Anything to the power 0 = 1
$1^x = 1$	1 to the power of anything = 1

These laws can be applied if the bases are the same

$x^a \times x^b = x^{a+b}$ $z^3 \times z^7 = z^{10}$	When multiplying powers with the same base – Add the powers
$x^a \div x^b = x^{a-b}$ $s^2 \div s^5 = s^{-3}$	When dividing powers with the same base – Subtract the powers
$(x^a)^b = x^{a\times b}$ $(e^4)^3 = e^{12}$	When raising the power (brackets) – Multiply the powers

Advanced Laws of Indices

Negative Indices

$$x^{-n} \Rightarrow 1/x^n$$

Find Reciprocal

Apply Positive Power

Apply top and bottom

$$z^{-3} = \left(\frac{1}{z}\right)^3 = \frac{1}{z^3} \qquad 6^{-2} = \left(\frac{1}{6}\right)^2 = \frac{1}{36}$$

Fractional Indices

$$x^{\frac{m}{n}} = \left(\sqrt[n]{x}\right)^m$$

Root by denominator first

Then power of numerator

$$x^{\frac{1}{2}} = \sqrt{x} \qquad x^{\frac{1}{3}} = \sqrt[3]{x} \qquad x^{\frac{1}{4}} = \sqrt[4]{x} \qquad x^{\frac{2}{3}} = \left(\sqrt[3]{x}\right)^2$$

$$64^{\frac{2}{3}} = \left(\sqrt[3]{64}\right)^2 = (4)^2 = 16$$

Negative Fractional Indices

$$x^{-\frac{a}{b}} = \frac{1}{\left(\sqrt[b]{x}\right)^a}$$

Negative Fractional Powers: Apply reciprocal first!

$$9^{-\frac{3}{2}} = \frac{1}{9^{\frac{3}{2}}} = \frac{1}{\left(\sqrt[2]{9}\right)^3} = \frac{1}{(3)^3}$$

Algebra – Expanding brackets

EZY MATHS

Single Brackets

Multiply terms outside by all terms inside

$$10\,(x + y + 4) = 10x + 10y + 40$$

$$3x\,(6x - 2) = 18x^2 - 6x$$

Expanding brackets often the first step in simplifying algebra

$$2\,(x + 3y)\underbrace{-\,7}_{\text{Include sign in multiplication}}(2x - y) = 2x + 6y - 14x + 7y$$

$$= -12x + 13y$$

Double brackets

FOIL Method

F	First terms	x^2
O	Outside terms	$11x$
I	Inside terms	$4x$
L	Last terms	44

$$(x + 4)(x + 11)$$

Then simplify

$$x^2 + 15x + 44$$

Multiply each term in first bracket by each term in second

Grid Method

$$(x + 4)(x - 3)$$

Split brackets up around grid

Multiply each term in the grid

	x	$+4$
x	x^2	$4x$
-3	$-3x$	-12

Then simplify

$$x^2 + x - 12$$

Algebra – Factorising

EZY MATHS

The process where an expression has common factors removed and brackets introduced

Highest common factor method

Look at whole expression, identify HCF and divide out

$12x - 6y + 3z$ HCF $= 3$

$3(4x - 2y + z)$

$ax + aby + 4az$ HCF $= a$

$a(x + by + 4z)$

HCF $= 6xy$

$18x^2y + 6xy - 24xy^2z$

$6xy(3x + 1 - 4yz)$

Look at each term separately, divide numbers first then the algebraic terms

Ladder method

Divide out simple common factors repeatedly

$12x - 6y + 3z$

$3(4x - 2y + z)$

$$\begin{array}{r|l} 3 & 12x - 6y + 3z \\ \hline & 4x - 2y + 1z \end{array}$$

$ax + aby + 4az$

$a(x + by + 4z)$

$$\begin{array}{r|l} a & ax + aby + 4az \\ \hline & x + by + 4z \end{array}$$

$$\begin{array}{r|l} 2 & 18x^2y + 6xy - 24xy^2z \\ \hline 3 & 9x^2y + 3xy - 12xy^2z \\ \hline x & 3x^2y + 1xy - 4xy^2z \\ \hline y & 3xy + 1y - 4y^2z \\ \hline & 3x + 1 - 4yz \end{array}$$

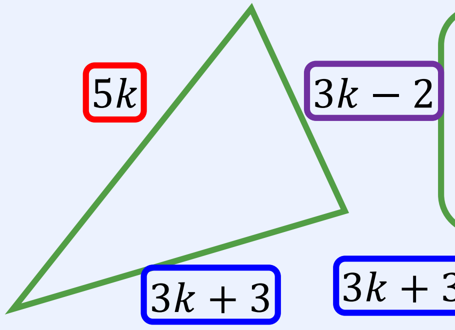
$6xy(3x + 1 - 4yz)$

Algebra – Linear Equations

EZY MATHS

A linear equation has the unknown variables as a power of one in the form $ax + b = 0$

Creating equations



The perimeter of the triangle is 67cm. Form an equation in x.

$$3k + 3 + 3k - 2 + 5k = 67$$

Once the equation has been created, it can be solved using a balance method or inverse method.

Derived from the word 'Equal' → This means that the = symbol is involved → The left side has the same value as the right side.

Left Side = Right Side

The equation is balanced

Solving linear equations

General 4 step process

ORDER:

1. Expand brackets and simplify (collect like terms)
2. If x is on both sides, eliminate smallest value
3. Eliminate excess number
4. Divide and solve for x

$$3(x + 1) = 2(x + 2)$$

$$3x + 3 = 2x + 4$$
$$-2x \qquad -2x$$

$$x + 3 = 4$$
$$-3 \qquad -3$$

$$x = 1$$

Advanced equations

Equations where fractions are involved

Fractions are divisions and can be eliminated by multiplying

$$\frac{x}{2} = 5 \qquad x = 10$$
$$\times 2 \qquad \times 2$$

Remove variable from denominator

$$\frac{2y}{(3 - y)} = 4 \Rightarrow 2y = 4(3 - y)$$
$$\times (3 - y) \qquad \times (3 - y)$$

Cross-multiplying allows us to move terms in a fraction from one side of an equation to the other

$$\frac{x + 1}{3} = \frac{x}{2} \Rightarrow 2(x + 1) = 3x$$

Algebra – Quadratics

EZY MATHS

An equation where the highest power of the variable is 2

$$ax^2 + bx + c$$

Factorising $a = 1$ Quadratics

Aim: Convert quadratic into double brackets

$$(x \pm \quad)(x \pm \quad)$$

Sum and product rule

$$x^2 + \boxed{b}x + \boxed{c}$$

Add to make b

Multiply to make c

Establish Signs

If c is positive — Signs are same

$x^2 + 5x \boxed{+ 6}$ — $(x + 3)(x + 2)$

If c is negative — Signs are different

$x^2 + 5x \boxed{- 6}$ — $(x + 6)(x - 1)$

Example

$$x^2 \boxed{- 7}x \boxed{+ 12}$$

Positive c → Signs Same

Negative b → Both Minus

$(x - \quad)(x - \quad)$

Factors of 12

12×1

6×2

4×3

Which pair make 7?

$(x - 4)(x - 3)$

Factorising $a \neq 1$ Quadratics

$$(?\,x \pm \quad)(?\,x \pm \quad)$$

Factors of a to find possible values

$$5x^2 - 14x - 3$$

$$= (5x \pm \quad)(1x \pm \quad)$$

$6x^2 + x - 2 =$ $(3x \pm \quad)(2x \pm \quad)$ OR $(6x \pm \quad)(1x \pm \quad)$

Then find factors of c and see which satisfy b

Difference of Two Squares (DOTS)

$$a^2 - b^2 = (a + b)(a - b)$$

$$x^2 - 81 = (x + 9)(x - 9)$$

$$4y^2 - 25 = (2y + 5)(2y - 5)$$

Algebra – Sequences

EZY MATHS

Introduction

A series of numbers, patterns that follow a given rule

3 8 13 18 … 23 28 33

+5 +5 +5 +5 +5 +5

Each number in the sequence is known as a 'term'

Identify what is happening between each term to generate the rule

General rule is known as n^{th} term.

Triangular Numbers	Square Numbers	Cube Numbers
1	1	1
3	4	8
6	9	27
10	16	64
15	25	125

Arithmetic progressions

Finding the n^{th} term rule

+3 +3 +3

Sequence	5	8	11	13
Times table	3	6	9	12
Extra bit	+2	+2	+2	+2

$$3n + 2$$

Find the common difference (this will be your n coefficient)

Write times table underneath sequence (of your n coefficient)

Sequence minus times table (this is your extra bit)

General formula

$$n^{th}\ term = 1^{st}\ term + common\ difference \times (n-1)$$

Quadratic sequences

Has the form $an^2 + bn + c$.
A second layer difference

5 9 15 23 33 …

+4 +6 +8 +10

+2 +2 +2

Halve 2nd layer difference for n^2 coefficient

$1n^2 \boxed{+ bn + c}$ → Find linear sequence

Sequence	5	9	15	23
$1n^2$	1	4	9	16
Subtract	4	5	6	7

n^{th} term rule of this $= n + 3$

$$1n^2 + 1n + 3$$

Statistics – Mean, Median, Mode and Range

EZY MATHS

Mean

$$\text{Mean} = \frac{\textit{Total of all values}}{\textit{number of values}}$$

$3, 3, 4, 5, 5, 8, 9, 15$

$$\text{Mean} = \frac{52}{8} = 6.5$$

Collect it all together and share it out evenly

Using the mean to find the total amount

$$\textit{Mean} \times \textit{Number of values}$$

Ezytown FC have scored an average of 3.8 goals per game in their last 15 matches. How many goals have they scored?

$3.8 \times 15 = 57 \textit{goals}$

Median

Median = Middle value (Numbers written in order)

$\cancel{3}, \cancel{3}, \cancel{4}, \boxed{5, 5}, \cancel{8}, \cancel{9}, \cancel{15}$

$$\text{Median} = 5$$

Finds the middle value

Use of formula to find location of median

$$\textit{Location} = \frac{n+1}{2}$$

The median of 45 values would be the 23rd number when written in order

$$\frac{45+1}{2} = 23$$

Mode

Mode = Most common value/item

$\boxed{3, 3}, 4, \boxed{5, 5}, 8, 9, 15$

$$\text{Mode} = 3 \textit{ and } 5$$

Average usually used for qualitative data

Occurrence of no mode

If every value appears equally, there is no mode

$1, 1, 3, 3, 7, 7$

Each value appears twice so there is no mode

Range

Range = Largest - Smallest

$3, 3, 4, 5, 5, 8, 9, 15$

$$\text{Range} = 15 - 3 = 12$$

Reveals how close/far apart the values are

Interpreting measures of spread

The Smaller the range, the closer and more 'consistent' the values are.

The Larger the range, the more varied and more 'inconsistent' the values are.

Statistics – Averages from a frequency table

EZY MATHS

Cars		Frequency		fx
0	×	4	→	0
1	×	11	→	11
2	×	12	→	24
3	×	7	→	21
4	×	6	→	24
Sum		40		80

This column is created by multiplying the frequency (f) by the number in the category (x)

This enables us to find out the total amount which is needed for the mean

Mean

Mean = $\frac{\textit{Total of all values}}{\textit{number of values}}$

Mean = $\frac{\textit{Total of } fx \textit{ column}}{\textit{Total frequency}}$

$$\text{Mean} = \frac{80}{40} = 2$$

Mode

Mode = Most common value/item

The category with the highest frequency

2

Range

Range = Largest - Smallest

$$4 - 0 = 4$$

Median

Median = Middle value (Numbers written in order)

$$Location = \frac{n+1}{2}$$

$$Location = \frac{40+1}{2} = 20.5$$

The median lies between 20th and 21st value

Cars	Frequency	
0	4	4
1	+ 11	15
2	+ 12	27
3	7	
4	6	
Sum	40	

Add down the frequency column. When location value has been exceeded, that is the group where the median lies.

$$\text{Median} = 2$$

Statistics – Averages from a grouped frequency table

EZY MATHS

Weight	Frequency	Mid-point	fx
$0kg \le x \le 2kg$	12	1	12
$2kg < x \le 4kg$	3	3	9
$4kg < x \le 6kg$	9	5	45
$6kg < x \le 8kg$	10	7	70
Sum of Freq	34	Sum of fx	136

Two extra columns are created, the midpoint of each group and the fx column

This enables us to find out an **estimate** for the mean

Mean

$$\text{Mean} = \frac{Total\ of\ all\ values}{number\ of\ values}$$

$$\text{Mean} = \frac{Total\ of\ fx\ column}{Total\ frequency}$$

$$\text{Mean} = \frac{136}{34} = 4kg$$

Median

Median = Middle value
(Numbers written in order)

$$Location = \frac{n+1}{2}$$

$$Location = \frac{34+1}{2} = 17.5$$

The median lies between 17th and 18th value

Weight	Frequency	
$0kg \le x \le 2kg$	12	12
$2kg < x \le 4kg$	3	15
$4kg < x \le 6kg$	9	22
$6kg < x \le 8kg$	10	
Sum of Freq	34	

Add down the frequency column. When location value has been exceeded, that is the group where the median lies.

Median class $=$

$$4kg < x \le 6kg$$

Mode

Mode = Most common value/item

The category with the highest frequency

$$0kg \le x \le 2kg$$

Range

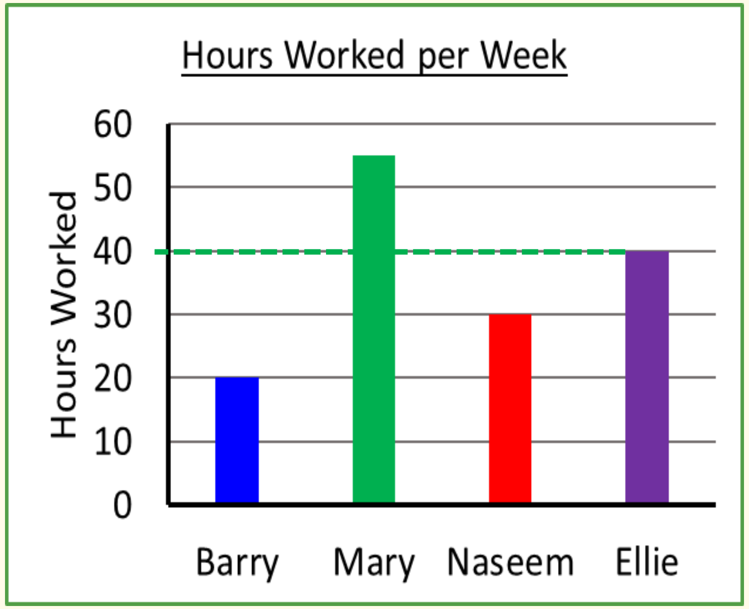
Range = Largest - Smallest

$$8 - 0 = 8kg$$

Statistics – Representing data

EZY MATHS

Bar Charts



Title

Axes

Scales

Labels

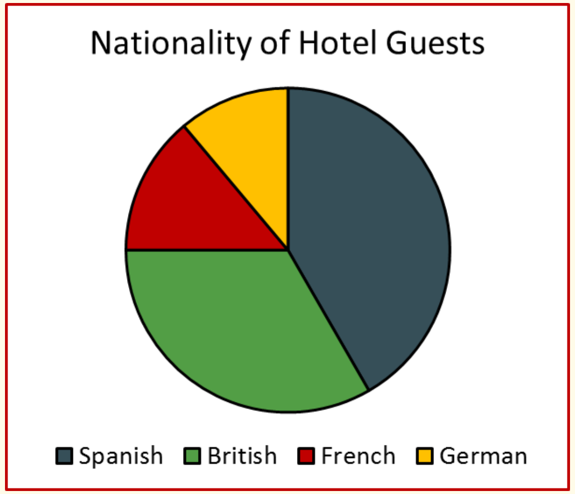
Bars
(equal width)
(equal gaps)

Read off bars
Ellie $=$ 40 hours

Pie Charts

Nationality	Guests	Angle
Spanish	30	$150°$
British	24	$120°$
French	10	$50°$
German	8	$40°$
Total	72	

Find degrees per value
$(360° \div total)$

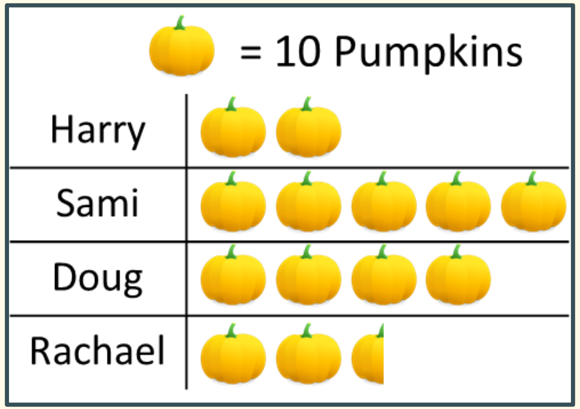


$$1\ Guest = \frac{360°}{72} = 5°$$

Pictograms

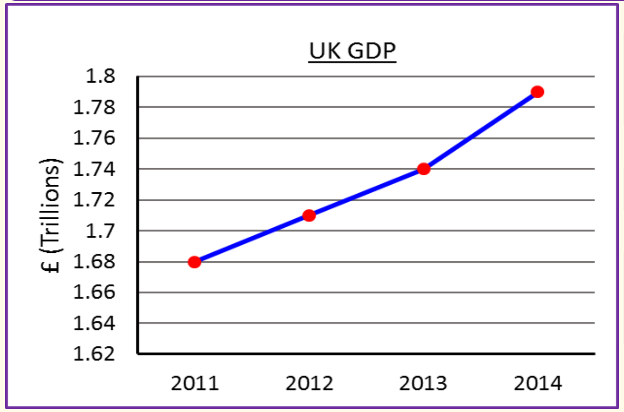
Using a symbol to represent certain amount

Farmer	Pumpkins
Harry	20
Sami	50
Doug	40
Rachael	25



Line Charts

Line charts are useful for displaying time series data



Data points within the lines are important

Lines visualise change

Extract required data carefully

Statistics – Cumulative Frequency tables and graphs

EZY MATHS

Cumulative Frequency tables

Running total (add up as we go down)

Time	Frequency	Cumulative Frequency
$45 \le x < 48$	3	3
$48 \le x < 50$	8	11
$50 \le x < 52$	16	27
$52 \le x < 55$	12	39
$55 \le x < 60$	11	50
$60 \le x < 70$	2	52

Add
Equals

The difference between cumulative frequency values will tell you the frequency

May be asked to calculate percentages

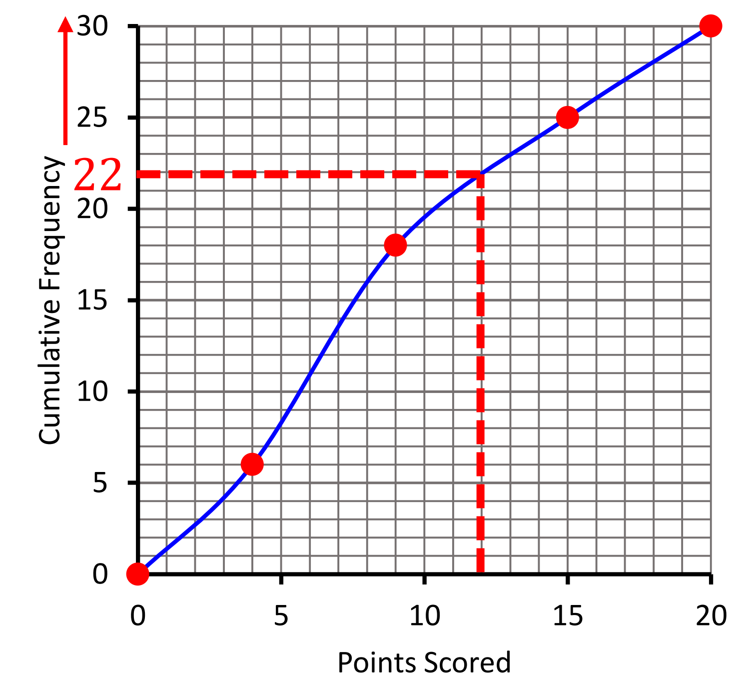
$$\% = \frac{Amount}{Total} \times 100$$

Cumulative Frequency graphs

Points	Cumulative Frequency
0 – 4	6
5 – 9	18
10 – 15	25
16 – 20	30

Always plot each point at the **end of the group**

Draw a smooth line through the points



How many games did the player score more than 12 points?

$30 - 22 = 8\ games$

What percentage of games does the player score less than 12 points?

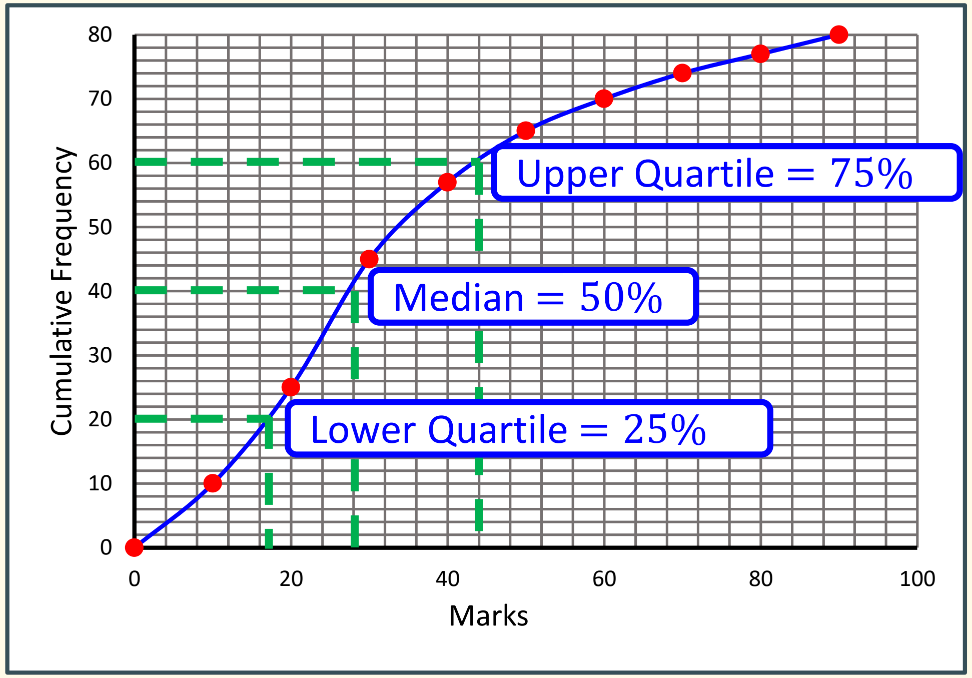
$$\frac{22}{30} \times 100 = 73\%$$

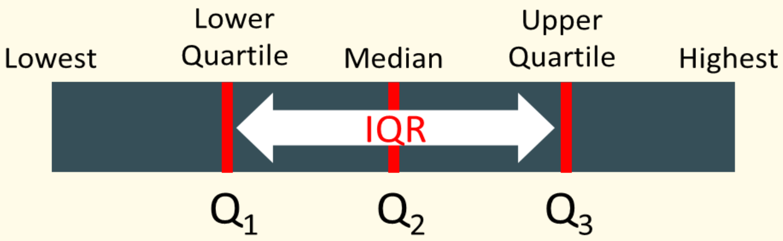
Statistics – Quartiles and box plots

EZY MATHS

Quartiles

Interested in specific points along the distribution





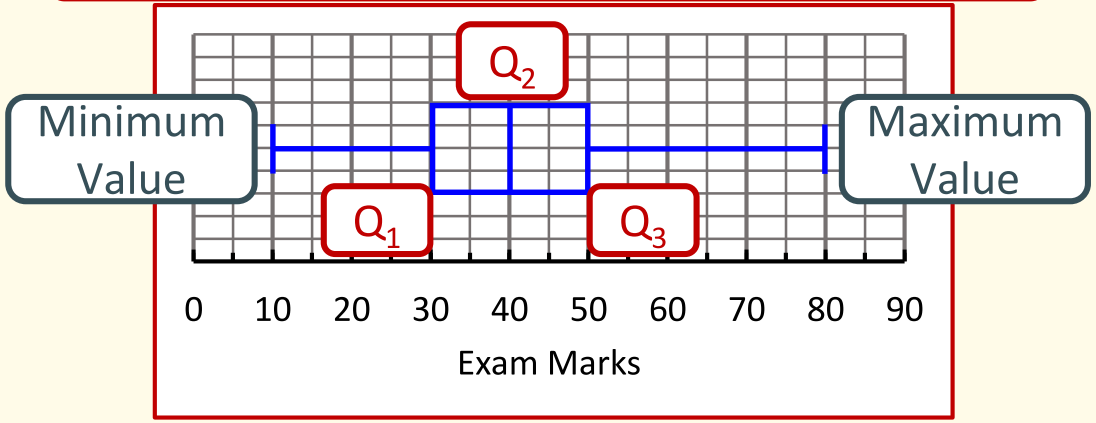
IQR = $Q_3 - Q_1$

The IQR provides a measure of the spread of the middle 50% of the data.

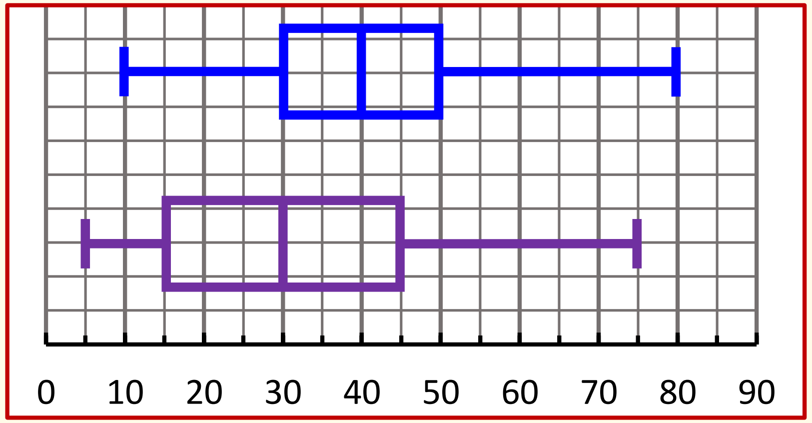
It is less affected by outliers than the range.

Box plots

Box plots allow us to visualise the spread of a dataset.



Also used to compare two or more different datasets



Look to compare medians, IQR and range

Statistics – Histograms

EZY MATHS

A special type of bar chart for grouped data

Frequency Density on the vertical axis

Bars can be different widths depending on the group size

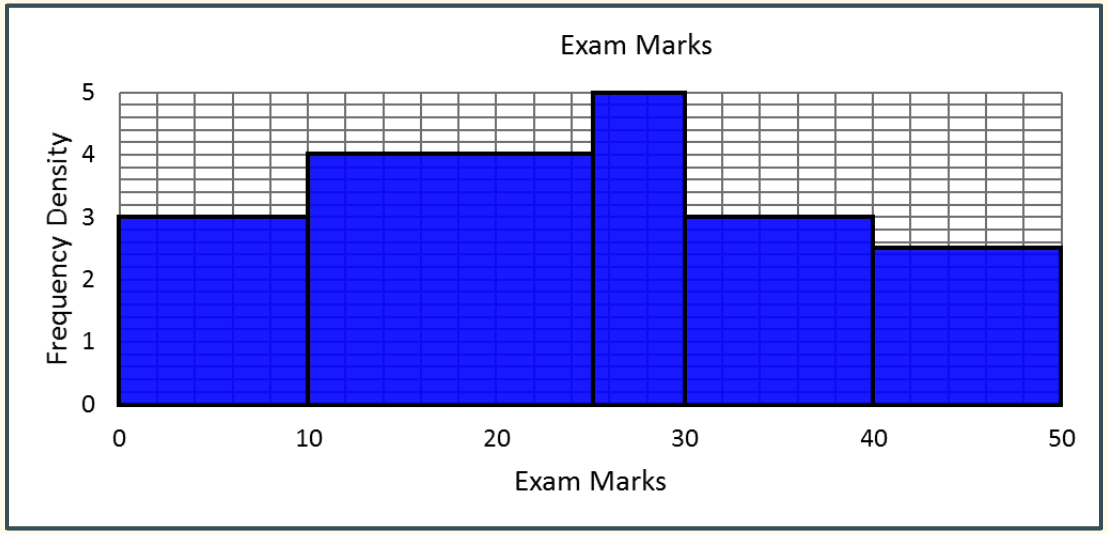
$$Frequency\ Density = \frac{Frequency}{Class\ Width}$$

Age	Frequency	Class Width	F.D.
$20 \le x < 25$	300	5	60
$25 \le x < 30$	150	5	30
$30 \le x < 40$	100	10	10
$40 \le x < 60$	100	20	5

Create the Frequency density column

Calculating Frequency from Histograms

$$Frequency = F.D. \times Class\ Width$$



Age	F.D.	Class Width	Frequency
$0 < x \le 10$	3	10	30
$10 < x \le 25$	4	15	60
$25 < x \le 30$	5	5	25
$30 < x \le 40$	3	10	30
$40 < x \le 50$	2.5	10	25

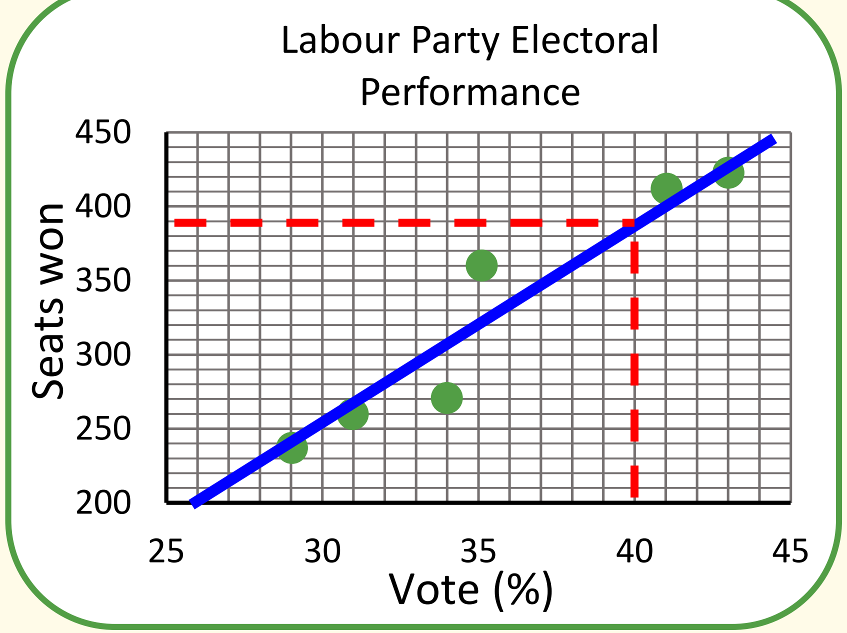
Statistics – Scatter Graphs

EZY MATHS

We use scatter graphs when we are interested in the relationship between two variables

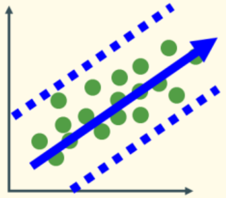
Points plotted like coordinates

Labour Party		
Year	Vote (%)	Seats
2015	31	232
2010	29	258
2005	35	356
2001	41	413
1997	43	418
1992	34	271

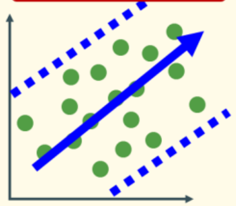


Positive Correlation

Strong



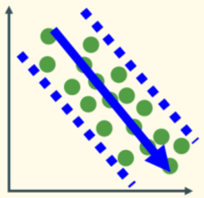
Weak



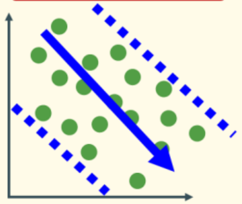
As one value increases, so does the other

Negative Correlation

Strong

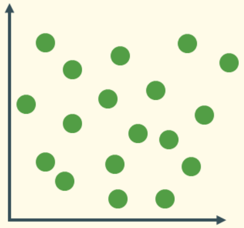


Weak



As one value increases, the other decreases

No Correlation



No link between the two variables

One of the main incentives for drawing lines of best fit is to make predictions

Place line of best fit through the middle of the data. (Ignore Outliers)

Predict values by reading off the line

$40\% = 390\ seats$

Interpolation → Predictions made within the dataset

Extrapolation → Predictions made outside of the dataset

Number – Fractions – Simplifying, Improper, Mixed

EZY MATHS

Simplifying

Divide both the numerator and denominator by the same value

Repeat the process until the fraction is in its simplest form

$\frac{x}{y}$ x → NUMERATOR, y → DENOMINATOR

Even = ÷ 2

Odd = ÷ 3,5,7 …

$$\frac{32}{80} \xrightarrow{\div 2} \frac{16}{40} \xrightarrow{\div 2} \frac{8}{20} \xrightarrow{\div 2} \frac{4}{10} \xrightarrow{\div 2} \frac{2}{5}$$

These are all equivalent fractions.

$$\frac{126}{72} \xrightarrow{\div 2} \frac{63}{36} \xrightarrow{\div 3} \frac{21}{12} \xrightarrow{\div 3} \frac{7}{4}$$

Improper Fractions

The numerator is larger than the denominator

Turning into a mixed number

$\frac{13}{5}$ Divide numerator by denominator to get whole number

2^{r3} Remainder forms **new** numerator

$2\frac{3}{5}$ Denominator remains the same

Mixed Number

The combination of a WHOLE number and a Fraction

Turning into an improper fraction

$7\frac{3}{8}$ Multiply whole number by denominator

$56 + 3$ Add on the numerator

$\frac{59}{8}$ Denominator remains the same

Don't forget to simplify your answers where necessary!

Useful skills for adding/subtracting/multiplying and dividing fractions

Number – Fractions – Addition and Subtraction

EZY MATHS

Adding or Subtracting fractions requires a common denominator

$$\frac{2}{9} \xrightarrow{+} \frac{5}{9} \Rightarrow \frac{7}{9}$$

When denominators are the same, simply add the numerators

$$\frac{7}{9} - \frac{1}{6}$$

$$\times 2 \qquad \times 3$$

When denominators are different, multiply the fractions

$$\frac{14}{18} \xrightarrow{-} \frac{3}{18} \Rightarrow \frac{11}{18}$$

Remember to simplify your answers

Adding/Subtracting Mixed Numbers

Method 1 – Deal with whole numbers and fractions separately

$$3\frac{1}{2} + 4\frac{1}{4} \Rightarrow \boxed{3+4} + \boxed{\frac{1}{2} + \frac{1}{4}} \Rightarrow = 7\frac{3}{4}$$

$$5\frac{2}{3} - 2\frac{1}{9} \Rightarrow \boxed{5-2} + \boxed{\frac{2}{3} - \frac{1}{9}} \Rightarrow = 3\frac{5}{9}$$

Method 2 – Convert to improper fractions first then calculate

$$6\frac{1}{5} - 4\frac{3}{4} \Rightarrow \frac{31}{5} - \frac{19}{4} \Rightarrow \frac{124}{20} - \frac{95}{20} \Rightarrow \frac{29}{20} = 1\frac{9}{20}$$

$$3\frac{1}{5} + 5\frac{9}{10} \Rightarrow \frac{16}{5} + \frac{59}{10} \Rightarrow \frac{32}{10} + \frac{59}{10} \Rightarrow \frac{91}{10} = 9\frac{1}{10}$$

Number – Fractions – Multiplying and Dividing

EZY MATHS

Multiplying

$$\frac{2}{3} \times \frac{5}{7} = \frac{10}{21}$$

Multiply across the top and bottom

Check to see if you can cross cancel

$$\frac{4}{15} \times \frac{3}{8} \Rightarrow \frac{\overset{1}{\cancel{4}}}{\underset{5}{\cancel{15}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{8}}} \Rightarrow \frac{1}{5} \times \frac{1}{2} \Rightarrow \frac{1}{10}$$

Convert if mixed numbers → Simplify vertically and diagonally → Multiply across top and bottom → Convert and simplify if required

Dividing

$$\frac{10}{3} \div \frac{2}{3} \xrightarrow{\text{Multiply by the Reciprocal}} \frac{10}{3} \times \frac{3}{2}$$

Finding the reciprocal of a fraction swaps the numerator and denominator

$$\frac{4}{7} \div \frac{6}{5} \Rightarrow \frac{\overset{2}{\cancel{4}}}{7} \times \frac{5}{\underset{3}{\cancel{6}}} \Rightarrow \frac{10}{21}$$

$$2\frac{1}{5} \div 3\frac{3}{10} \Rightarrow \frac{11}{5} \div \frac{33}{10} \Rightarrow \frac{\overset{1}{\cancel{11}}}{\underset{1}{\cancel{5}}} \times \frac{\overset{2}{\cancel{10}}}{\underset{3}{\cancel{33}}} = \frac{2}{3}$$

Convert mixed numbers → Find the reciprocal → Change sign → Simplify and Multiply

Number – Fractions – Converting Decimal to Fraction

EZY MATHS

Decimal -> Fraction Conversions you should know

Decimal	Fraction
0.5	$^1/_2$
$0.\dot{3}$	$^1/_3$
0.25	$^1/_4$
0.2	$^1/_5$
0.125	$^1/_8$
0.1	$^1/_{10}$

$0.3 \rightarrow {}^1/_{10} + {}^1/_{10} + {}^1/_{10} \longrightarrow {}^3/_{10}$

0.1	$^1/_{10}$

Know your place values

Place Decimal part over 10/100/1000 etc.

Simplify the Fraction

Put back whole numbers if you had them

Using place value

Hundredths $^x/_{100}$ Ten Thousandths $^x/_{10000}$

$$0.1234$$

Tenths $^x/_{10}$ Thousandths $^x/_{1000}$

Write 0.32 as a fraction in its simplest form

Hundredths $^x/_{100}$

$$0.32 \Rightarrow \frac{32}{100} \begin{matrix}\div 2\\ \div 2\end{matrix} \frac{16}{50} \begin{matrix}\div 2\\ \div 2\end{matrix} \boxed{\frac{8}{25}}$$

Number – Fractions – Converting Fraction to Decimal

EZY MATHS

Division method

Divide the numerator by the denominator.
Using Bus shelter division

$$\frac{1}{7} \Rightarrow \begin{array}{r} 0.1428 \\ 7\overline{)\cancel{1}.{}^{1}0{}^{3}0{}^{2}0{}^{6}0} \end{array} \Rightarrow 0.143$$

Equivalent Fraction method

Multiply or Divide the fractions so that they can be converted to decimals easily

$$\frac{13}{20} \xrightarrow{\times 5} \frac{65}{100} = 0.65$$

Mixed Numbers and Improper Fractions

Process does not change.

Try and work with mixed numbers where possible.

$$\frac{86}{25} \Rightarrow 3\frac{11}{25} \xrightarrow{\times 4} \frac{44}{100} = 0.44$$

$$3.44$$

Decimal -> Fraction Conversions you should know

Decimal	Fraction
0.5	$^{1}/_{2}$
$0.\dot{3}$	$^{1}/_{3}$
0.25	$^{1}/_{4}$
0.2	$^{1}/_{5}$
0.125	$^{1}/_{8}$
0.1	$^{1}/_{10}$

$^{3}/_{4} \rightarrow 0.25 + 0.25 + 0.25 \rightarrow 0.75$

0.25	$^{1}/_{4}$

Number – Fractions – Converting recurring decimals

EZY MATHS

What is a recurring decimal?

A decimal number that will after a certain point, repeat itself indefinitely.

Written with a little dot above the number/s

$0.\dot{6} \longrightarrow 0.6666666666666\ldots$

$0.21\dot{3} \longrightarrow 0.2133333333333\ldots$

$0.\dot{8}4\dot{1} \longrightarrow 0.841841841841\ldots$

A recurring decimal as a fraction

$0.\dot{x} \longrightarrow$ A single recurring digit will be a fraction over 9 $\quad \frac{x}{9}$

$0.\dot{x}\dot{y} \longrightarrow$ A double recurring digit will be a fraction over 99 $\quad \frac{xy}{99}$

$0.\dot{x}y\dot{z} \longrightarrow$ A triple recurring digit will be a fraction over 999 $\quad \frac{xyz}{999}$

More complex recurring decimals

Be in a position to eliminate the Decimal numbers

$$0.20\dot{5}$$

$$(\times 100)\ 0.20\dot{5} = x\ (\times 100)$$

Move recurring decimal up to the decimal point

$$100x = 20.\dot{5}$$

$(\times 10) \quad (\times 10)$

$$1000x = 205.\dot{5}$$

Be in a position to eliminate the Decimal numbers

$$1000x = 205.\cancel{\dot{5}}$$
$$-\ 100x = 20.\cancel{\dot{5}}$$
$$900x = 185$$

$$x = \frac{185}{900} = \boxed{\frac{37}{180}}$$

Create equation by labelling the decimal x

Move recurring decimal up to the decimal point by multiplying by 10/100 etc.

Be in a position to eliminate the recurring decimal by multiplying again by 10/100 etc.

Subtract two equations to eliminate recurring decimals and convert into fraction

Simplify your answer

RPR – Quantities as fractions/percentages of each other

EZY MATHS

Quantities as fractions of each other

Express 5 as a fraction of 25 ➡ $\frac{5}{25} \xrightarrow{\div 5} \frac{1}{5}$

Think of it as a test score

Decide on numerator ➡ Decide on denominator ➡ Simplify

Nigel earns £90 and saves £30. Sanjay earns £100 and saves £35. Who has saved a greater proportion of their earnings?

$\text{Nigel} = \frac{30}{90} = \frac{1}{3} \rightarrow 0.\dot{3}$

$\text{Sanjay} = \frac{35}{100} = \frac{7}{20} \rightarrow \underline{\underline{0.35}}$

Fractions of amounts

Divide amount by denominator ➡ Then multiply by the numerator

$\frac{3}{5}$ of 60 ➡ $60 \div 5 = 12$ ➡ $12 \times 3 = \underline{\underline{36}}$

Quantities as percentages of each other

Express 5 as a percentage of 20

Convert to fraction or decimal then to percentage

Method 1 — Equivalent fraction over 100: $\frac{5}{20} \xrightarrow{\times 5} \frac{25}{100}$ ➡ 25%

Method 2 — Convert to decimal: $5 \div 20$ ➡ $0.25 \xrightarrow{\times 100} 25\%$

Percentages of amounts

Find 35% of 40

Method 1- Unitary method

Find 1%, 10%, 5% etc.

$10\% = 4 \quad (\div 10)$

$30\% = 12$

$+\ 5\% = 2$

14

Method 2- Decimal method

Turn % to a decimal ($\div$ 100) Then multiply by amount

$35\% \div 100 = 0.35$

$0.35 \times 40 = 14$

RPR – Percentages and percentage change

EZY MATHS

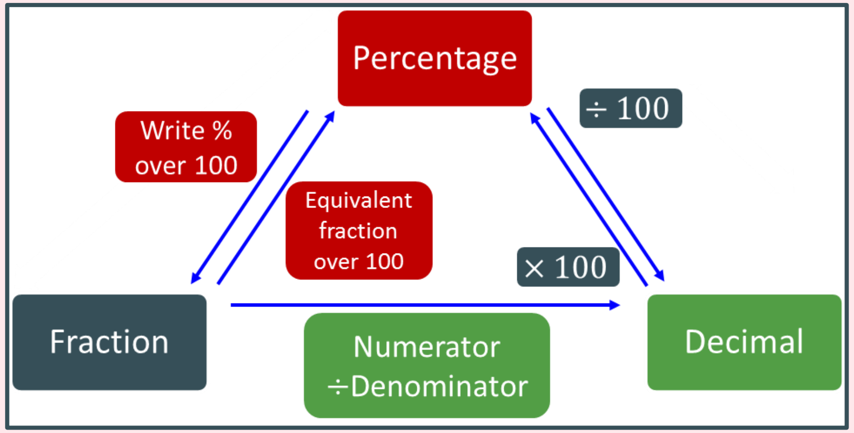
Introduction to percentages

Means out of 100

Fraction	Decimal	Percent
$\frac{1}{2}$	0.5	50%
$\frac{1}{3}$	$0.\dot{3}$	$33.\dot{3}$ %
$\frac{1}{4}$	0.25	25%
$\frac{1}{5}$	0.2	20%
$\frac{1}{8}$	0.125	12.5%
$\frac{1}{10}$	0.1	10%

The common conversions you should really know

Conversion Flow Chart



Percentage increase/decrease

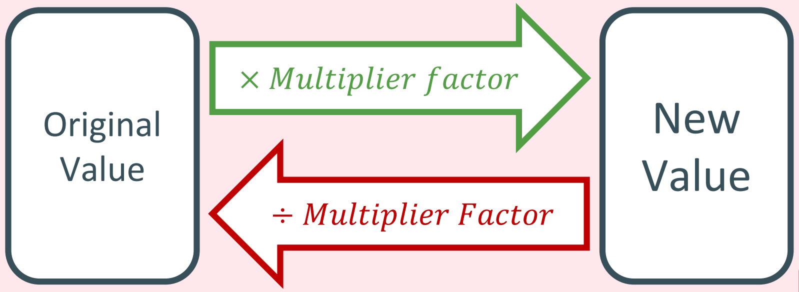
Increase 70 by 15%

Calculate percentage of amount.
Add on for increase. Subtract for decrease.

Method 1- Unitary method: $10\% = 7, 5\% = 3.5 \Rightarrow 15\% = 10.5$

Method 2- Decimal method: $15\% = 0.15 \Rightarrow 0.15 \times 70 \Rightarrow 15\% = 10.5$

Method 3- Calculator method



To find original amount, work backwards and divide by multiplier factor

Increase or Decrease?

Add % to 100 for increase	Subtract % from 100 for decrease

Convert to decimal (÷ 100)

This is your multiplier factor

$$Increase\ of\ 23\% = \times 1.23$$

$$Decrease\ of\ 42\% = \times 0.58$$

$$Percentage\ change = \frac{difference}{orignal\ amount} \times 100$$

RPR – Simple interest and Compound Growth and Decay

EZY MATHS

Simple Interest

Follows the I=PRY formula

Interest ⇨ Principal × Rate × Years

Calculate the Simple Interest earned on £350 at a rate of 9% p.a for 4 years?

Interest ⇨ $£350 \times 0.09 \times 4 = £126$

Remember to convert the % to a decimal

To calculate other parts of the formula, you will need to change the subject

How many years would it take for £45000 to receive £19800 Simple Interest at a rate of 5.5% p.a?

$$\frac{\text{Interest}}{\text{Principal} \times \text{Rate}} \Rightarrow \text{Years}$$

Compound Growth

An amount is increased or decreased by a percentage

The process is repeated several times at each interval

The most efficient way to do this is using a Multiplier

The general formula for compound growth and decay

$$\text{Interest} \Rightarrow \text{Principal} \times (1 \pm \text{Rate})^{\text{Years}}$$

Growth (+), Decay (−)

£4000 is invested at a rate of 5% p.a for three years. Calculate Final value of the investment after three years.

$$£4000 \times 1.05^3 = £4630.50$$

A car worth £15000 depreciates in value at a rate of 15% p.a. What is the depreciated value of the car after 4 years

$$£15000 \times 0.85^4 = £7830.09$$

To calculate other parts of the formula, you will need to change the subject

RPR – Ratio

EZY MATHS

Introduction

Is the relationship between two or more quantities

It is written in the form $\underline{a : b}$

Compares one part to another part

The ratio of red to blue is 4 : 5

The ratio of blue to red is 5 : 4

Sentence structure is important!

Simplifying Ratios

$12 : 8 \xrightarrow{\div 2} 6 : 4 \xrightarrow{\div 2} \underline{\underline{3 : 2}}$

Divide all numbers by the same value

The ratio of boys to girls in a Geography class is 15 : 5

What fraction of the class is girls?

$$\frac{5\ girls}{20\ total\ parts} \Rightarrow \underline{\underline{\frac{1}{4}}}$$

Sharing in a given ratio

Find total number of parts → Find value of one part → Multiply by original ratio

Share \$40 in the ratio 3 : 5

Find total number of parts	Add the ratio parts together

$$3 + 5 = 8$$

Find value of one part	Divide amount by number of parts

$$\$40 \div 8 = \$5$$

Each part of the ratio is worth \$5

Multiply by original ratio

$$3 : 5 \xrightarrow{\times \$5} \underline{\underline{\$15 : \$25}}$$

Mark and John have sweets in the ratio 3 : 4, If Mark has 27 sweets. How many does John have?

$$27 \div 3 = 9\ sweets\ per\ part$$

$$4 \times 9 = 36\ (John's\ sweets)$$

Map scale factors

It is the ratio of a distance on the map/model to the corresponding size in real life.

Written in the form $\underline{1 : n}$

Map or Model $\xrightarrow{\times\ scale\ factor}$ Real life

Real life $\xrightarrow{\div\ scale\ factor}$ Map or Model

Know your conversions

$$10mm = 1cm$$
$$100cm = 1m$$
$$1000m = 1km$$

A map has a scale of 1:25000. Michael is 6cm from his home. How far from home is he? Give your answer in km

$$6cm \times 25000 = 150000cm$$

$$150000cm \xrightarrow{\div 100} 1500m \xrightarrow{\div 1000} \underline{\underline{1.5km}}$$

RPR – Proportion

EZY MATHS

Direct Proportion

As one value increases, the other increases at the same rate

Three Coffees cost £7.50,
How much would five Coffees cost?

Find the value of one coffee then multiply by quantity needed

$£7.50 \div 3 = £2.50\ per\ coffee$

$£2.50 \times 5 = \underline{\underline{£12.50}}$

Inverse Proportion

As one value increases, the other decreases at the same rate

It takes 3 men 4 days to build a wall.
How long would it take 2 men?

Find the time taken by one man then divide by quantity stated

$3men \times 4\ days = 12\ days$

$12\ days \div 2\ men = \underline{\underline{6\ days}}$

Direct Proportion

y is directly proportional to x

$y \propto x$ Constant of proportionality

$y = k \times x$ k is the rate of change

Solving direct proportion problems

p is directly proportional to t.

$p = 24, t = 8$

a) Find p when $t = 7$

b) Find t when $p = 39$

Compare two values

$p = k \times t \Rightarrow 24 = k \times 8$

Work out the value of k

$24 = k \times 8 \xrightarrow{\div 8} \frac{24}{8} = k \Rightarrow 3 = k$

Form equation to solve problems

$p = 3 \times t$ $a)\ p = 3 \times 7 = \underline{\underline{21}}$

$b)\ 39 = 3 \times t \xrightarrow{\div 3} t = \underline{\underline{13}}$

Inverse Proportion

y is inversely proportional to x

$y \propto \frac{1}{x} \rightarrow y = \frac{k}{x}$ Constant of proportionality k

Solving inverse proportion problems

p is inversely proportional to t.

$p = 16, t = 2$

a) Find p when $t = 8$

b) Find t when $p = 64$

Compare two values

Work out the value of k

$p = \frac{k}{t}$ $16 = \frac{k}{2} \xrightarrow{\times 2} 32 = k$

Form equation to solve problems

$p = \frac{32}{t}$ $a)\ p = \frac{32}{8} = \underline{\underline{4}}$

$b)\ 64 = \frac{32}{t} \Rightarrow t = \frac{32}{64} = \underline{\underline{0.5}}$

RPR – Graphical representations of Proportion

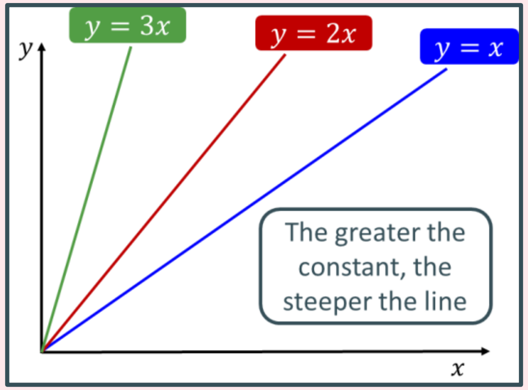
EZY MATHS

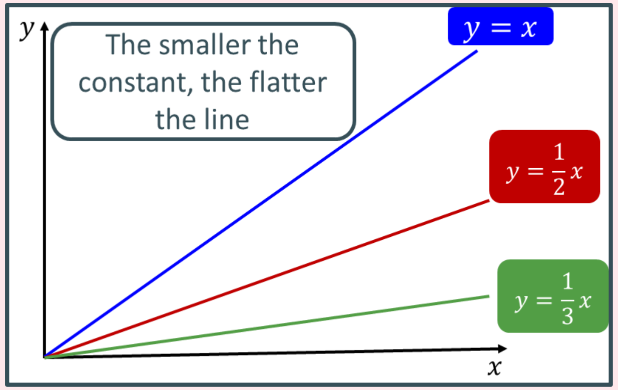
Direct Proportion

$y \propto x$ → $y = k \times x$

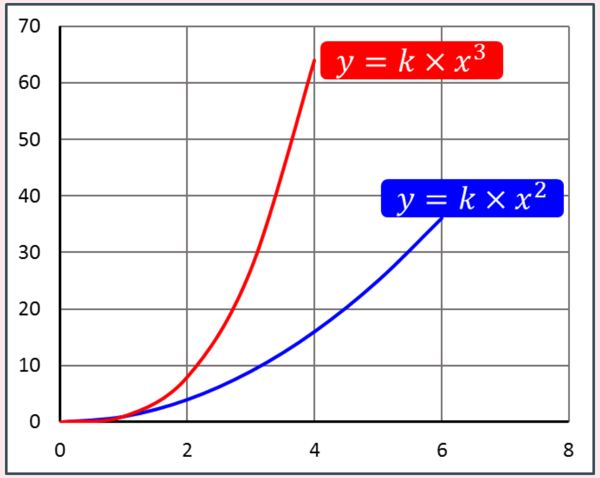
As one value increases, so does the other

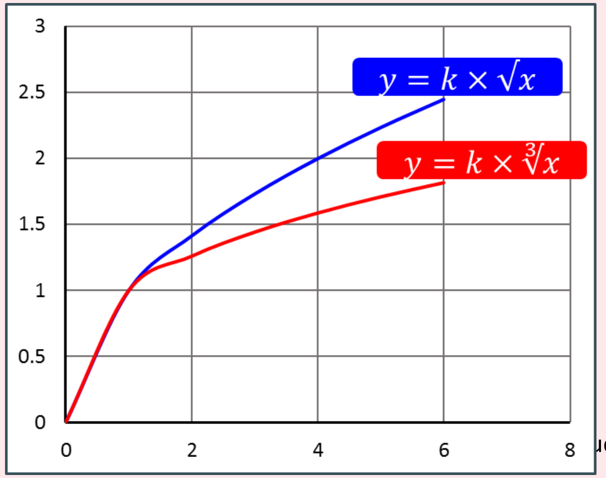
Linear relationships





Non - linear relationships



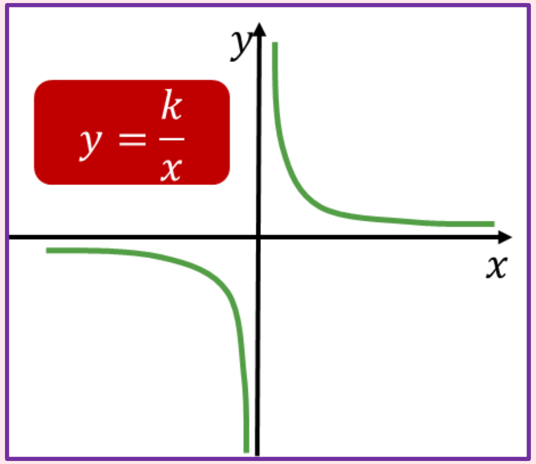


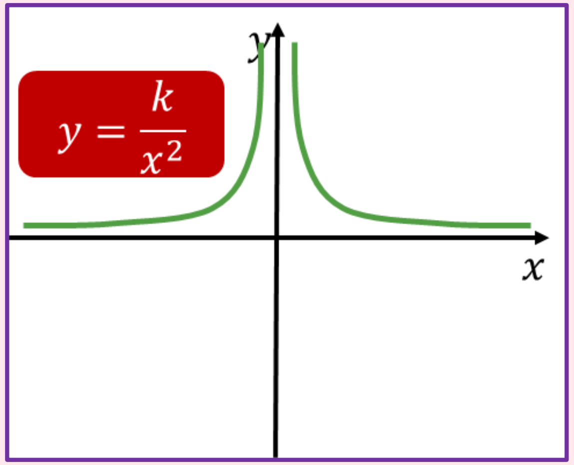
Inverse Proportion

$y \propto \frac{1}{x}$ → $y = \frac{k}{x}$

As one value increases, the other decreases

Inverse proportion will always give a **<u>Curved</u>** graph

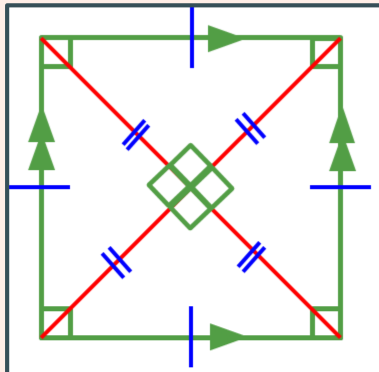




Geometry – Quadrilaterals

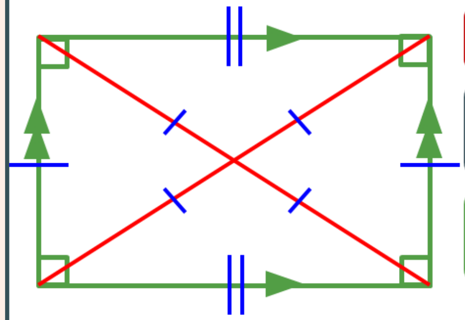
EZY MATHS

Know the names of these Quadrilaterals and their properties



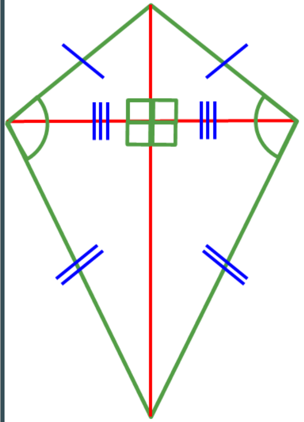
SQUARE

- All sides equal
- All angles 90°
- Diagonals equal length
- Diagonals Perpendicular
- Opposite sides parallel



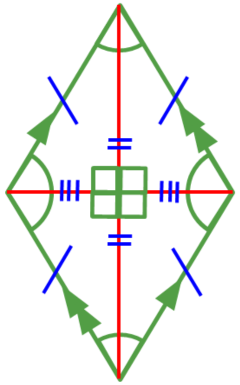
Rectangle

- Opposite sides equal
- All angles 90°
- Diagonals equal length
- Opposite sides parallel



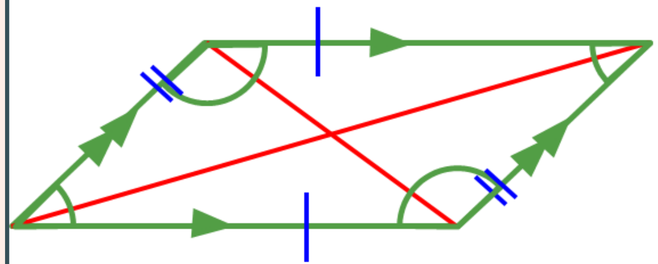
Kite

- Two pairs of adjacent equal sides
- One pair of equal angles
- Diagonals bisect angles
- Diagonals Perpendicular



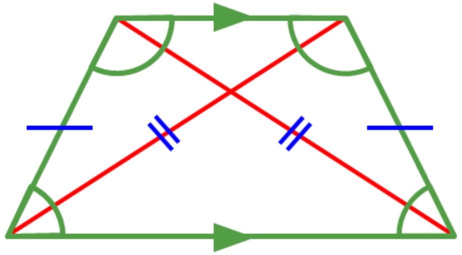
Rhombus

- All sides equal
- Opposite angles equal
- Diagonals Bisect each other
- Diagonals Perpendicular
- Opposite sides parallel



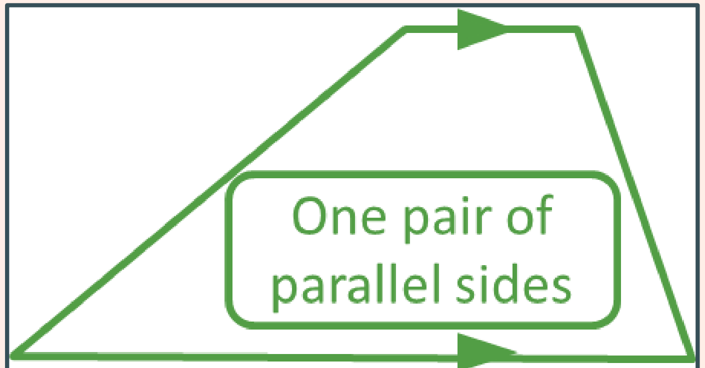
Parallelogram

- Opposite sides equal
- Opposite angles equal
- Diagonals Bisected
- Opposite sides parallel



Isosceles Trapezium

- One pair of equal sides
- Two pairs of equal angles
- Diagonals equal length
- One pair of parallel sides



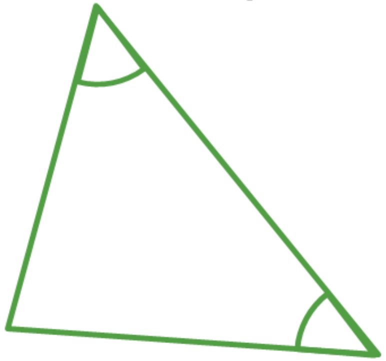
Trapezium

Geometry – Triangles

EZY MATHS

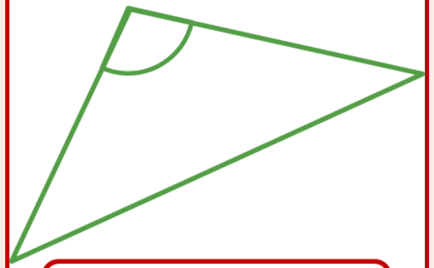
Know the names of these Triangles and their properties

Acute angled



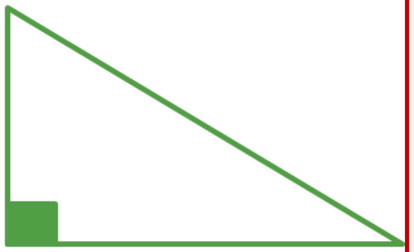
All angles are less than 90°

Obtuse angled



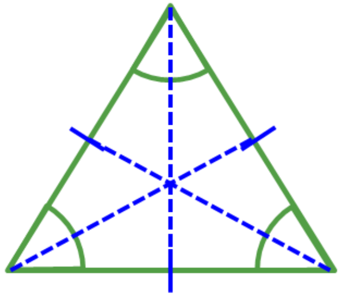
One angle is greater than 90°

Right Angled



One angle is 90°

Equilateral

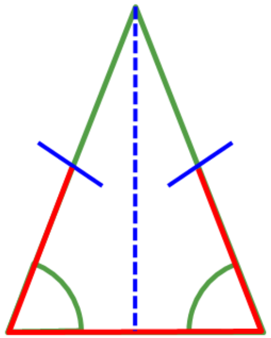


All sides equal

All angles equal (60°)

Three lines of symmetry

Isosceles

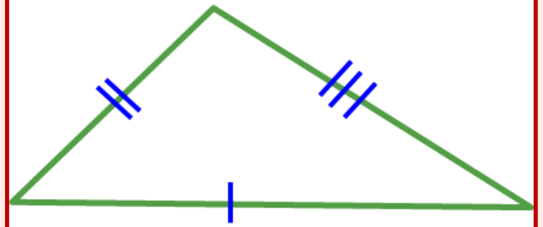


Two equal sides

Two equal angles

One line of symmetry

Scalene



No equal sides

No equal angles

No line of symmetry

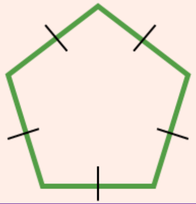
Geometry – Polygons

EZY MATHS

Regular

Side lengths are the same

Interior and exterior angles the same



Irregular

Side lengths are not **ALL** the same

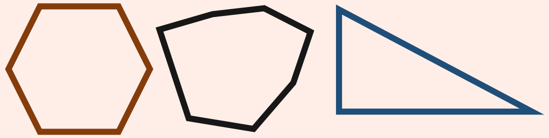
Interior and exterior angles the not **ALL** the same

Equal sides are marked with a dash through the line

Convex

The shape is 'bulging' outwards

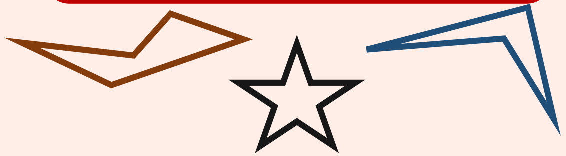
All angles less than 180°



Concave

The shape has 'caved' inwards

One or more angles is greater than 180°



They are classified by the number of sides they have

Number of sides	Name of shape
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon

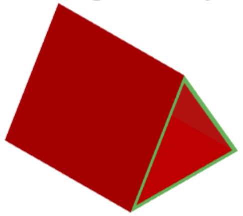
Geometry – 3D shapes

EZY MATHS

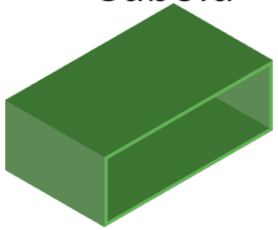
Prisms

The same cross sectional area throughout

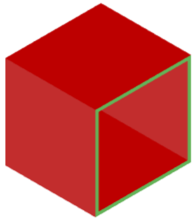
Triangular prism

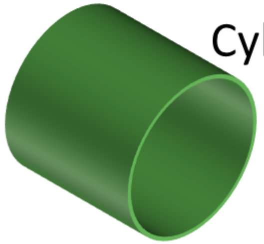


Cuboid

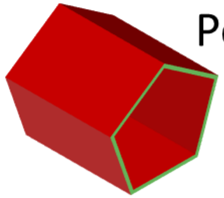


Cube





Cylinder



Pentagonal faced prism

Cone

Square based Pyramid

Sphere

Hemisphere

Pyramids are classified according to their base

Faces, Edges and Vertices

Faces	The flat surface of a 3-D shape
Edges	Where two Faces meet
Vertices	Where two or more edges meet at a point

Faces + Vertices – Edges = 2

Shape	Faces	Edges	Vertices
Cube	6	12	8
Cuboid	6	12	8
Triangular Prism	5	9	6
Pentagonal Prism	7	15	10
Square based Pyramid	5	8	5
Cone	2	1	1
Cylinder	3	2	0
Sphere	1	0	0

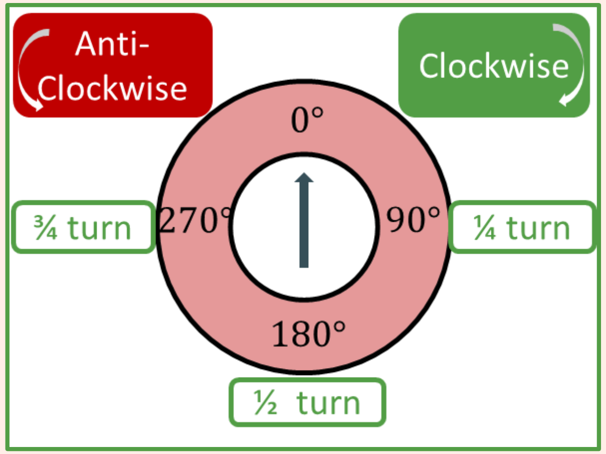
Geometry – Angle facts

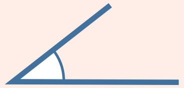
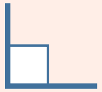
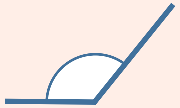
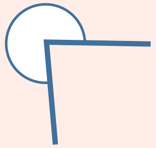
EZY MATHS

Introduction

A measure of turn.
Using a protractor

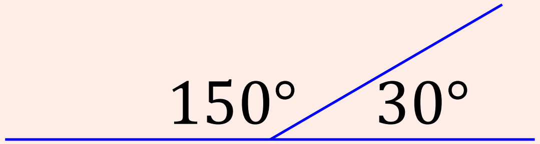
It has the unit degrees (°)



Acute	Greater than 0° less than 90°
	Looks like a book closing or crocodile jaws
Right	Exactly 90°
	Has a square in the angle to indicate that it is 90°
Obtuse	Greater than 90° less than 180°
	Looks like a book falling open
Straight	Exactly 180°
	A half turn to create a straight line
Reflex	Greater than 180° less than 360°
	The larger angle outside the acute or obtuse angle
Full turn	Exactly 360°
	A movement around a point to create a circle

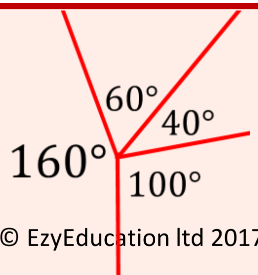
A straight line

All angles on a straight line will add up to make 180°



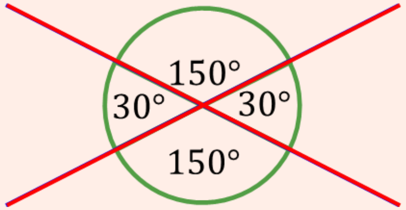
Around a point

All angles around a point will add up to make 360°



Vertically opposite

Where two straight lines cross, opposite angles are equal

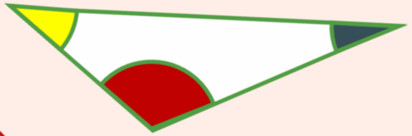


Geometry – Angles in triangles and polygons

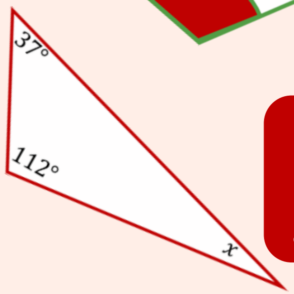
EZY MATHS

Triangles

All three angles can be orientated to fit on a straight line → All angles in a triangle make 180°



$Straight\ line = 180°$



Calculate what you already know.

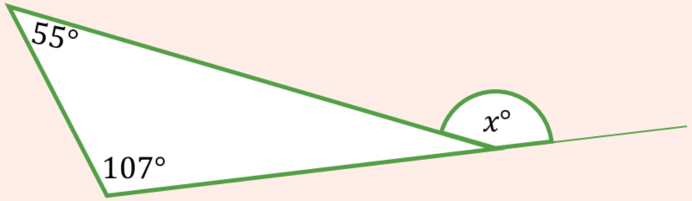
$$\begin{array}{r} 112° \\ +\ \ 37° \\ \hline 149° \end{array}$$

Subtract from 180°

$180° - 149°$

$x = \underline{\underline{31°}}$

Exterior angle = Sum of two angles on opposite side.



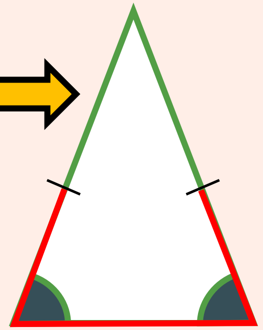
$55° + 107° = x°$

$\underline{\underline{162°}} = x°$

Isosceles triangle

It has two equal lengths

It has two equal angles

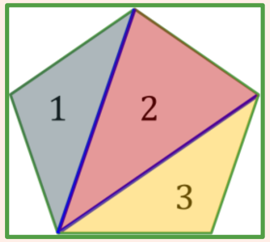


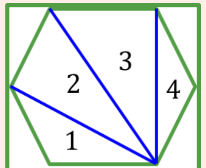
Polygons

Knowledge of triangles is important

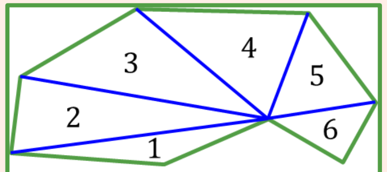
Number of sides	Number of △	Sum of interior angles	Regular interior angle	Regular exterior angle
3	1	180°	60°	120°
4	2	360°	90°	90°
5	3	540°	108°	72°
6	4	720°	120°	60°
7	5	900°	129°	51°
8	6	1080°	135°	45°
n	$(n-2)$	$(n-2) \times 180°$	$\frac{(n-2) \times 180°}{n}$	$360° \div n$

The number of triangles in a shape will always be **TWO** less than the number of sides





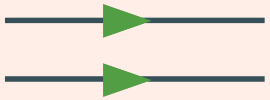
Hexagon



Octagon

Geometry – Angles Parallel lines

EZY MATHS

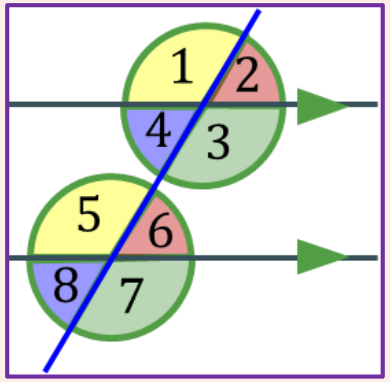


Two lines that travel side by side keeping the same distance apart at all times, never intersecting.

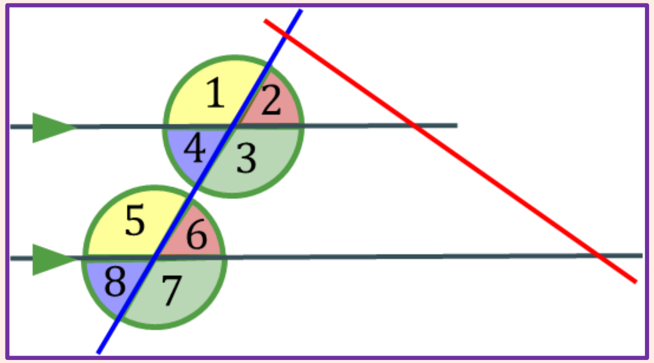
When a line intersects the parallel lines

Eight angles are created

One **Acute** angle
One **Obtuse** angle

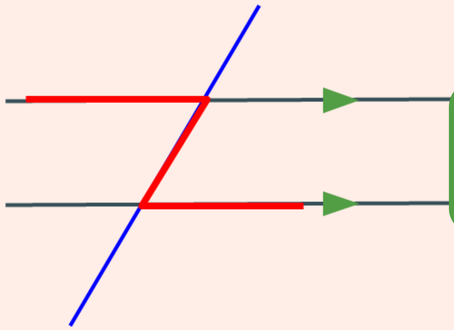


The angles on one parallel line match up with the angles on the other parallel line



Different lines will create different sets of angles

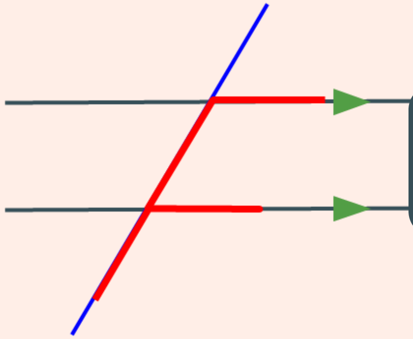
Alternate



'Z' shape

Alternate angles are the same

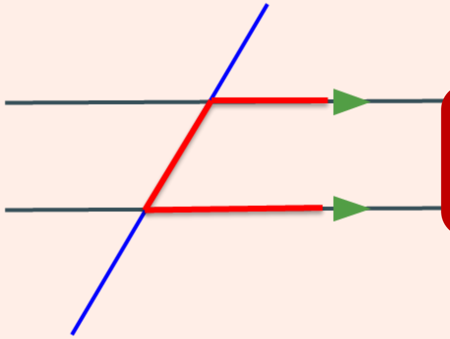
Corresponding



'F' shape

Corresponding angles are the same

Co-Interior



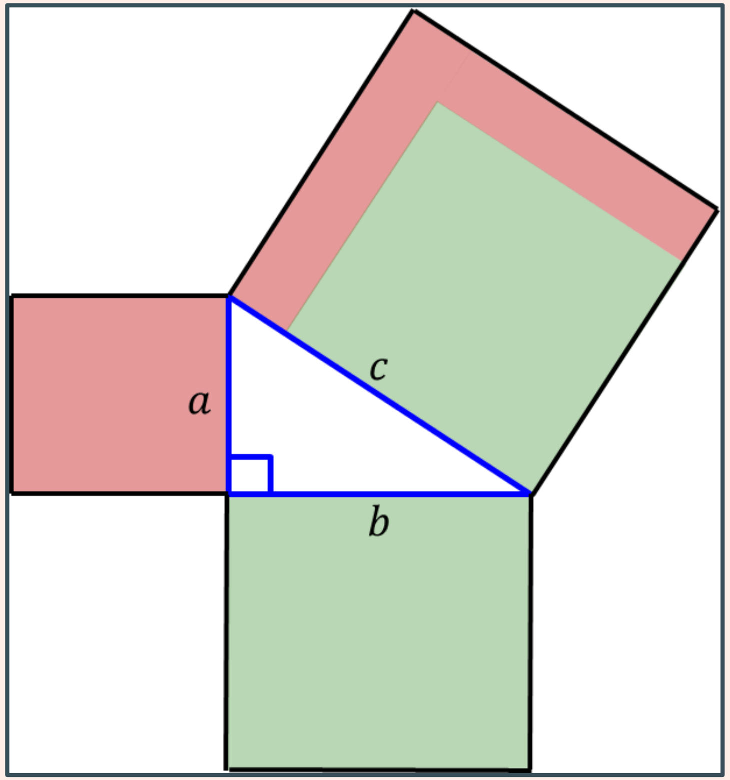
'C' shape

Co-interior angles make 180°

Geometry – Pythagoras' Theorem

EZY MATHS

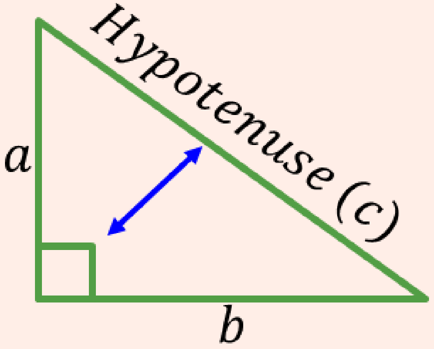
$$c^2 = a^2 + b^2$$



For any right angled triangle, the area of the square drawn on the hypotenuse is equal to the sum of the areas drawn on the other two sides.

Finding the Hypotenuse

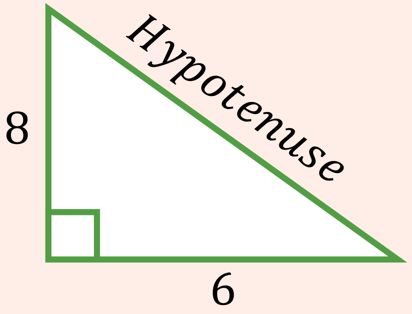
If you know the lengths of the **two shorter sides**, you can calculate the length of the hypotenuse.



Square the two sides

<u>Add</u> them

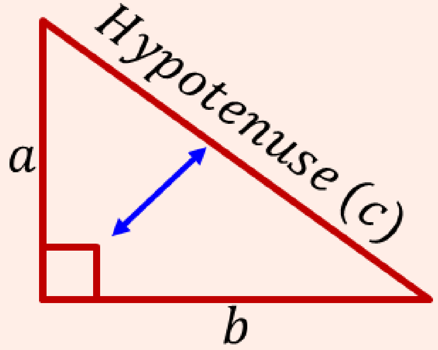
Square root for answer



$$c^2 = 8^2 + 6^2$$
$$c^2 = 64 + 36$$
$$c^2 = 100 \ (\sqrt{\ })$$
$$\boxed{C = 10}$$

Finding the Shorter side

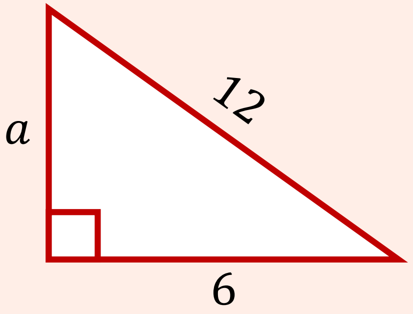
If you know the **Hypotenuse** and a **shorter side**, you can calculate the length of the other shorter side.



Square the two sides

<u>Subtract</u> them

Square root for answer



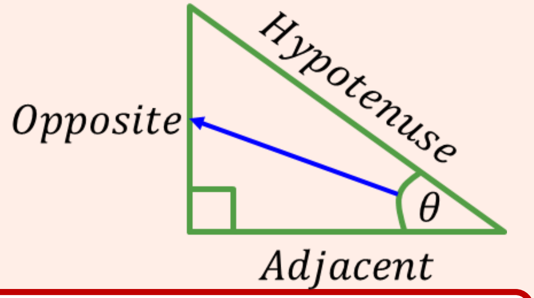
$$c^2 - b^2 = a^2$$
$$12^2 - 6^2 = a^2$$
$$144 - 36 = a^2$$
$$108 = a^2 \ (\sqrt{\ })$$
$$\boxed{10.39 = a}$$

Geometry – Trigonometry functions

EZY MATHS

Trigonometry is used to calculate sides lengths and angles in triangles using three important ratios. Sine, Cosine and Tangent

The angle is often described as theta which is the Greek letter (θ)



Sine Function

Work out side lengths and angles when given the **opposite** and **hypotenuse**

$$\sin\theta = \frac{opposite}{hypotenuse}$$

$\sin^{-1}$

sin

$\sin\theta$	Value
0°	0
30°	0.5
45°	$\frac{\sqrt{2}}{2} = 0.707$
60°	$\frac{\sqrt{3}}{2} = 0.866$
90°	1

To get $\sin^{-1}$ you have to press shift first

Only used to calculate an angle

Cosine Function

Work out side lengths and angles when given the **adjacent** and **hypotenuse**

$$\cos\theta = \frac{adjacent}{hypotenuse}$$

$\cos^{-1}$

cos

$\cos\theta$	Value
0°	1
30°	$\frac{\sqrt{3}}{2} = 0.866$
45°	$\frac{\sqrt{2}}{2} = 0.707$
60°	0.5
90°	0

To get $\cos^{-1}$ you have to press shift first

Only used to calculate an angle

Tangent Function

Work out side lengths and angles when given the **adjacent** and **opposite**

$$\tan\theta = \frac{opposite}{adjacent}$$

$\tan^{-1}$

tan

$\tan\theta$	Value
0°	0
30°	$\frac{\sqrt{3}}{3}$
45°	1
60°	$\sqrt{3}$

To get $\cos^{-1}$ you have to press shift first

Only used to calculate an angle

Geometry – SohCahToa

EZY MATHS

When to use trigonometry

Find a missing side when given a length and an angle

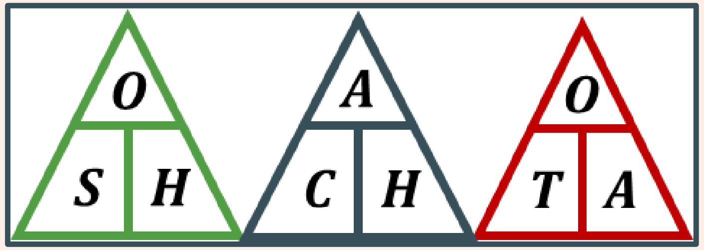
or

Find an angle when given two lengths

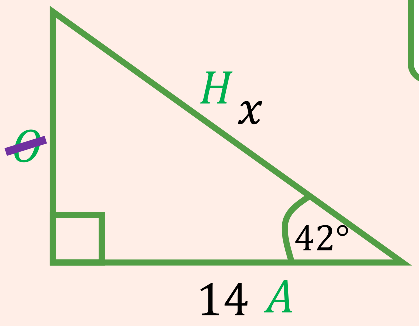
Label the triangle accurately with O,A,H

Decide which ratio to apply

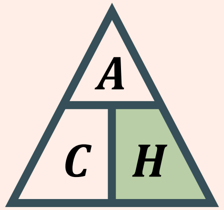
Use the triangles to help



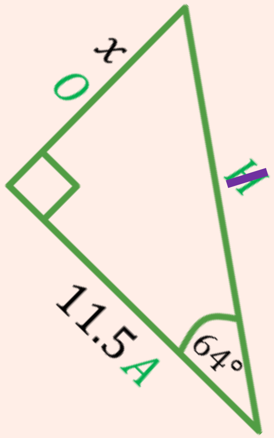
Calculating a missing side



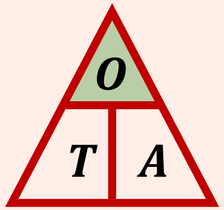
Cover up what you need to find



$$x = 14 \div \cos(42°)$$
$$= \underline{\underline{18.84}}$$

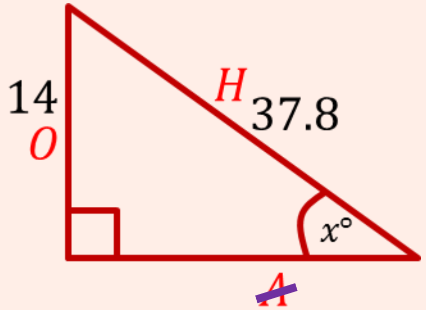


Cover up what you need to find

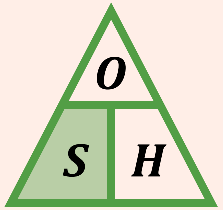


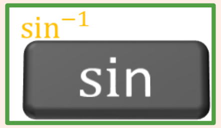
$$x = \tan(64°) \times 11.5$$
$$= \underline{\underline{23.58}}$$

Calculating a missing angle



Cover up what you need to find





x is an angle so use inverse

$$x = \sin^{-1}(14 \div 37.8)$$
$$= \underline{\underline{21.7°}}$$

Alternate process

Use the functions and substitute

$$\sin\theta = \frac{opposite}{hypotenuse}$$

$$\tan\theta = \frac{opposite}{adjacent}$$

$$\cos\theta = \frac{adjacent}{hypotenuse}$$

Graphs – Coordinates

EZY MATHS

A set of values that indicate the position of a point.

They normally occur in pairs in the form (x, y)

(x, y)

Direction along the x - axis

Along the corridor

Direction up/down the y - axis

Up, Down the stairs

Start from a central point $(0,0)$ - Origin

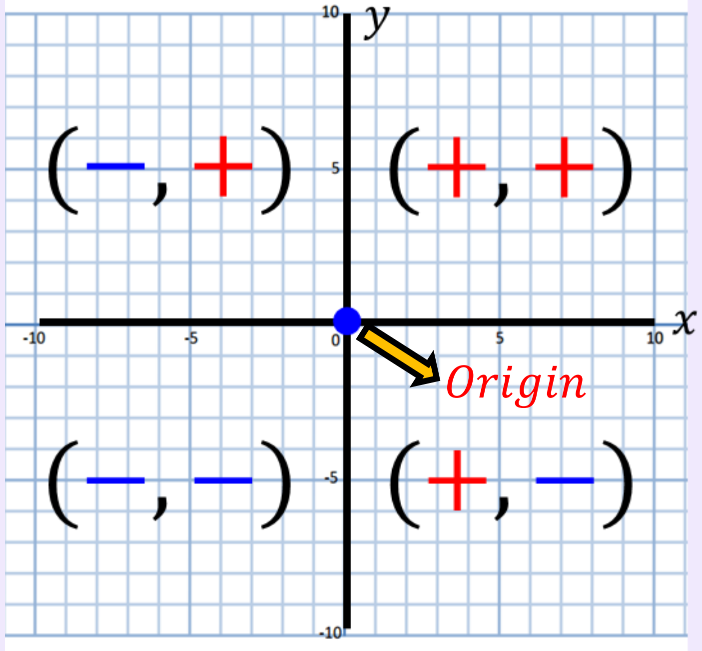
Reading the coordinates will lead you to the exact position.

$(7, -4)$ ⟹ Seven units right, Four units down

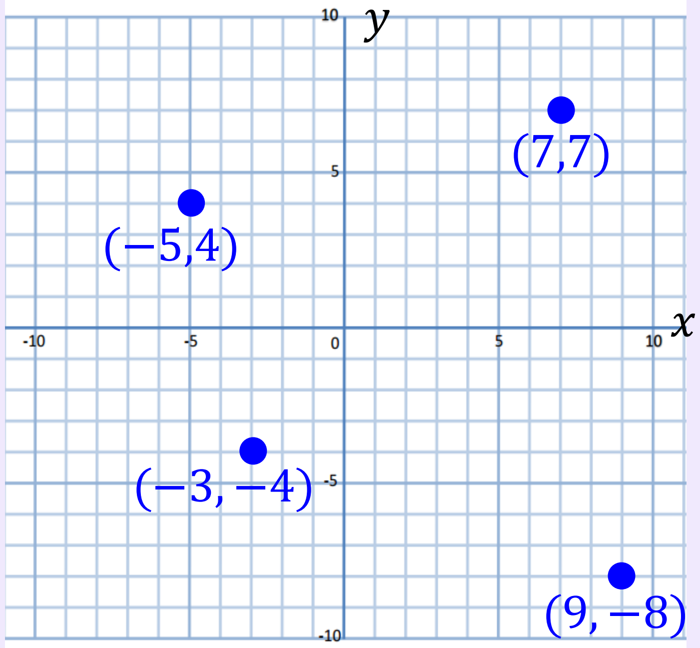
$(-2,6)$ ⟹ Two units left, Six units up

$(-5, -2)$ ⟹ Five units left, Two units Down

Four Quadrants



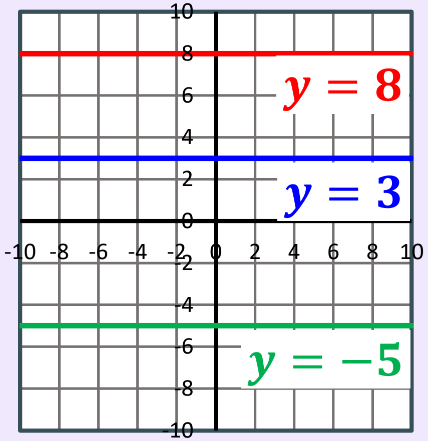
Plotting coordinates



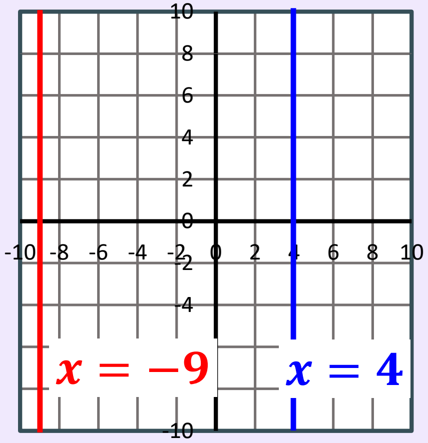
Graphs – Equation of a Straight line

EZY MATHS

Horizontal lines
→ $y = ?$



Vertical lines
→ $x = ?$



All straight line graphs follow the same rule

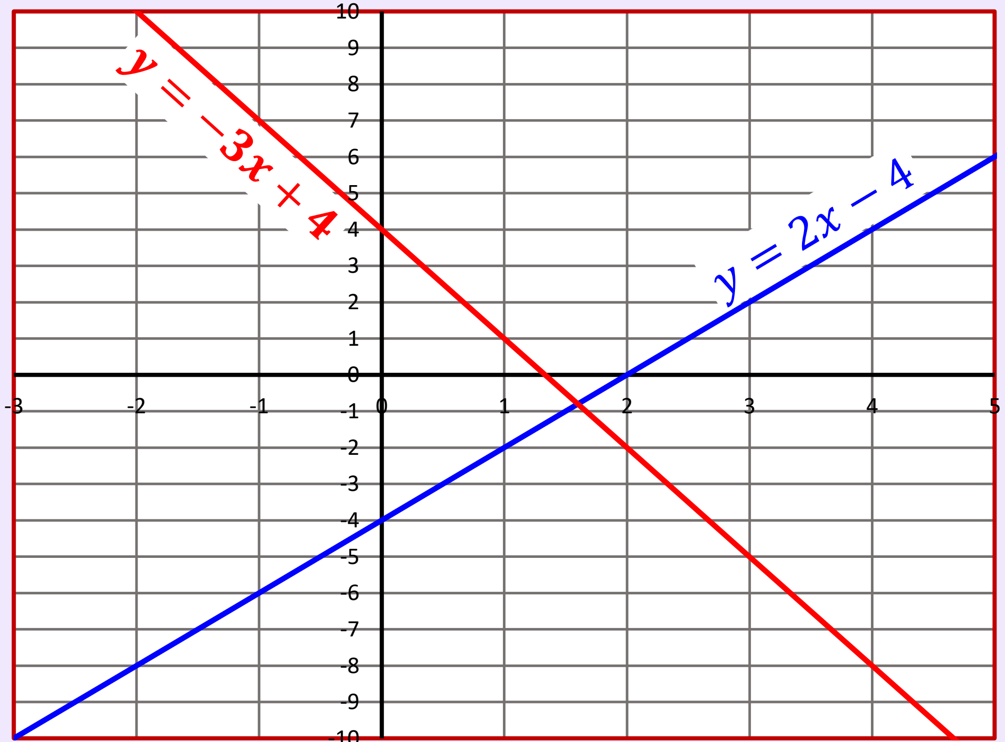
$$y = mx + c$$

m → Gradient

c → y intercept

Gradient is the 'steepness' of the line

Calculated by $\dfrac{\textit{Change in } y}{\textit{Change in } x}$ or $\dfrac{\textit{Rise up}}{\textit{Run along}}$



Equation of line from coordinates

Calculate gradient between points (m)

Substitute in points and solve (c)

Find the equation of the line that passes through $(0,2)$ and $(3,8)$

Gradient$= \dfrac{y_2 - y_1}{x_2 - x_1} \Rightarrow \dfrac{6}{3} = 2 \Rightarrow (m)$

$y = 2x + c \xrightarrow{\text{substitute}} 8 = 2(3) + c$

$8 = 6 + c \xrightarrow{\text{solve}} 2 = c$

$$y = mx + c$$
$$\underline{\underline{y = 2x + 2}}$$

Graphs – Midpoints, Parallel lines and Perpendicular lines

EZY MATHS

Midpoints

A midpoint is the **halfway point** between two end points of a line segment

$$\left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2}\right)$$

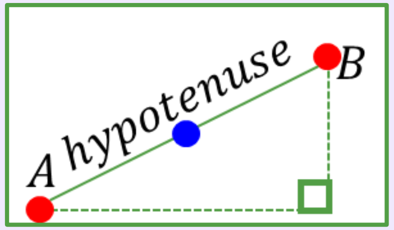
Add up the x coordinates and halve it

Add up the y coordinates and halve it

Find the coordinate of the midpoint joining the points $(6,11)$ and $(15,-9)$

$$x = \frac{6+15}{2} \qquad y = \frac{11+-9}{2}$$

$x = 10.5$ ⇨ $(10.5, 1)$ ⇦ $y = 1$



The distance between two points will always be the hypotenuse

Parallel lines

Parallel lines are lines that run equidistant to each other and **never** intersect (cross)

Parallel lines have the **same** gradient. Different y – intercepts

$$y = mx + c$$

Same (m) Different (c)

Find the equation of the line parallel to $y = 2x + 4$ that passes through $(4,2)$

Substitute in point and solve (c)

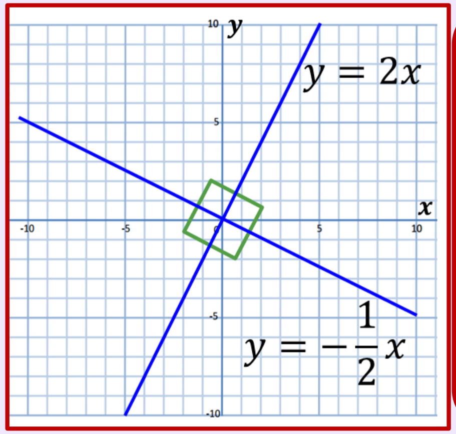
$y = 2x + c$ ⇨ $2 = 2(4) + c$

$2 = 8 + c$ $\xrightarrow{-8}$ $-6 = c$

$$y = 2x - 6$$

Perpendicular lines

Perpendicular lines are lines that **intersect** (cross) to **form 90° angles**



Gradients that are negative reciprocals of each other.

$$m_1 \times m_2 = -1$$

Invert = Reciprocal

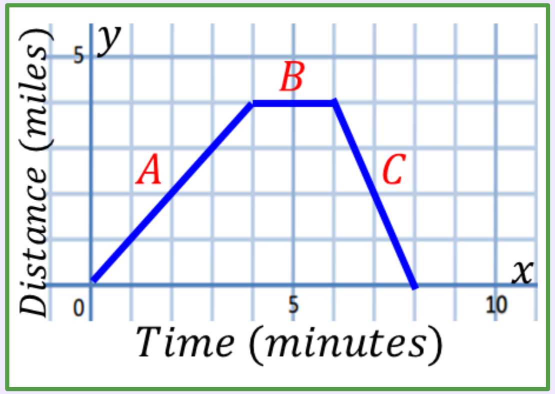
Use gradients and coordinate points to calculate the equation of the line

Graphs – Contextual graphs

EZY MATHS

Distance – Time graphs

Distance - Time graphs record the journey of an object as it begins to move away from and return to a point.



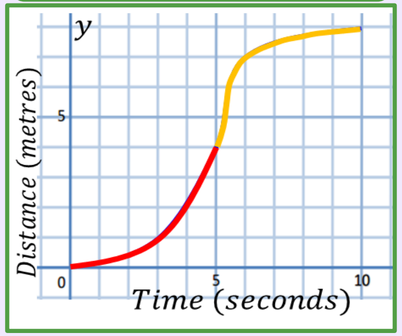
A
Moving away
B
Stationary
C
Returning

Gradient = Speed

$$Gradient = \frac{rise}{run}$$

Not all objects travel at a constant speed

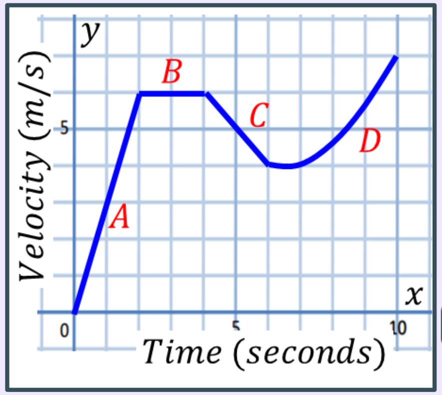
$$Speed = \frac{Distance}{Time}$$



Calculate speed at a specific point by creating a tangent.

Velocity – Time graphs

Velocity - Time graphs record the velocity of a particle moving along a straight line



Stage A
Constant rate of acceleration

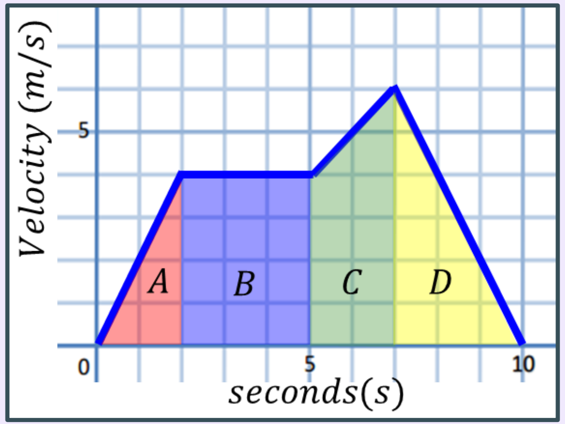
Stage B
Steady 'constant' speed

Stage C
Constant rate of deceleration

Stage D
Increasing rate of acceleration

The gradient of the line is the acceleration.

Area under graph = Distance travelled

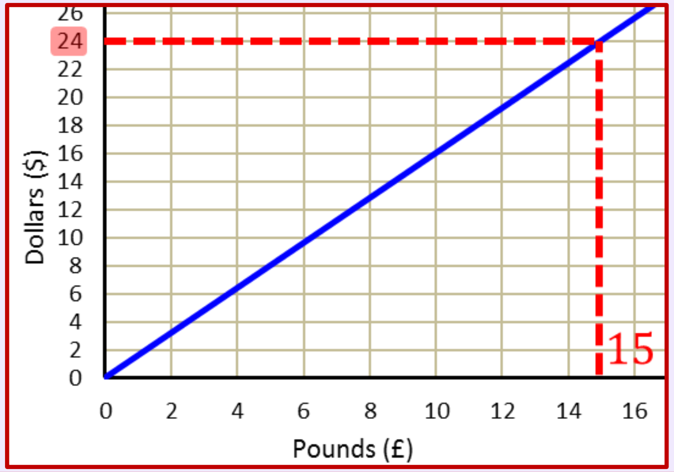


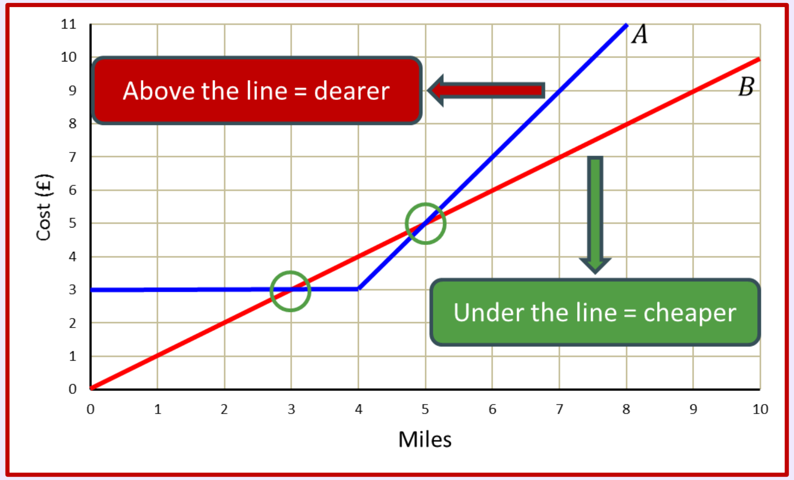
Financial graphs

Currency Conversions

Predict future costs

Cost Comparisons

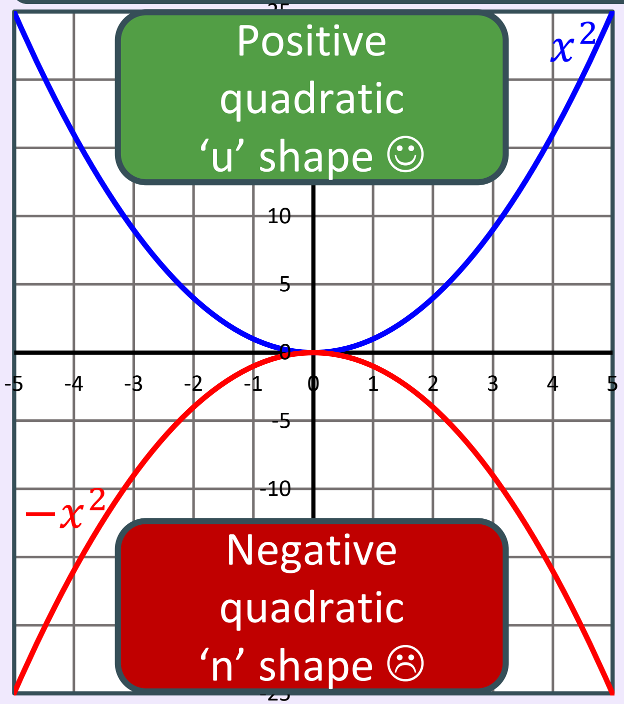


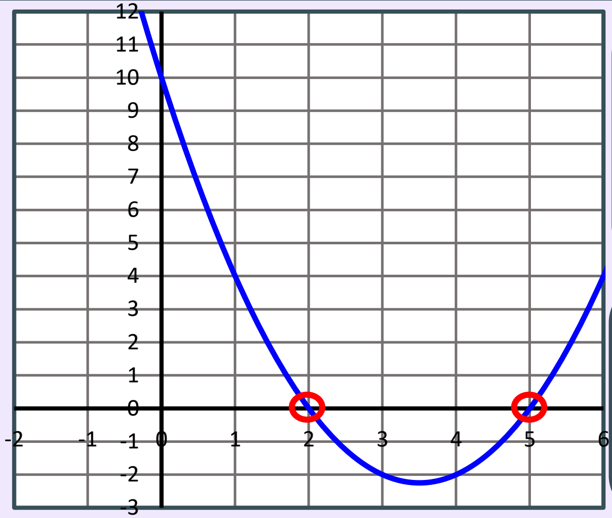


Graphs – Quadratic and Cubic graphs

EZY MATHS

Quadratic graphs





The points where the graph crosses the x axis are known as roots.

These are the solutions to the quadratic when it equals zero

To plot graphs

Substitute the x values into equation to get y and plot points like coordinates

Maximum and minimum points

The line of symmetry runs through these points

Read off coordinate point from graph

Find the midpoint of the roots and substitute into equation to calculate y −coordinate

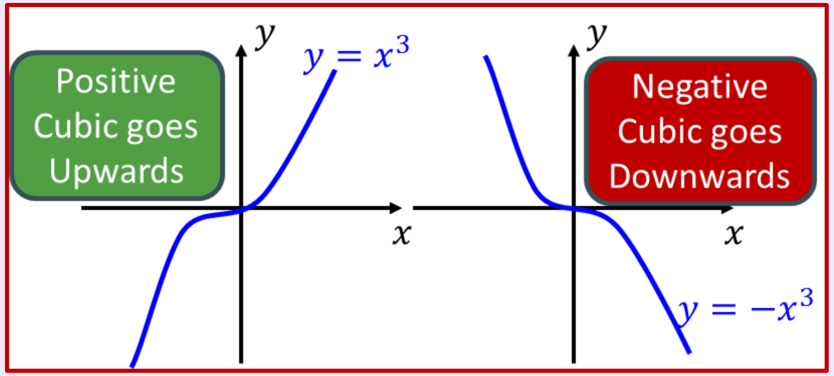
Complete the square in the form $(x-p)^2+q$

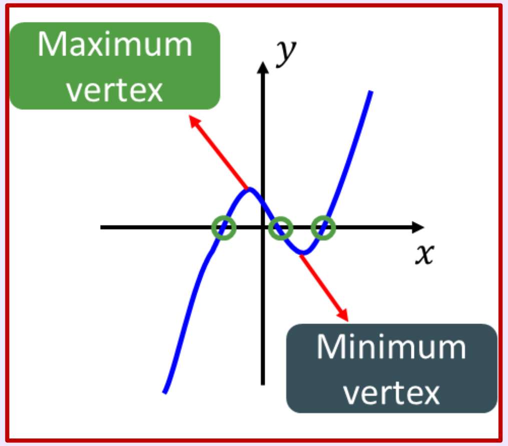
Max/min point can be found by (p, q)

Cubic graphs

Can be defined as an equation where the highest power of the variable (usually x) is 3

$$y = x^3 + 3x^2 + 5x - 20$$

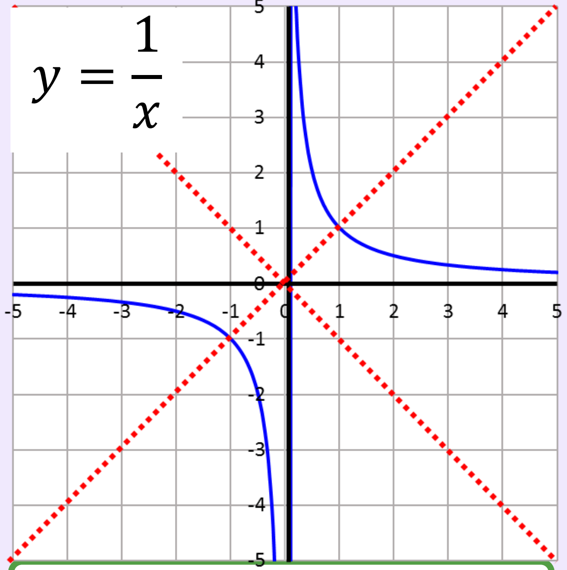


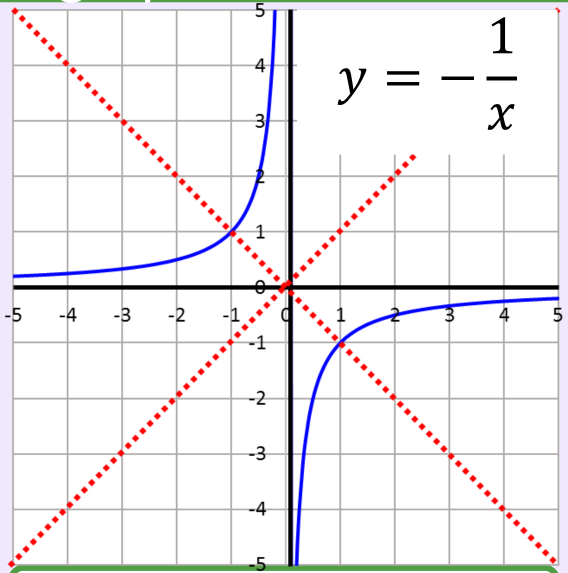


Graphs – Reciprocal and Exponential graphs

EZY MATHS

Reciprocal graphs





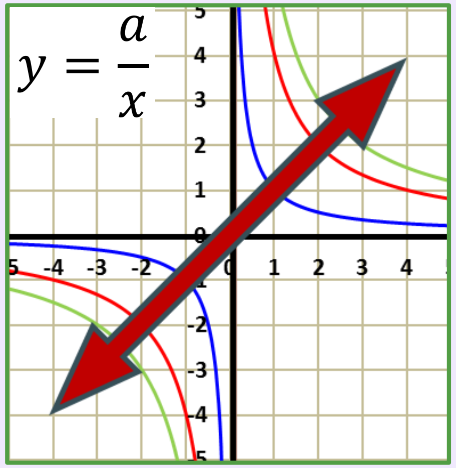
x-axis and y-axis are asymptotes

Two lines of symmetry

$y = \frac{1}{x}$

$y = \frac{4}{x}$

$y = \frac{6}{x}$

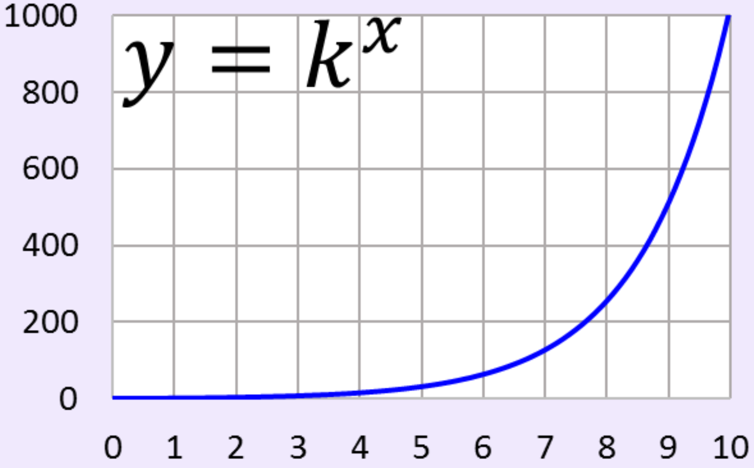


As a increases, the graphs move further away from the origin.

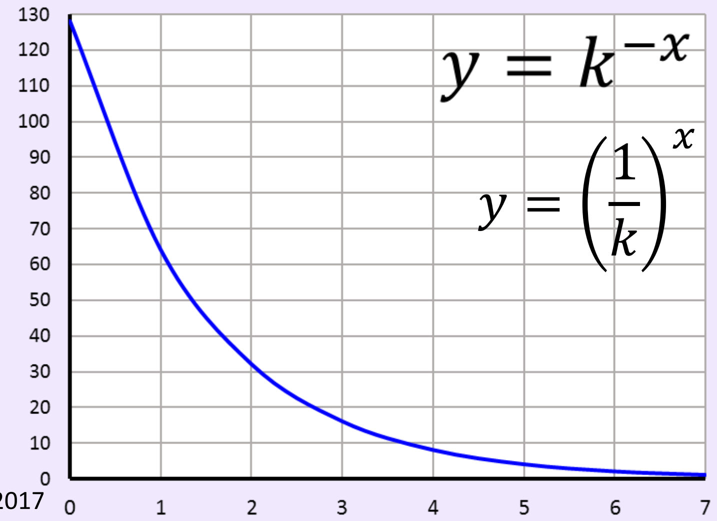
Points can be found by substitution

Exponential graphs

As x continues to increase, y continues to rise or fall at a continually faster or slower rate.



When $k > 1$, and x is positive, the graph will curve upwards



When $0 < k < 1$, or x is negative, the graph will curve downwards

Graphs – Equation of a circle

EZY MATHS

There is a specific general formula for the equation of a circle

$$x^2 + y^2 = r^2$$

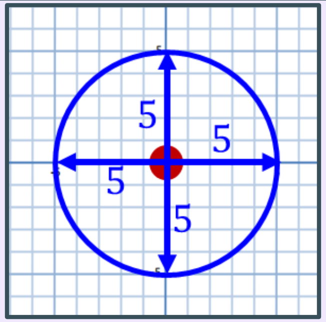
$$\underbrace{x^2 + y^2}_{\text{Algebra}} = \underbrace{r^2}_{\text{Number}}$$

$$\overbrace{x^2 + y^2}^{\text{Algebra}} = \overbrace{25}^{\text{Number}}$$

Radius $= 5$

Centered at Origin

Find the radius (r)



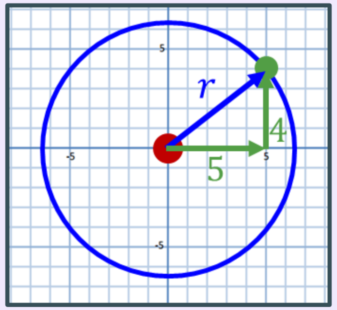
Finding the radius and equation using Pythagoras

$$5^2 + 4^2 = r^2$$

$$41 = r^2$$

$$r = \sqrt{41}$$

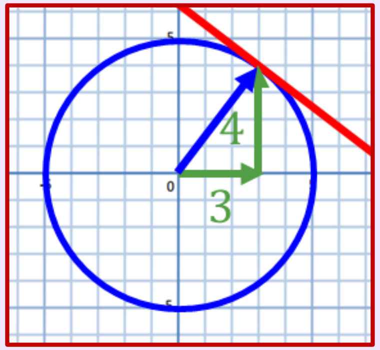
$$x^2 + y^2 = 41$$



We may be asked to find the equation of a tangent to a circle at a given point.

Gradient of Radius → Gradient of Tangent → Substitute in Point

What is the tangent to the circle $x^2 + y^2 = 25$ at the point $(3,4)$?



$$\text{Gradient (Radius)} = \frac{4}{3}$$

$$\text{Gradient (Tangent)} = -\frac{3}{4}$$

Circle Theorem: A radius always meets a tangent at a right-angle.

Perpendicular line

$$y = -\frac{3}{4}x + c \Rightarrow \text{Point } (3,4) \Rightarrow (4) = -\frac{3}{4}(3) + c \Rightarrow \frac{25}{4} = c$$

$$y = -\frac{3}{4}x + \frac{25}{4}$$

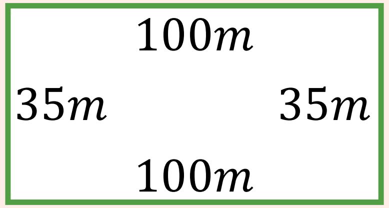
Geometry – Perimeter and Area

EZY MATHS

Perimeter

The total distance **AROUND** a 2D shape

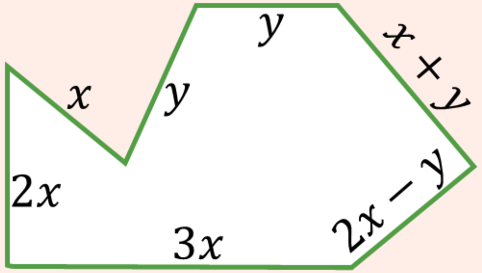
Adding all the side lengths together



$100 + 100 + 35 + 35$

$270m$

The process does not change if we have algebraic terms



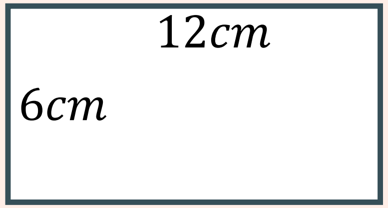
$= 9x + 2y$

Rectangular areas

The total **space** taken up by a 2D shape

Multiplying two side lengths together

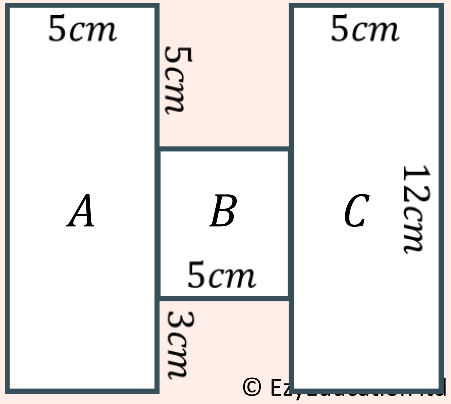
Area of rectangle $\square = l \times w$



Area $= 6cm \times 12cm$

$72cm^2$

With compound shapes, break it down.



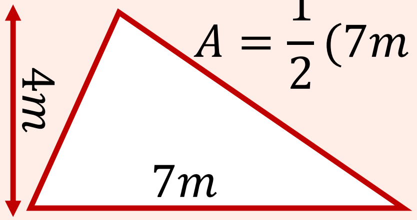
$A = 60cm^2$
$B = 20cm^2$
$C = 60cm^2$

$140cm^2$

Triangular areas

The area of a triangle takes up **half** the space of the rectangle that is formed around it

Area of triangle $= \frac{1}{2}(b \times h)$



$A = \frac{1}{2}(7m \times 4m) = \frac{1}{2}(28m^2)$

$14m^2$

Be sure to use perpendicular heights

Calculate base × height first

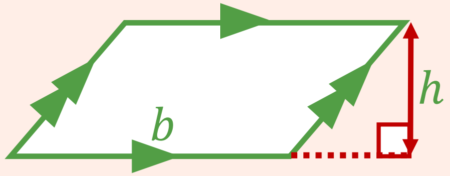
Remember to **halve** your answer!

Geometry – Advanced areas

EZY MATHS

Parallelogram

Imagine a tilted rectangle



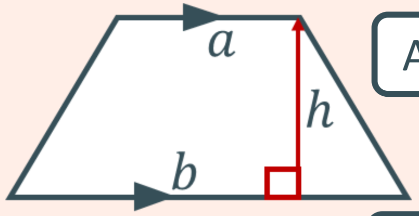
▱ $= b \times h$

Be sure to use **perpendicular heights**

Trapezium

A more complex formula to know

⏢ $= \frac{1}{2}(a+b)\times h$



Add the parallel sides

Halve it

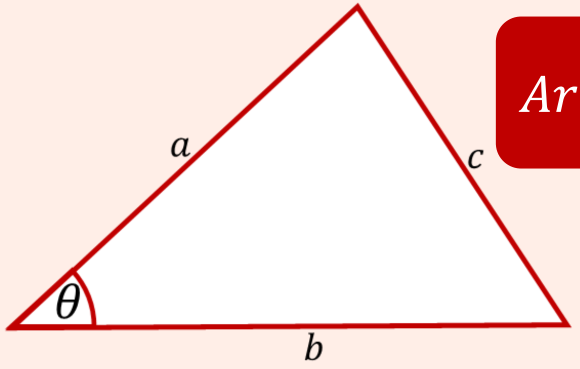
Multiply by height

Calculating area of a triangle using $^1/_2\, ab\, sinC$

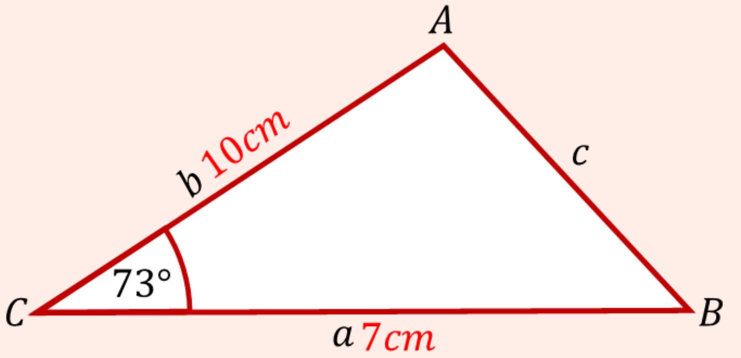
Two sides and the included angle

Label triangle

Substitute values in and calculate



$Area = {}^1/_2\, ab\, sinC$



We have two sides and the included angle

$$Area = {}^1/_2\,(7)(10)\sin(73°)$$

$$Area = {}^1/_2 \times 7 \times 10 \sin(73°)$$

$$Area = \underline{\underline{33.47cm^2}}$$

Geometry – Circle Definitions

EZY MATHS

Circumference

The perimeter **around** the circle

Diameter

The distance **across the centre** of the circle

Radius

The distance from the **centre to the edge** of the circle

Sector

Part of the area of a circle, enclosed by two radii

Arc

Part of the circumference of a circle

Tangent

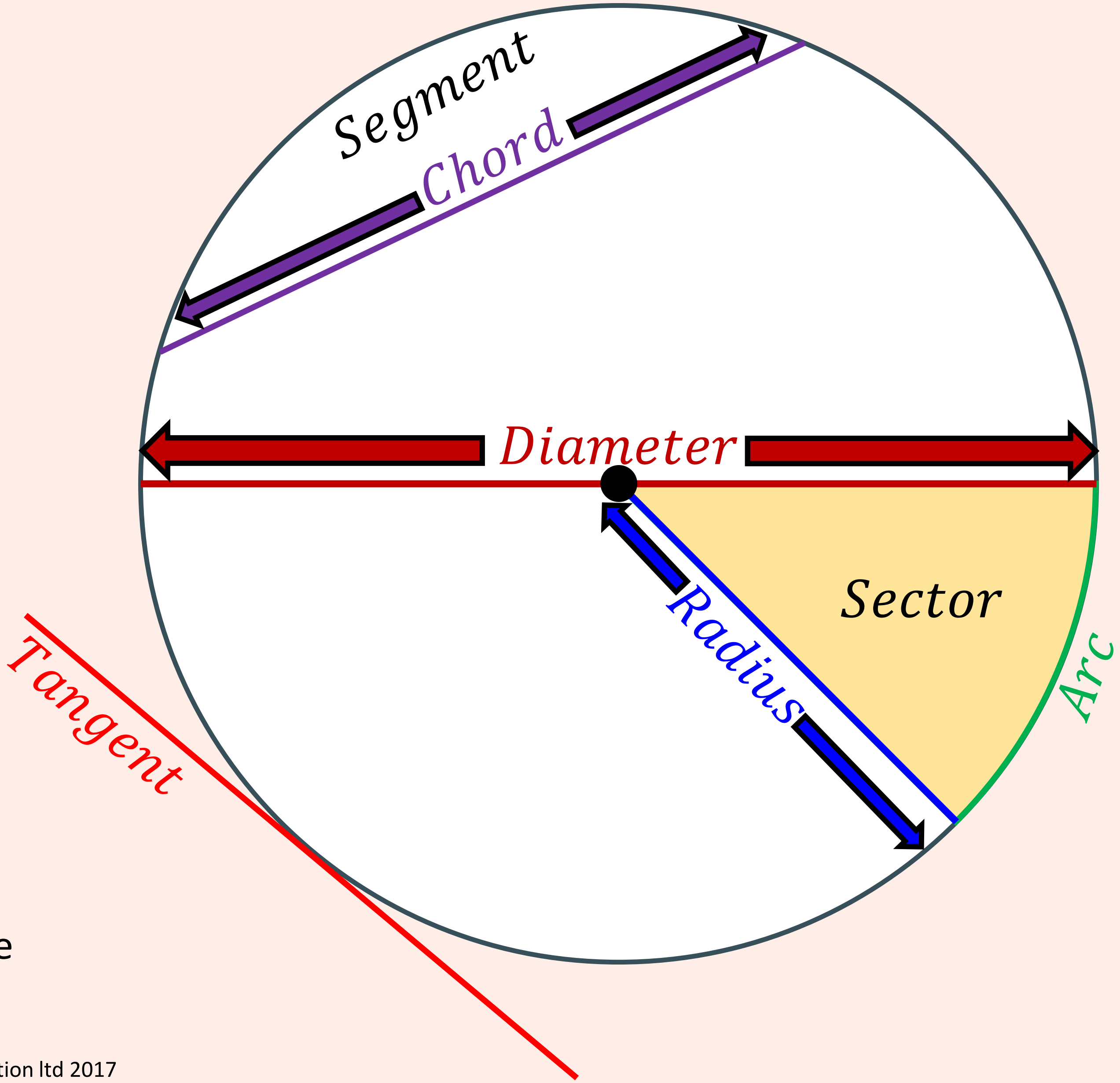
A straight line that touches the curve of the circle at a point

Chord

A straight line segment between two points on the circle edge

Segment

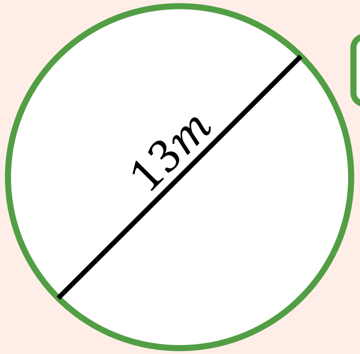
The area created by the chord



Geometry – Area and Circumference

Area

$A = \pi r^2$ Pi times the radius squared



Diameter is double the radius

$A = \pi \times 6.5^2$

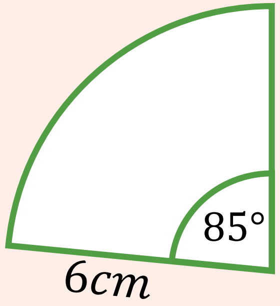
$A = \pi \times 42.25$

$A = 132.73m^2$

Sector

$A = \frac{n°}{360}\pi r^2$

Calculate the **proportion** of the circle required then use area formula



$\frac{85°}{360°}(\pi \times 6^2)$

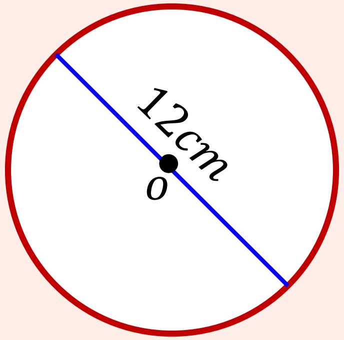
$26.7cm^2$

Circumference

$C = \pi d$

$C = 2\pi r$

The circumference is always about three time the length of the diameter



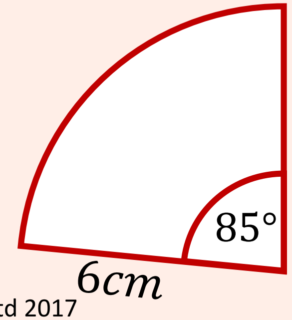
$C = \pi \times 12cm$

$C = 37.7cm$

Arc length

$L = \frac{n°}{360}\pi d$

Calculate the **proportion** of the circle required then use circumference formula



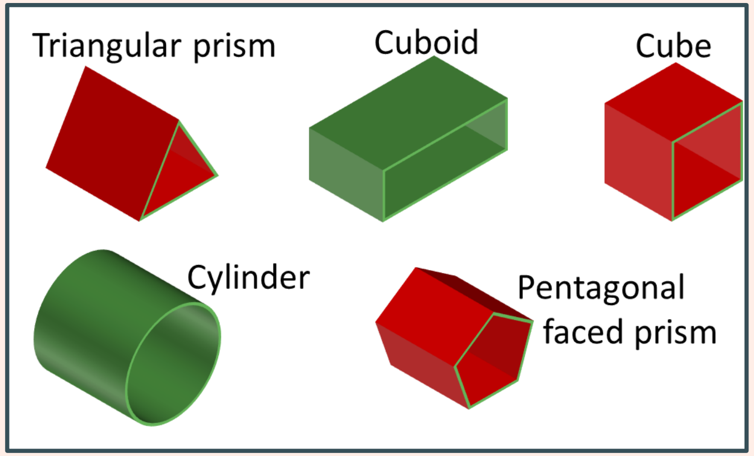
$\frac{85°}{360°}(\pi \times 12)$

$8.90cm$

Geometry – Volume

EZY MATHS

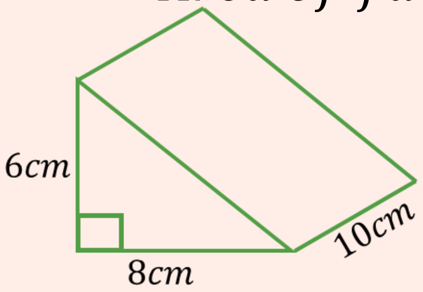
Prisms



The same cross sectional area throughout

$$Volume = Area\ of\ face \times depth$$

$$Area\ of\ face = \frac{1}{2}(8 \times 6)$$



$$24cm^2$$

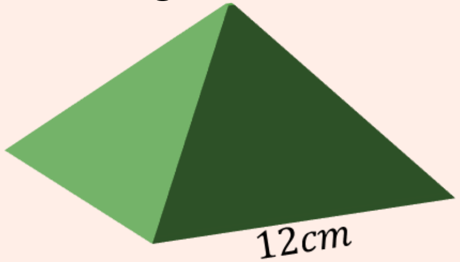
$$24cm^2 \times 10cm = 240cm^3$$

Pyramids

The volume of a pyramid is always $^1/_3$ of the prism that surrounds it

$$Volume = \frac{Area\ of\ base \times height}{3}$$

Calculate the volume of the square based pyramid if its height measures $15cm$

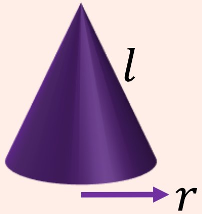


$$12cm^2 \times 15cm = 2160cm^3$$

$$2160cm^3 \div 3$$

$$720cm^3$$

Curved area of cone



$$Area = \pi rl$$

Spheres

$$Volume = \frac{4}{3}\pi r^3$$

Find the volume of the sphere with diameter $16cm$. Give your answer to 3 significant figures

$$Volume = \frac{4}{3} \times \pi \times (8)^3$$

$$2144.660585cm^3$$

$$2140cm^3$$

For a **hemisphere**, don't forget to **halve** your answer

Number – Approximation and Error Intervals (Bounds)

EZY MATHS

Approximation

Estimates tell us the rough value of a calculation

$$\frac{103.5 \times 1.92}{51.36} \approx \frac{100 \times 2}{50}$$

Rounding off makes it easier to calculate

Round values to 1s.f.

Perform Calculation

$$\frac{8.41 \times 3.2}{0.00216} \approx \frac{8 \times 3}{0.002} \Rightarrow \frac{24}{0.002} \Rightarrow \frac{24000}{2}$$

$$= 12000$$

Error Intervals

By definition, a rounded number does not give us the exact value

Lower Bound The minimum a value might be

Upper Bound The maximum a value might be

Continuous Values (Decimal values)

Halve accuracy level

Add on for Upper bound

Subtract for Lower bound

$240m \text{ to nearest } 10m$

$$235m \leftarrow \boxed{240m} \rightarrow 245m$$

$$235m \leq x < 245m$$

Discrete values (Whole values)

The number of people on a train is 400 to the nearest 100

$$350 \leftarrow \boxed{400} \rightarrow 449$$

Geometry – Similarity and Congruence

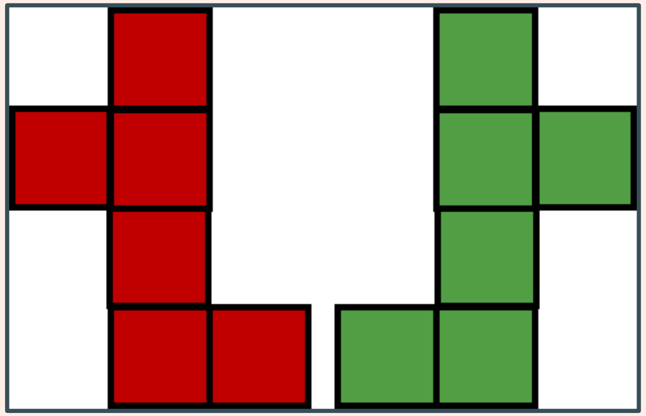
EZY MATHS

Congruence

Congruent shapes are just **exact replicas** of the original

The angles and side lengths **remain the same**

The shapes may well be orientated differently



The two shapes are congruent.
They are reflections of each other.

Similarity in 1D

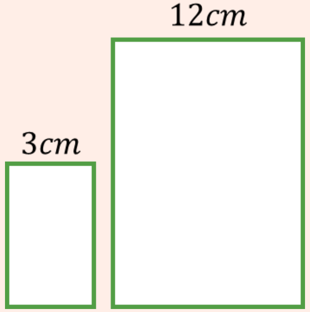
Similar shapes are just **enlargements** of the original

The angles remain the same but the **lengths** have been **scaled up or down**

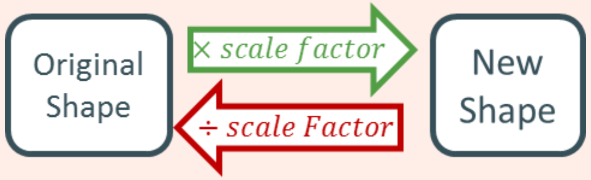
This **scale factor** needs to be calculated in order to solve problems involving similar shapes.

Find two comparative lengths.

$$Scale\ factor = \frac{New\ shape}{Original\ shape}$$



$$Scale\ factor = \frac{12cm}{3cm} = 4$$

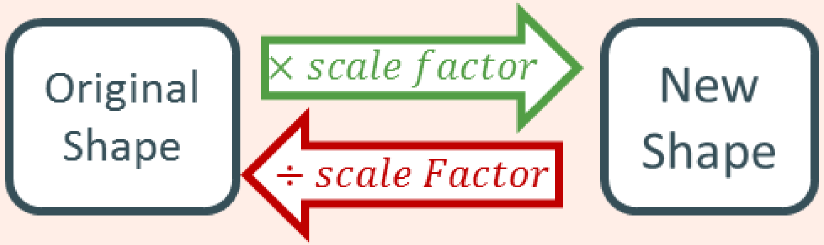


Similarity in more than 1D

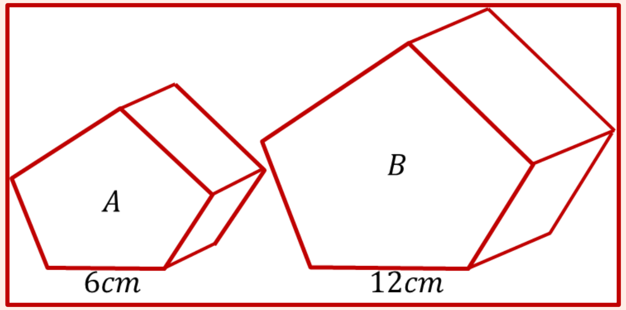
Area and Volume scale factors will need to be calculated

Area scale factor = $Scale\ factor^2$

Volume scale factor = $Scale\ factor^3$



Work out **all** scale factors first



Scale factor $= 2$

Area scale factor $= (2)^2 = 4$

Volume scale factor $= (2)^3 = 8$

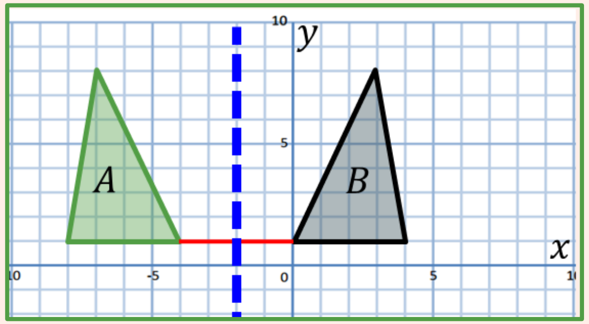
Geometry – Transformations

EZY MATHS

Reflection

Reflection: the replacement of each point on one side of a line by the point symmetrically placed on the other side of the line.

Need a mirror line



Reflection in the line $x = -2$

Rotation

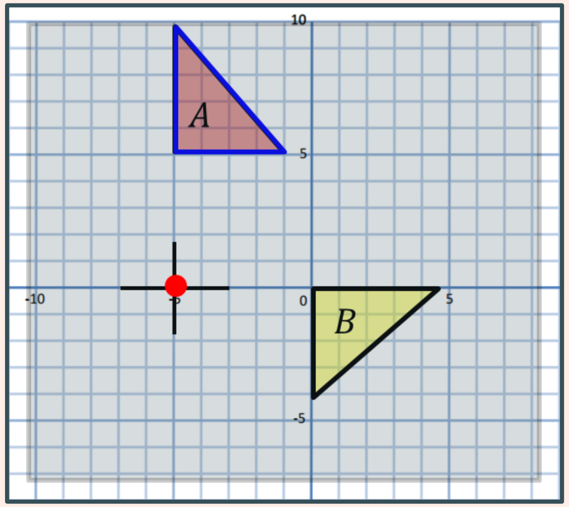
Rotation: the action of rotating a shape about an axis or central point.

Need a Centre point

Need a Direction

Need a measure of turn

Use of tracing paper



Rotation

$90°$ Clockwise

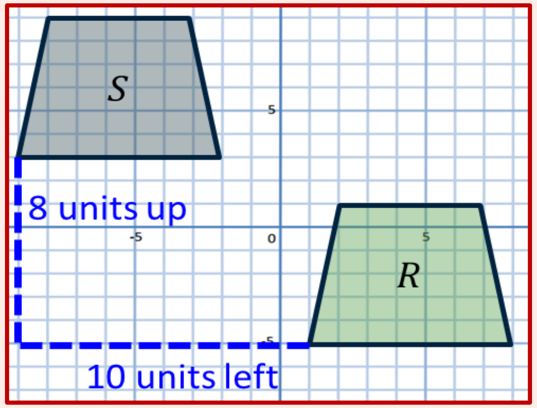
About $(-5,0)$

Translation

Translation: the action of 'sliding' a shape to a new position.

Described using Vector Notation

$\begin{pmatrix} x \\ y \end{pmatrix}$ x → Direction along; y → Direction up/down



$\boldsymbol{R \rightarrow S} =$ Translation $\begin{pmatrix} -10 \\ 8 \end{pmatrix}$

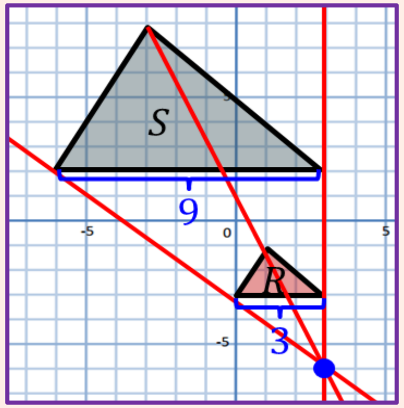
Enlargement

Enlargement: the action of resizing a shape to the scale factor given from a specific point.

Need a Centre point

Need a scale factor

$\frac{\textit{New shape}}{\textit{Original shape}}$



$\boldsymbol{R \rightarrow S}$

Use projection lines

Enlargement

Scale factor 3

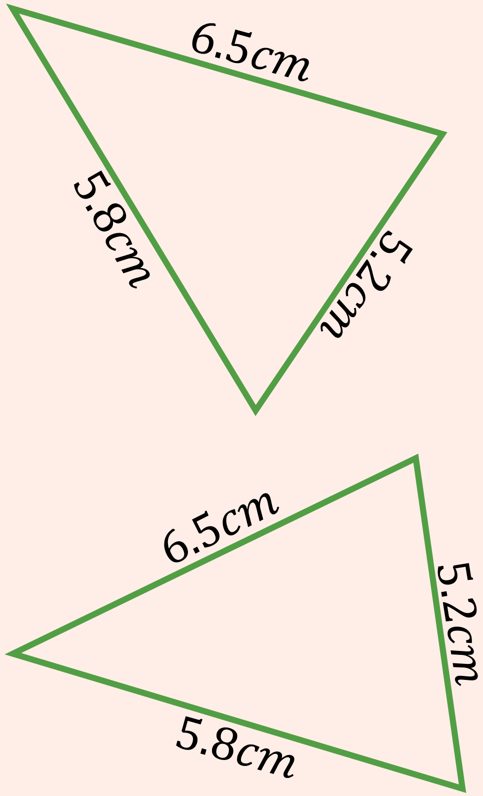
From $(3, -6)$

Geometry – Congruence criteria for triangles

EZY MATHS

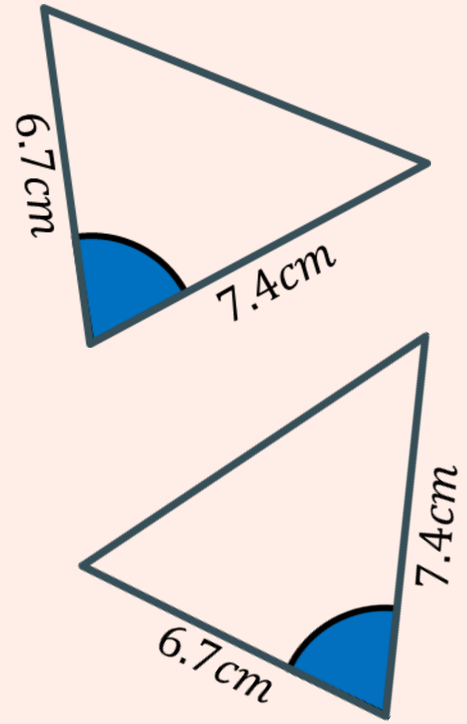
Side, Side, Side

All three sides of one triangle are equal to the corresponding sides of the other triangle.



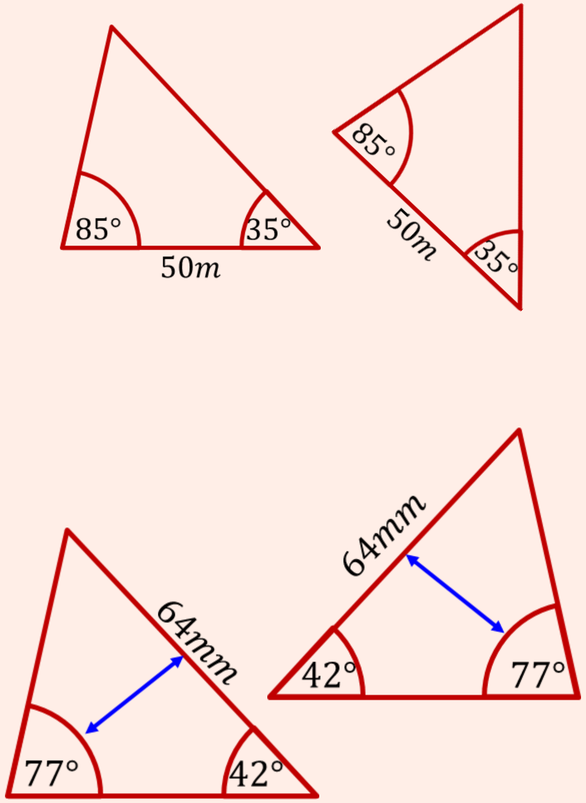
Side, Angle, Side

Two sides and the included angle are equal to the corresponding sides and included angle of the other triangle



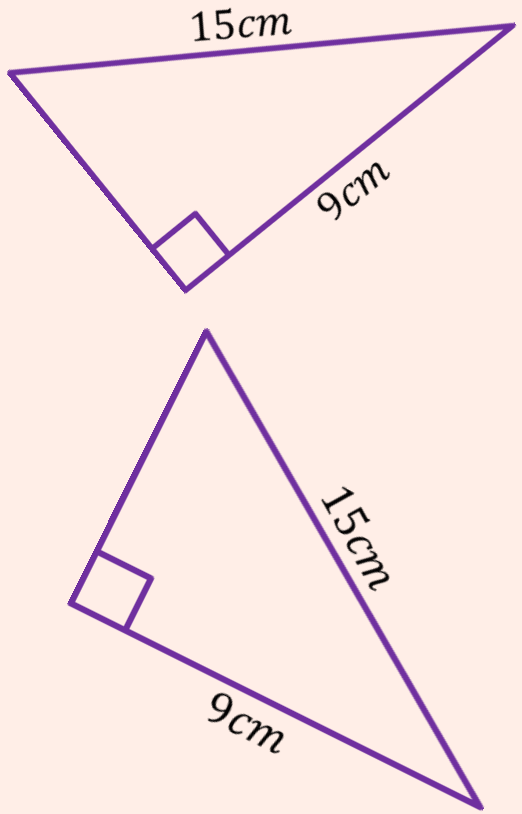
Angle, Side, Angle

Two angles and one side of a triangle are equal to the corresponding angles and side of the other triangle

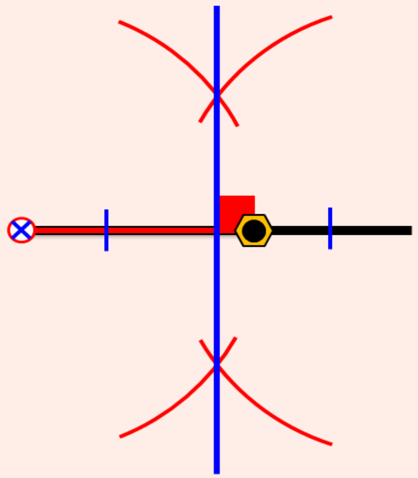


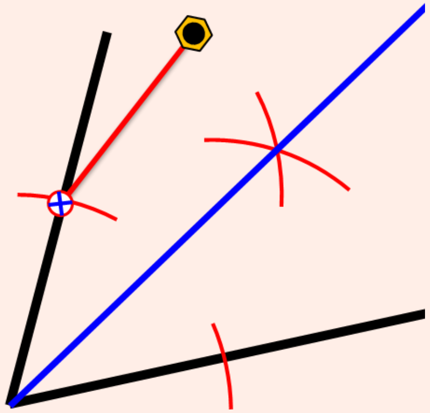
Right, Hypot, Side

Each triangle contains a right angle, the hypotenuses are equal in length as well as another equal comparative side.



Geometry – Constructing bisectors and Loci





Set compass to over halfway

Mark off each line segment

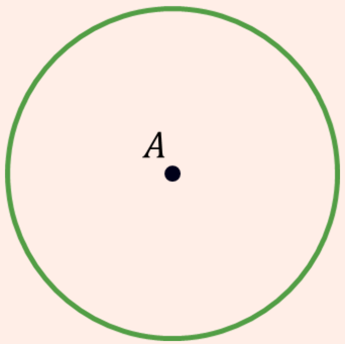
Create arcs above and below the line

Create arcs within the angle

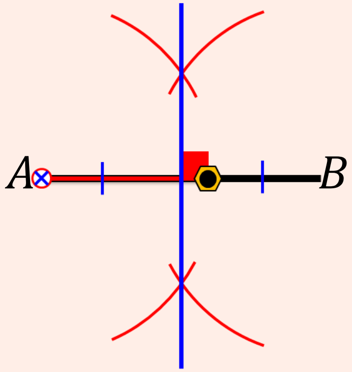
Do not alter the compass at all!

Do not alter the compass at all!

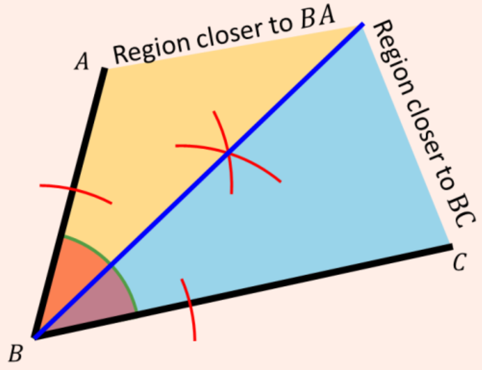
Sometimes you will need to create a separate line segment



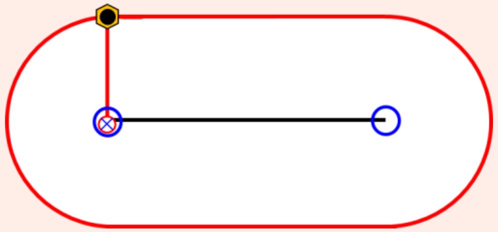
The locus of points from a point is a circle



The locus of points between two points is the perpendicular bisector



The locus of points between two lines is the angle bisector



Notice how the locus of points around a corner is CURVED

A locus is a series of points that satisfy a particular condition. Loci is the plural and will often involve several conditions.

Algebra – Solving by factorising and the Quadratic formula

EZY MATHS

Solving by factorising

$ax^2 + bx + c = 0 \Rightarrow (x \pm \quad)(x \pm \quad) = 0$

Factorise the quadratic – You may need to rearrange first

$x^2 + 8x + 7 = 0 \Rightarrow (x + 7)(x + 1) = 0$

$x = -7 \text{ or } -1$

Find values for x that will make each bracket $= 0$

$2x^2 - 2x = 3(1 - x) \Rightarrow 2x^2 - 2x = 3 - 3x$

Expand and rearrange to $= 0$

$2x^2 + x - 3 = 0 \Rightarrow (2x + 3)(x - 1) = 0$

$x = -\frac{3}{2} \text{ or } +1$

The quadratic formula

The formula you **need** to know

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substitute values into the formula to generate two answers for x

$5x^2 + 8x - 4$ (a = 5, b = 8, c = −4)

Identify values of a, b and c

$$x = \frac{-(8) \pm \sqrt{(8)^2 - 4(5)(-4)}}{2(5)}$$

Substitute and simplify

$$x = \frac{-8 \pm \sqrt{144}}{10}$$

Carry out two calculations

$x = 0.4$ or -2

Algebra – Completing the square and solving quadratics

EZY MATHS

Changing the form of the quadratic

$$ax^2 + bx + c \Rightarrow (x+p)^2 + q$$

$x^2 + bx + c$

Halve the coefficient of b

$\left(x + {}^{b}/_{2}\right)^2 - \left({}^{b}/_{2}\right)^2 + c$

Simplify

$(x+p)^2 + q$

$$x^2 + 6x - 2 \Rightarrow (x+3)^2 - (3)^2 - 2$$

$$(x+3)^2 - 11$$

Solving equations by completing the square

Complete the square	Solve equation

$$x^2 - 10x + 15 = 0 \Rightarrow (x-5)^2 - 10 = 0$$

$$(x-5)^2 = +10 \Rightarrow x - 5 = \pm\sqrt{10}$$

$$x = 5 \pm \sqrt{10} \Rightarrow \begin{array}{l} x = 5 + \sqrt{10} \\ x = 5 - \sqrt{10} \end{array} \quad \boxed{\begin{array}{l} = 8.16 \\ = 1.84 \end{array}}$$

Completing the square $a \neq 1$

$$3x^2 + 6x - 4 = 0$$

Factor out the three

$$3\underbrace{\left(x^2 + 2x - \frac{4}{3}\right)} = 0$$

Complete the square on this expression

$$3\left((x+1)^2 - (1)^2 - \frac{4}{3}\right) = 0$$

Complete the square.

$$3\left((x+1)^2 - \frac{7}{3}\right) = 0$$

Expand

$$3(x+1)^2 - 7 = 0$$

Solve for x.

$$3(x+1)^2 - 7 = 0$$

Add 7 to both sides

$$3(x+1)^2 = 7$$

Divide both sides by 3

$$(x+1)^2 = \frac{7}{3}$$

Square root both sides

$$x + 1 = \pm\sqrt{\frac{7}{3}}$$

Subtract 1 from both sides

$$\Rightarrow x = -1 \pm \sqrt{\frac{7}{3}}$$

$$\boxed{x = 0.528 \text{ or } -2.528}$$

Algebra – Simultaneous equations

EZY MATHS

Simultaneous equations

Equations involving two or more unknowns that are to have the same values in each equation

$$4x + 3y = 5 \qquad 2x - 3y = 4 \qquad 3y + 10x = 7$$
$$3x + 2y = 4 \qquad 5x + 2y = 1 \qquad y = 2x + 1$$

Linear equations (Elimination method)

Multiply equations to get matching coefficients → Add/subtract equations → Substitute to find second variable

$$\begin{aligned} 4x + 3y &= 5 \;\times 3 \\ 3x + 2y &= 4 \;\times 4 \end{aligned} \Rightarrow \begin{aligned} 12x + 9y &= 15 \\ -\;12x + 8y &= 16 \\ \hline y &= -1 \end{aligned}$$

Substitute $y = -1$ into equation 2

$$3x + 2(-1) = 4 \Rightarrow 3x - 2 = 4 \Rightarrow \underline{x = 2}$$

<u>Matching coefficients:</u>

Same signs (Subtract the equations)

Opposite signs (Add the equations)

Linear equations (Substitution method)

$$\begin{aligned} 3y + 10x &= 7 \\ y - 2x &= 1 \end{aligned} \Rightarrow \begin{aligned} 3y + 10x &= 7 \\ y &= 2x + 1 \end{aligned}$$

Rearrange to get a single variable on its own

Substitute $y = 2x + 1$ into equation 1

$$3(2x + 1) + 10x = 7$$
$$16x + 3 = 7$$
$$\underline{x = 0.25}$$

Substitute equations to find first variable

Substitute $x = 0.25$ into equation 2

$$y = 2(0.25) + 1 \Rightarrow \underline{y = 1.5}$$

Substitute to find second variable

Quadratic equations (Substitution method)

$$\begin{aligned} y &= x + 6 \\ y &= x^2 - 2x + 2 \end{aligned} \Rightarrow \begin{aligned} x + 6 &= x^2 - 2x + 2 \\ 0 &= x^2 - 3x - 4 \end{aligned}$$

Substitute equation into quadratic and rearrange to $= 0$

$$(x + 1)(x - 4) = 0 \qquad x = -1 \; or + 4$$

Factorise and find two solutions for variable

$$\begin{aligned} y &= (-1) + 6 \\ y &= 5 \end{aligned} \qquad \begin{aligned} y &= (4) + 6 \\ y &= 10 \end{aligned}$$

Substitute each answer to find other pair of solutions

Algebra – Inequalities

EZY MATHS

Symbols

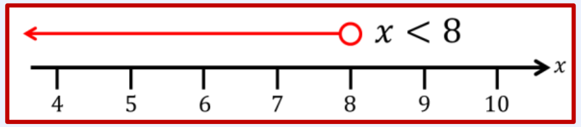
$x > y$	x is greater than y	$x < y$	x is less than y
$x \geq y$	x is greater than or equal to y	$x \leq y$	x is less than or equal to y

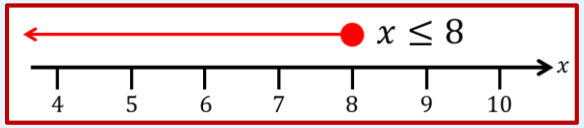
Inequalities on a numberline

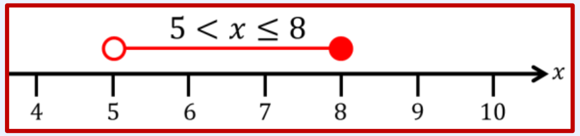
Shows the range of values for an inequality

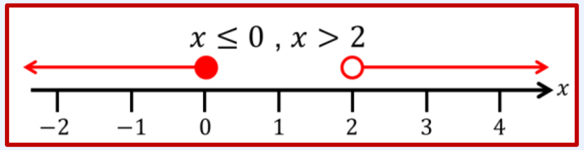
Open circle does **not include** value

Closed circle **includes** value









Solving Linear inequalities

Same process as solving linear equations

When we multiply or divide by a **negative number**, we must **flip** the inequality sign

$$x + 5 \leq 16$$
$$x \leq 16 - 5$$
$$x \leq 11$$

$$-2x > 4$$
$$x < -2$$

$$4 < 3x + 1 < 16$$
$$1 < x < 5$$

Solving Quadratic inequalities

Same process as solving equations

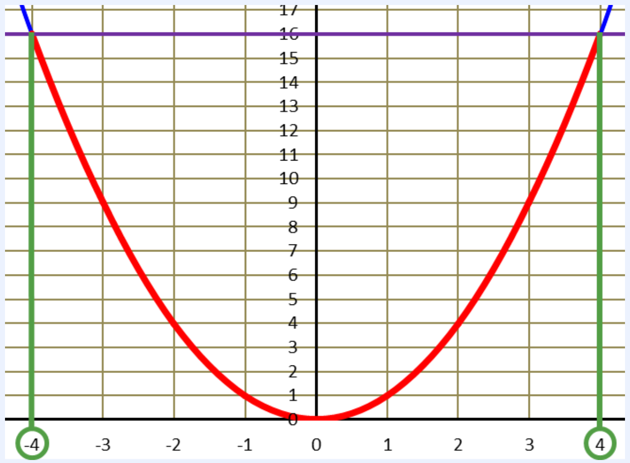
Sketch the graph to interpret the range of values required.

$$x^2 \leq 16$$
$$\sqrt{} \qquad \sqrt{}$$
$$x = \pm 4$$

The roots

$$-4 \leq x \leq 4$$

The range of values below the line



Two variable inequalities

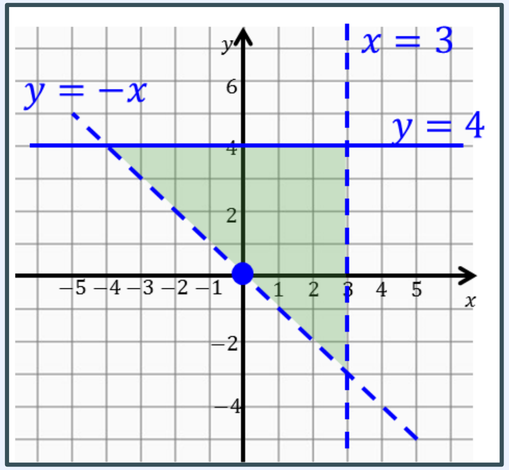
Inequalities with 2-variables need to be represented on a graph

Shade the region satisfied by the inequalities: $y > -x$, $y \leq 4$, $x < 3$

Draw the line graph for each inequality

$<$ - Dashed line

$\leq$ - Solid line



Statistics – Probability

EZY MATHS

Introduction

The likelihood of an event happening

0%	1/4 25%	1/2 50%	3/4 75%	100%
0	0.25	0.5	0.75	1
Impossible		Even Chance		Certain

Counting outcomes

Working out how many combinations there are

Rolling a die and flipping a coin

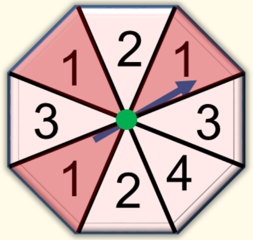
	1	2	3	4	5	6
Heads	H, 1	H, 2	H, 3	H, 4	H, 5	H, 6
Tails	T, 1	T, 2	T, 3	T, 4	T, 5	T, 6

This is a sample space diagram

There are 12 possible outcomes from this event

Calculating probability

$$P(\text{Event}) = \frac{number\ of\ successful\ outcomes}{total\ number\ of\ outcomes}$$



$$P(3) = \frac{2}{8} \Rightarrow \frac{1}{4}$$

Simplify answers where possible

The 'OR' rule (mutually exclusive)

$$P(a\ or\ b) = P(a) + P(b)$$

$$P(2\ or\ 4) = \frac{2}{8} + \frac{1}{8} \Rightarrow \frac{3}{8}$$

Add each probability

The 'AND' rule (independent)

$$P(a\ and\ b) = P(a) \times P(b)$$

Flip a coin twice

$$P(2\ tails) = \frac{1}{2} \times \frac{1}{2} \Rightarrow \frac{1}{4}$$

Multiply each probability

Types of events

Mutually exclusive

Events that cannot happen at the same time

Rolling a die → $P(1\ and\ 6)$

All probabilities from the event will **sum** to make **1**

Independent events

Events where the outcome of one **doesn't** affect the outcomes of the others

Picking a counter out of a bag, replacing it and repeating.

Dependent events

Events where the outcome of one **does** affect the outcomes of the others

Picking a counter out of a bag, **not** replacing it and repeating.

Calculating expected outcomes

$$P(event) \times number\ of\ trials$$

Statistics – Venn Diagrams and Probability trees

EZY MATHS

Venn diagrams

A set is a collection of things, called elements

The set of prime numbers less than 12 $\quad A = \{2, 3, 5, 7, 11\}$

Intersections (∩) and Unions (∪)

$A = \{2, 3, 5, 7, 11\} \qquad B = \{1, 3, 5, 7, 9\}$

$A \cap B = \{3, 5, 7\}$ – Intersection of A and B

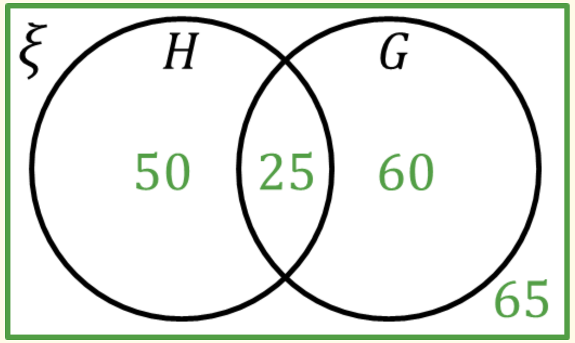
The crossover between sets

$A \cup B = \{1, 2, 3, 5, 7, 9, 11\}$ – Union of A and B

All values in the sets

$A' \cap B = \{1, 9\}$

Values in B but not in A



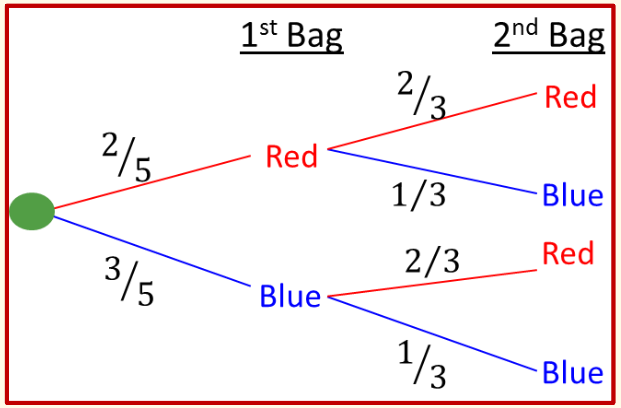
$P(G) = \frac{85}{200}$

$P(H \cap G) = \frac{25}{200}$

$P(H \cap G') = \frac{50}{200}$

Probability trees

Probability trees are really useful to calculate the probabilities of combined events happening



Multiply along branches

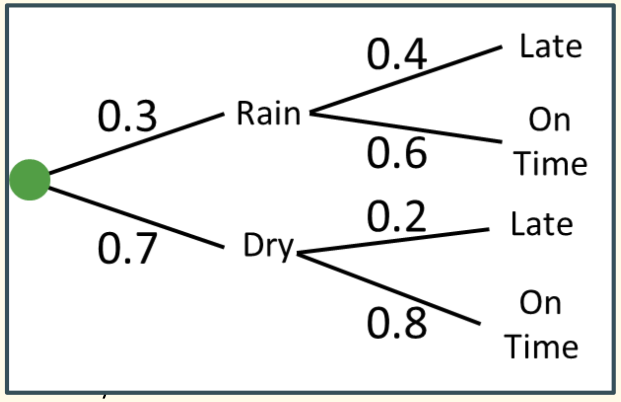
$P(Red\ and\ Red) = \frac{4}{15}$

$P(1\ Red\ and\ 1\ Blue) = \frac{2}{15} + \frac{6}{15} = \frac{8}{15}$

Add together all combinations

Dependent events

Probability trees where the outcome of one events affects the outcome of the next event e.g. no replacement, weather etc.



$P(Rain\ and\ late) = (0.3 \times 0.4) = 0.12$

$P(On\ time) = (0.18 + 0.56) = 0.74$

When dealing with **no replacement**, remember to reduce the denominator by **one** for the second event

Number – Standard Form

EZY MATHS

Basic Structure

$1 \leq a < 10 \longleftarrow a \times 10^{b} \longrightarrow$ *Whole number*

$$2.83 \times 10^{6} = 2830000$$

Positive power of 10 $=$ Large number

$$3.14 \times 10^{-4} = 0.000314$$

Negative power of 10 $=$ Small decimal number

Add/Subtract Standard form

Take numbers out of Standard form.
Add/Subtract values.
Convert answer back to Standard form.

$$(3.23 \times 10^{4}) + (8.2 \times 10^{3})$$
$$= 32300 + 8200$$
$$= 40500$$
$$= \underline{\underline{4.05 \times 10^{4}}}$$

Multiply/Divide Standard form

Separate the numbers and powers of 10.
Multiply/Divide numbers,
Apply laws of indices to power of 10s
Give answer in Standard form

$$(4.6 \times 10^{4}) \times (3 \times 10^{3})$$
$$\boxed{4.6 \times 3} \times \boxed{10^{4} \times 10^{3}}$$
$$13.8 \times 10^{7} ✗$$
$$\underline{\underline{1.38 \times 10^{8}}} ✓$$

$$(1.56 \times 10^{-4}) \div (7.5 \times 10^{-7})$$
$$\boxed{1.56 \div 7.5} \times \boxed{10^{-4} \div 10^{-7}}$$
$$0.208 \times 10^{3} ✗$$
$$\underline{\underline{2.08 \times 10^{2}}} ✓$$

Algebra – Algebraic fractions

EZY MATHS

Addition and Subtraction

Make sure the denominators are the same

$$\frac{x}{y}+\frac{3}{y}=\frac{x+3}{y}$$

Multiply fraction/s to achieve same denominator

$$\frac{x^2+5}{y^2}+\frac{x\times y}{y\times y} \Rightarrow \frac{x^2+5}{y^2}+\frac{xy}{y^2} \Rightarrow \frac{x^2+5+xy}{y^2}$$

Same rules apply in Subtraction

$$\frac{x^2}{9(x-5)}-\frac{x+4\times 9}{x-5\times 9} \Rightarrow \frac{x^2}{9(x-5)}-\frac{9(x+4)}{9(x-5)}$$

Expand brackets and simplify where possible on the **numerators**

$$\frac{x^2-9x-36}{9(x-5)}$$

$$\frac{2}{x+1}-\frac{5x}{x-4} \Rightarrow \frac{2(x-4)}{(x-4)(x+1)}-\frac{5x(x+1)}{(x-4)(x+1)}$$

$$\frac{2(x-4)-5x(x+1)}{(x-4)(x+1)} \Rightarrow \frac{-8-5x^2-3x}{(x-4)(x+1)}$$

Multiplication and Division

Multiply across the numerators and denominators

Cross cancel terms where possible | Simplify each fraction | Factorise expressions

$$\frac{2}{x}\times\frac{x^2}{y} \Rightarrow \frac{2}{1\cancel{x}}\times\frac{\cancel{x^2}x}{y} \Rightarrow \frac{2x}{y}$$

$$\frac{6a+6b}{2}\times\frac{1}{a+b} \xrightarrow{Factorise} \frac{6(a+b)}{2}\times\frac{1}{a+b}=3$$

To **divide**, multiply by reciprocal of 2nd fraction

$$\frac{4yz}{x}\div\frac{yz^2}{10} \xrightarrow{Keep, change, flip} \frac{4yz}{x}\times\frac{10}{yz^2}$$

$$\frac{4\cancel{yz}}{x}\times\frac{10}{\cancel{yz^2}z} \Rightarrow \frac{40}{xz}$$

Algebra – Functions

EZY MATHS

Think of it as a machine that has an input which is processed by the function to give an output.

$$f(x) = x + 2$$

Input ⇨ Function ⇨ Output

x ⇨ $x + 2$ ⇨ Output

$f(4) = 6 \quad f(-3) = -1$

Substitute input into the function to generate output value

Using functions

$$g(x) = \sqrt{4x - 3} \text{ find } g(21)$$

$$g(21) = \sqrt{4x - 3}$$

Substitute input into the function and calculate

$$g(21) = \sqrt{81} \Rightarrow g(21) = \pm 9$$

$$f(t) = 3t^2 + 2 \text{ find } f(2)$$

$$f(2) = 3t^2 + 2$$

Substitute input into the function and calculate

$$f(2) = 12 + 2 \Rightarrow f(2) = 14$$

Inverse functions

A function that performs the opposite process of the original function ➡ You have been given the output and need to work out the value of the input.

Normal function $f(x)$

Inverse function $f^{-1}(x)$

Two ways to solve problems involving inverse functions

$$f(x) = 5x + 2$$

Function machine method	Subject of Formula method
Input → $\times 5$ → $+2$ → Output	$y = 5x + 2 \Rightarrow y - 2 = 5x$
Input ← $\div 5$ ← -2 ← Output	$f^{-1}(x) = \dfrac{x-2}{5} \Leftarrow \dfrac{y-2}{5} = x$

Composite functions

The combination of two or more functions to create a new function

$f(x) = 2x + 2$ and $g(x) = x - 2$. Find $fg(x)$

The output of $g(x)$ will form the input of $f(x)$

$$g(x) = \boxed{x - 2}$$

$$f(x) = 2x + 2$$

$$f(x - 2) = 2x + 2$$

$$f(x - 2) = 2(x - 2) + 2$$

$$fg(x) = 2x - 2$$

RPR – Rates of change

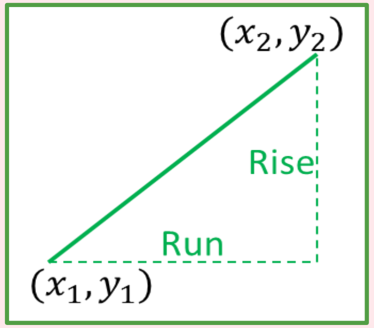
Rate of Change

A rate that describes how one quantity changes in relation to another quantity

It is represented by the Gradient of a line

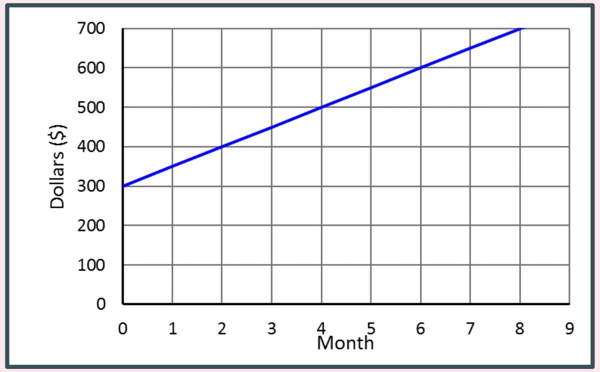
$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{Gradient} = \frac{Rise}{Run}$$



Interpreting Rates of Change

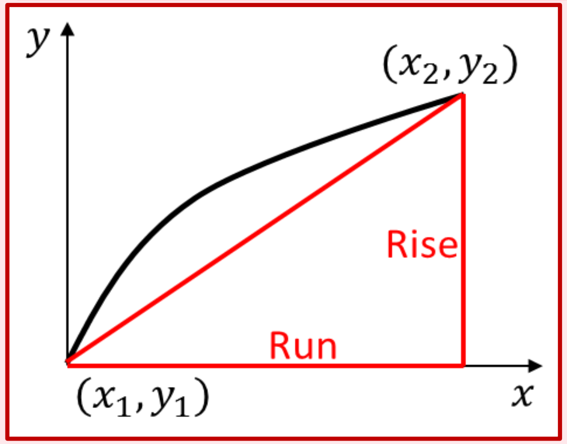
Gradient → Amount of (y) per Amount of (x)



Rate of change $=$ \$50 per month

Average rate of change

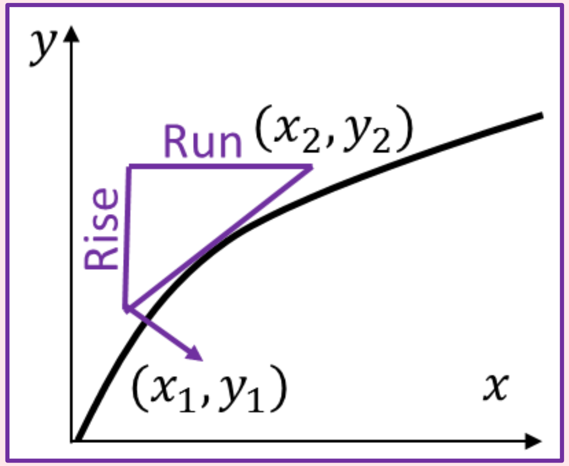
The rate of change **over a given interval**



- Create **chord** between two intervals
- Calculate gradient of chord
- Interpret gradient as a rate of change

Instantaneous rate of change

The rate of change at a **particular moment**



- Create **tangent** at specific point
- Calculate gradient of tangent
- Interpret gradient as a rate of change

Graphs – Translations and Reflections

EZY MATHS

Translation

A translation can be defined as the movement 'sliding' of a shape to a new position

$f(x) + a$

The graph shifts up/down the $y-axis$ by a units

$f(x + a)$

The graph shifts left/right along the $x - axis$ by a units

$+ =$ Left

$- =$ Right

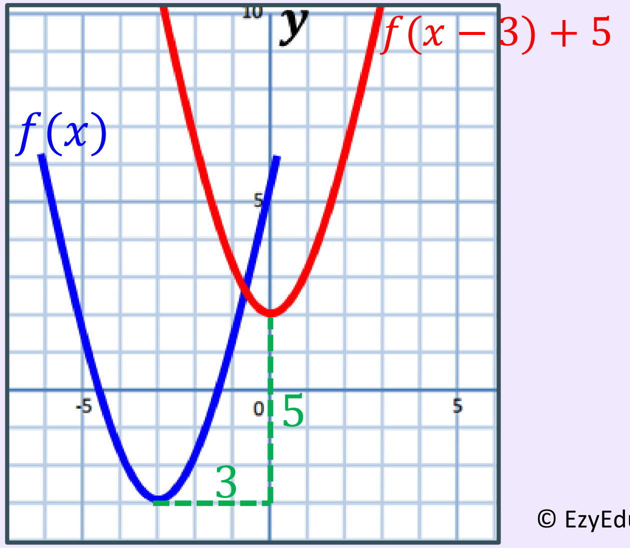
Adding to the function causes a Translation

Given $f(x)$,

Sketch $f(x - 3) + 5$

3 units right

5 units up



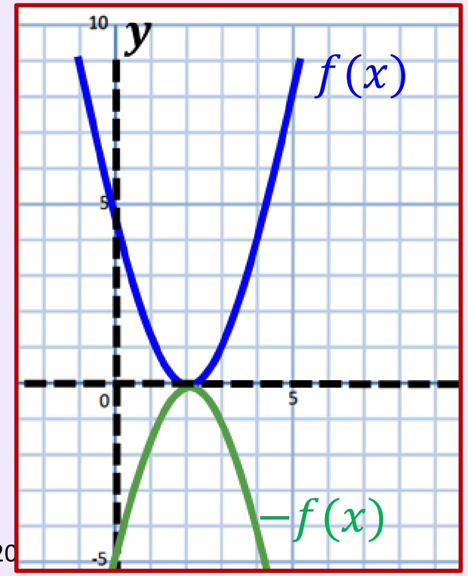
Reflection

Reflection: The replacement of each point on one side of a line by the point symmetrically placed on the other side of the line.

$y = -f(x)$

The **outputs** are reversed

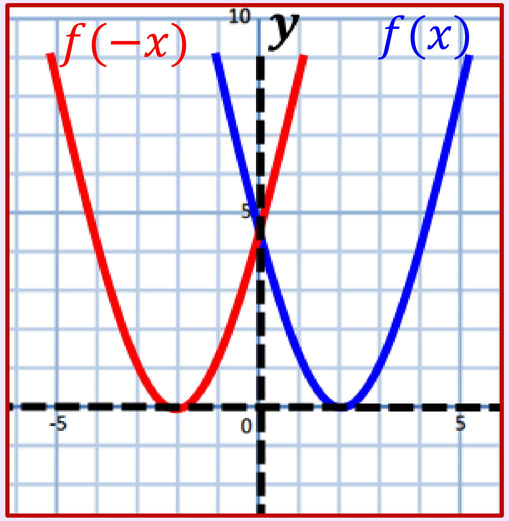
The graph is reflected in the $\boldsymbol{x - axis}$



$y = f(-x)$

The **inputs** are reversed

The graph is reflected in the $\boldsymbol{y - axis}$



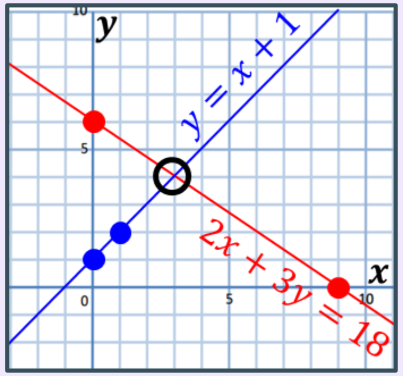
Graphs – Using graphs to find solutions

EZY MATHS

When we are given a system of equations, we can find the solutions to these equations algebraically (simultaneous equations) or **graphically.**

$$\left.\begin{aligned} 2x + 3y &= 18 \\ y &= x + 1 \end{aligned}\right\}$$

Solve simultaneously or **graph** each equation

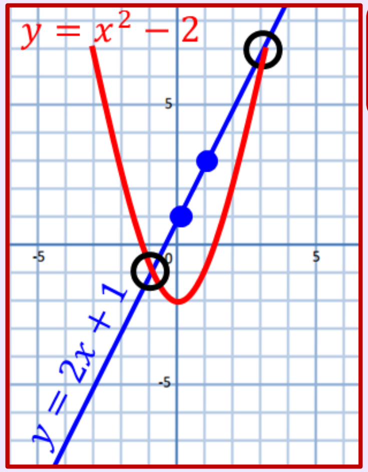


The point of **intersection** will be your **solutions**.
The point where x and y have the same value for each equation.

$x = 3 \quad y = 4$

$$\left.\begin{aligned} y &= x^2 - 2 \\ y &= 2x + 1 \end{aligned}\right\}$$

Solve simultaneously or **graph** each equation



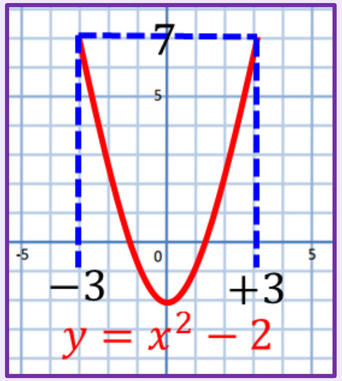
Two intersections = **two sets** of solutions

$x = 3 \quad y = 7$

or

$x = -1 \quad y = -1$

Find the solutions of x when $x^2 - 2 = 7$



Plot graph and read off points

$x = 3$

or

$x = -3$

Plot each equation separately

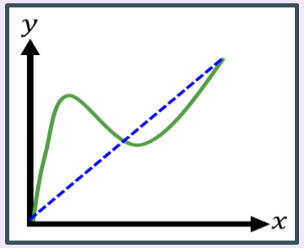
Identify and read off the points of intersections for your solutions

Use graph to read off specific values for x and y

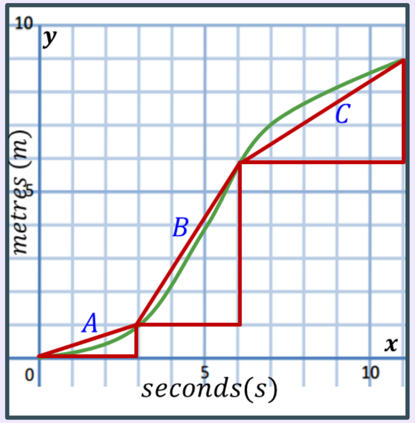
Graphs – Estimating gradients, Area under a curve

EZY MATHS

Estimating Gradients



Calculate average gradient from beginning to end.
These are not very accurate and do not show the full picture

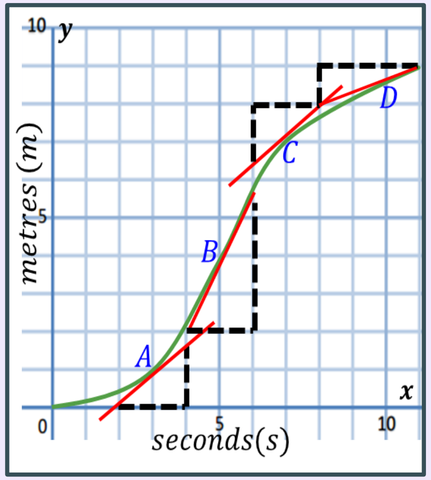


Break the graph down into smaller pieces to see what is happening

$Gradient\ A = {}^{1}/_{3} \Rightarrow 0.3m/s$

$Gradient\ B = {}^{5}/_{3} \Rightarrow 1.7\ m/s$

$Gradient\ C = {}^{3}/_{5} \Rightarrow 0.6m/s$



Find out what is happening at a particular point - **Tangents**

$Gradient\ A = \frac{1.5}{2}$

$0.75m/s$ at 3 seconds

$Gradient\ C = \frac{1.5}{2}$

$0.75m/s$ at 7 seconds

$Gradient\ B = \frac{3.5}{2}$

$1.75m/s$ at 5 seconds

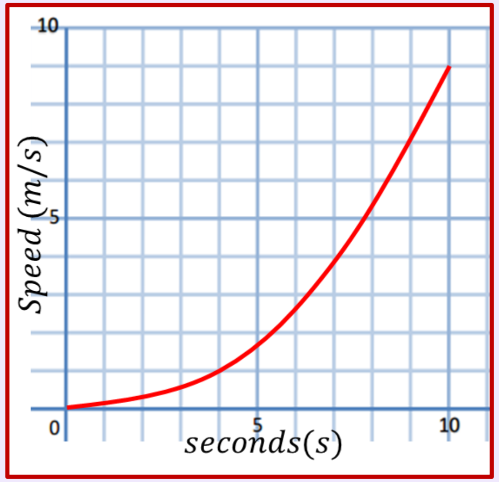
$Gradient\ D = \frac{1}{3}$

$0.3m/s$ at 10 seconds

Estimate because tangents vary

Area under a curve

The area under a curve will enable you to estimate the total distance travelled in velocity time graphs



Formulae needed

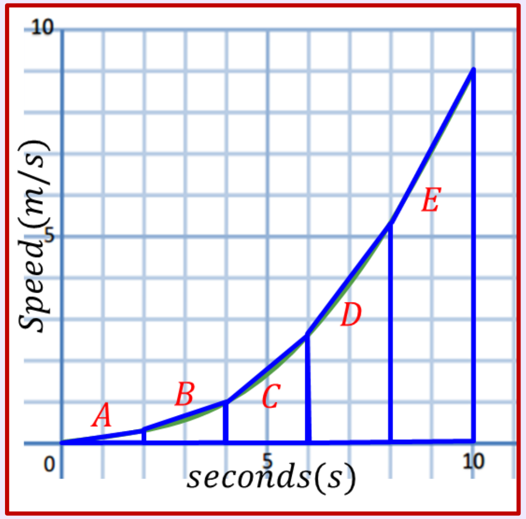
Trapezium

$$\frac{1}{2}(a+b)\times h$$

Triangle

$$\frac{1}{2}(b\times h)$$

Estimate the distance travelled for the first ten seconds



$Area\ A = 0.5\times 2\times 0.5$

$Area\ A = 0.5m$

$Area\ B = 0.5(0.5+1)\times 2$

$Area\ B = 1.5m$

$Area\ C = 0.5(1+2.5)\times 2$

$Area\ C = 3.5m$

$Area\ D = 0.5(3+5)\times 2$

$Area\ D = 8m$

$Area\ E = 0.5(5+9)\times 2$

$Area\ E = 14m$

$27.5m$

Geometry – Sine and Cosine rules

EZY MATHS

Using Trigonometry to calculate missing angles and side lengths in non right angled triangles

The Sine rule formula

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

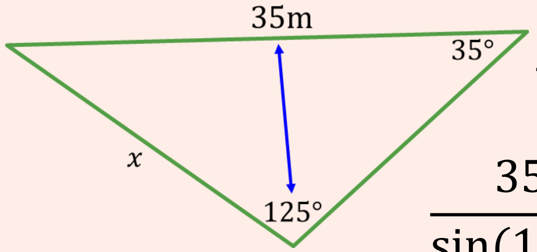
Missing length

or

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Missing angle

We tend to use the Sine rule if we know an **angle** and its **opposite length**

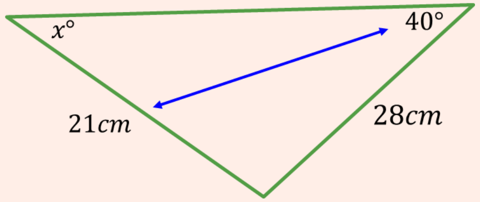


$$\frac{35}{\sin(125)} = \frac{x}{\sin(35)} \quad \times \sin(35)$$

$$\frac{35}{\sin(125)} \times \sin(35) = x$$

$24.51m$

$$\frac{\sin(x°)}{28} = \frac{\sin(40)}{21} \quad \times 28$$



$x = 59°$

$$\sin(x) = \frac{\sin(40) \times 28}{21}$$

$$x° = \sin^{-1}\left(\frac{\sin(40) \times 28}{21}\right)$$

The Cosine rule formula

$a^2 = b^2 + c^2 - 2bc \cos A$ → Missing length

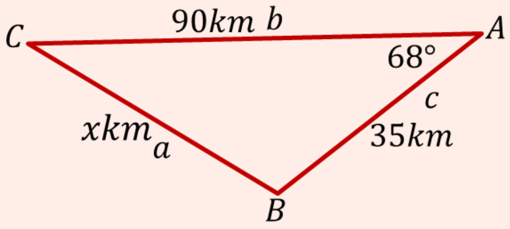
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$ → Missing angle

If we know two sides **AND** the included angle

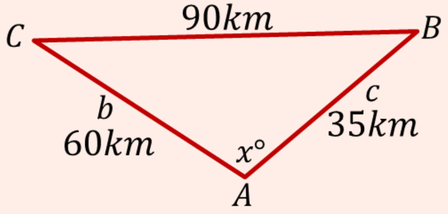
$$x^2 = (90)^2 + (35)^2 - (2(90)(35)\cos(68°))$$

$$x^2 = 6964.978 \ldots$$

$$x = 83.46km$$



$$\cos x = \frac{(60)^2 + (35)^2 - (90)^2}{2(60)(90)}$$



$$\cos x = -0.303 \ldots$$

$$x = \cos^{-1}(-0.303 \ldots)$$

$x = 107.7°$

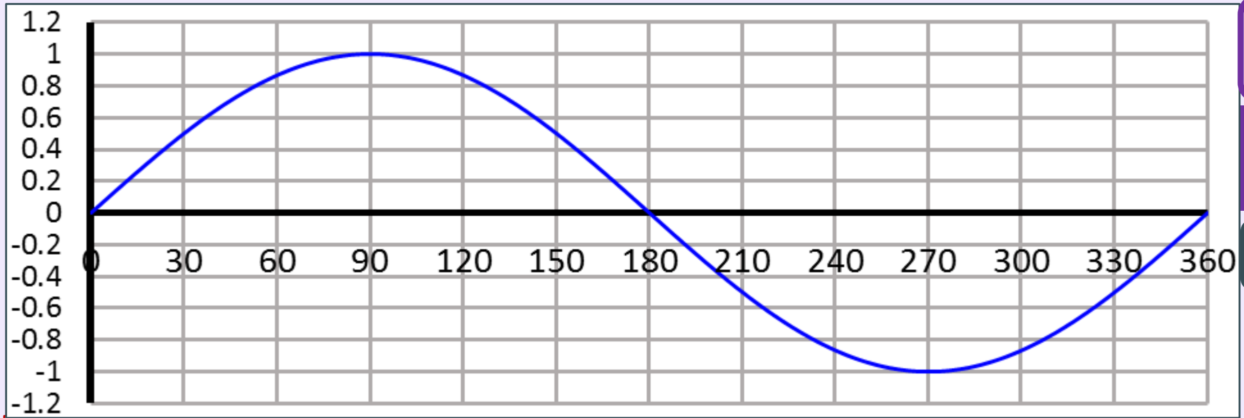
Graphs – Trigonometric Graphs

EZY MATHS

They are periodical
they will continue to repeat every $360°$ in both directions

There can be multiple answers for values of $Sin(\theta), Cos(\theta), Tan(\theta)$

Use symmetry of graphs to solve problems

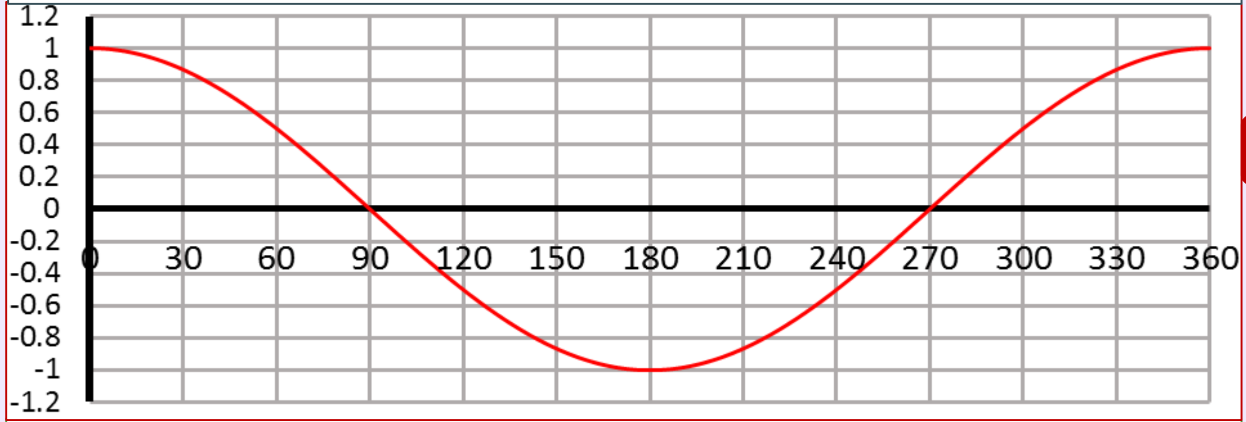


Sine

$$Sin(\theta) = \frac{opp}{hyp}$$

x axis intercept at $0°, 180°, 360°$

Turning points at $90°, 270°$

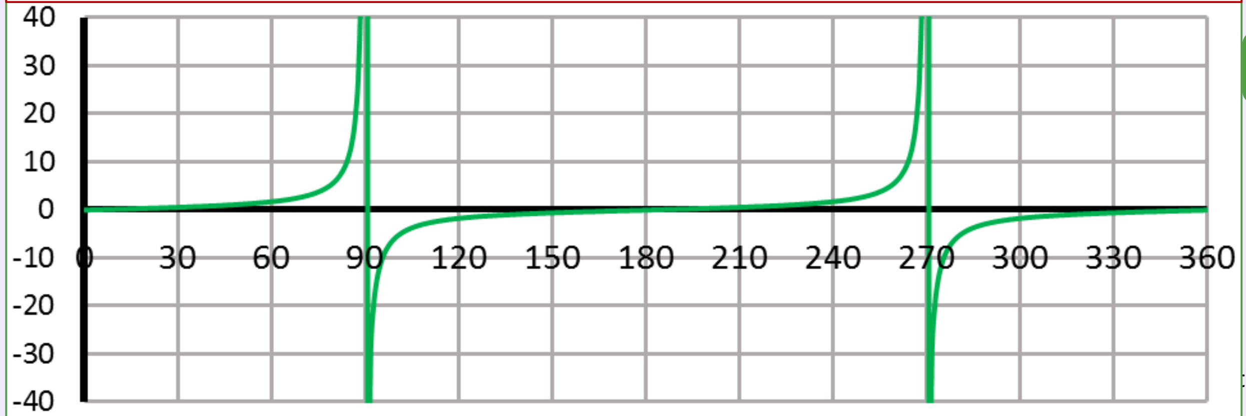


Cosine

$$Cos(\theta) = \frac{adj}{hyp}$$

x axis intercept at $90°, 270°$

Turning points at $0°, 180°, 360°$



Tangent

$$Tan(\theta) = \frac{opp}{adj}$$

x axis intercept at $90°, 270°$

Asymptotes at $90°, 270°$

Statistics – Types of data and Sampling

EZY MATHS

Types of data

Quantitative

Data that is **numeric**

Discrete

Data that can be **counted** and only has certain values

People on a bus
Shoe Size
Dress size

Continuous

Data that can be **measured** to various levels of accuracy

Height of a tree
Speed of a car
Mass of a person

Qualitative

Data that is **descriptive**

Categorical data

Data which can be grouped into categories

Hair colour
Favourite food
Sport

Grouped Data

Data which is organised into classes

Primary

Data collected by **you**

Secondary

Data gathered from **another source**

Sampling

We collect and analyse data to give us information about a **population**

Census

Data is collected from the **WHOLE** population

Can take a very long time to collect the information

Sample

Data is collected from **PART** of the population

Quicker to collect the data and the data can be used to describe the whole population

Random

Your sample is randomly selected

Each member assigned a number
Numbers randomly generated
Those numbers used in sample

Stratified

Proportionate numbers from each group selected to make sample

$$\frac{\text{Amount in group}}{\text{Total number}} \times \text{Sample size}$$

Bias

Some situations can cause bias and make the sample unrepresentative

When and where the sample is taken?
Is the sample large enough?
Who is in the sample?

Probability and Statistics – Frequency and Two-Way Tables

EZY MATHS

Favourite Colour

Yellow	Blue	Red	Red
Red	Blue	Green	Pink
Blue	Red	Blue	Green
Pink	Red	Blue	Red
Red	Blue	Green	Blue

Tally Up | Add Tally Marks

Colour	Tally	Frequency	Relative Frequency
Yellow	|	1	1/20 = 0.05
Blue	~~||||~~ ||	7	7/20 = 0.35
Red	~~||||~~ ||	7	7/20 = 0.35
Green	|||	3	3/20 = 0.15
Pink	||	2	2/20 = 0.1
	Sum	20	

Relative frequency describes what proportion selected that colour

If the sample size is large enough, we can interpret relative frequencies as probabilities

Two–Way Frequency Tables

Provides information about the frequency of two variables

	Year 12	Year 13	Total
Male	120	80	200
Female	70	100	170
Total	190	180	370

Number of girls in Year 12 is 70.

Rows and columns add up!

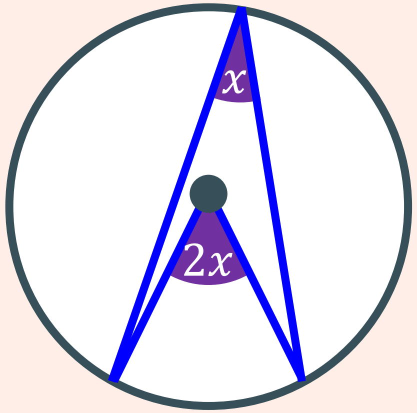
	Year 12	Year 13	Total
Male	120	+ 80	= 200
Female	+ 70	100	170
Total	= 190	180	370

Use to calculate missing values

Geometry – Circle Theorems

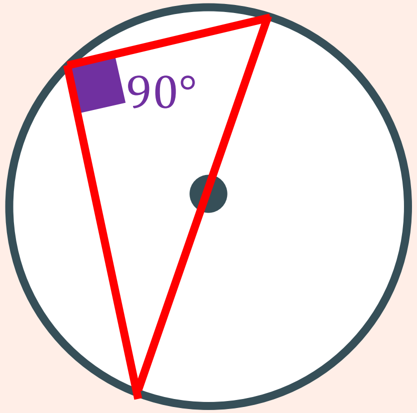
EZY MATHS

Angle at the centre is twice the angle at the circumference



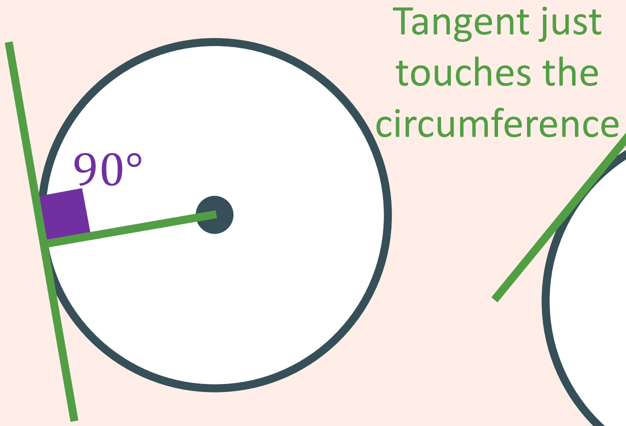
Look for the 'Arrow' Shape!

Angle subtended at circumference by a semicircle is 90°

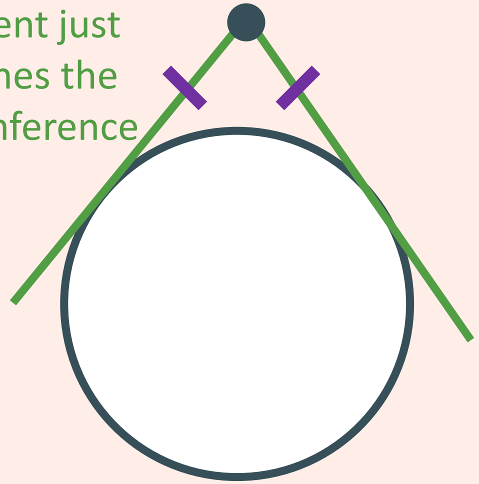


Opposite angle to the diameter!

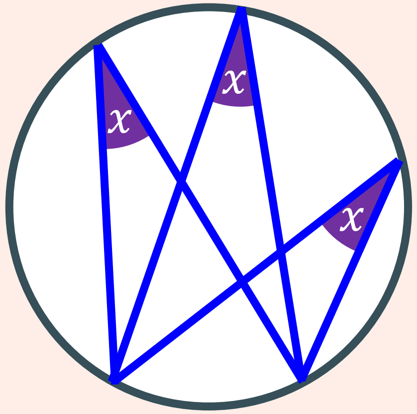
Tangents and radii meet at 90°



Tangents from a point have equal length

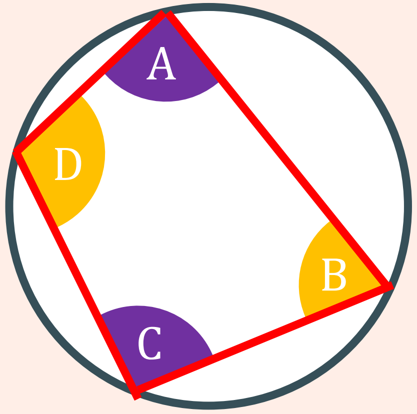


Angles in the same segment are equal



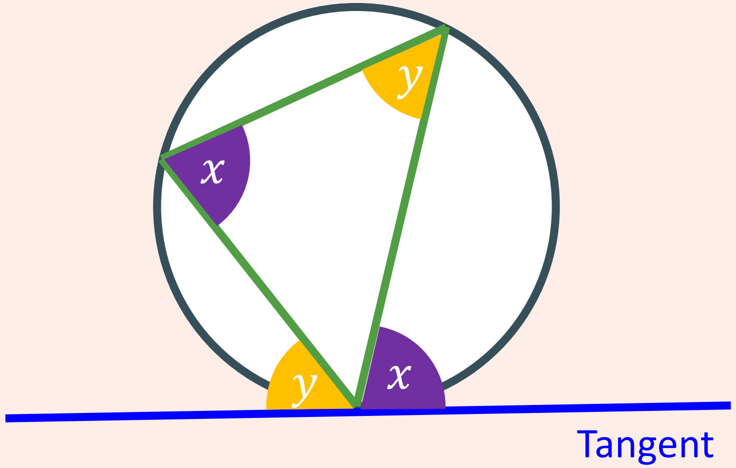
Look for the 'Bow' Shape!

Opposite angles in a cyclic quadrilateral sum to 180°



$A + C = 180°$ $B + D = 180°$

Alternate Segment Theorem



Number – Surds and Rationalising the denominator

EZY MATHS

Surds

Surds are expressions which contain an irrational square root

$\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$ $\quad \sqrt{3} \times \sqrt{7} = \sqrt{3 \times 7} = \sqrt{21}$

$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ $\quad \frac{\sqrt{6}}{\sqrt{10}} = \sqrt{\frac{\cancel{6}\,3}{\cancel{10}\,5}} = \sqrt{\frac{3}{5}}$

$\sqrt{a} + \sqrt{b} \neq \sqrt{a+b}$ $\quad \sqrt{5} + \sqrt{20} = \sqrt{25}$ ✘

Writing in the form $a\sqrt{b}$

Think square numbers

$\sqrt{200}$

$Square\ Factors = 4, 25, 100$

Choose the largest square factor

$\sqrt{100} \times \sqrt{2} = 10\sqrt{2}$

Rationalising the denominator

Rationalising the denominator involves removing all of the roots from the bottom of a fraction.

$\frac{6}{\sqrt{3}} \Rightarrow \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ Multiply top and bottom by irrational root $\Rightarrow \frac{6\sqrt{3}}{\sqrt{9}} \Rightarrow \frac{6\sqrt{3}}{3}$

A more complex denominator

$\frac{5}{3+\sqrt{2}} \Rightarrow \frac{5}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}}$ Multiply top and bottom by Conjugate (opposite root)

$= \frac{5(3-\sqrt{2})}{(3+\sqrt{2})(3-\sqrt{2})}$ Expand and simplify

$= \frac{15-5\sqrt{2}}{9\boxed{-3\sqrt{2}+3\sqrt{2}}-2} = \frac{15-5\sqrt{2}}{7}$

Geometry – Vectors

EZY MATHS

Vectors describe translations

$$\begin{pmatrix} x \\ y \end{pmatrix}$$

x → Direction along

y → Direction up/down

Notation

$\overrightarrow{AB}$ $\boldsymbol{a}$ $\underline{a}$

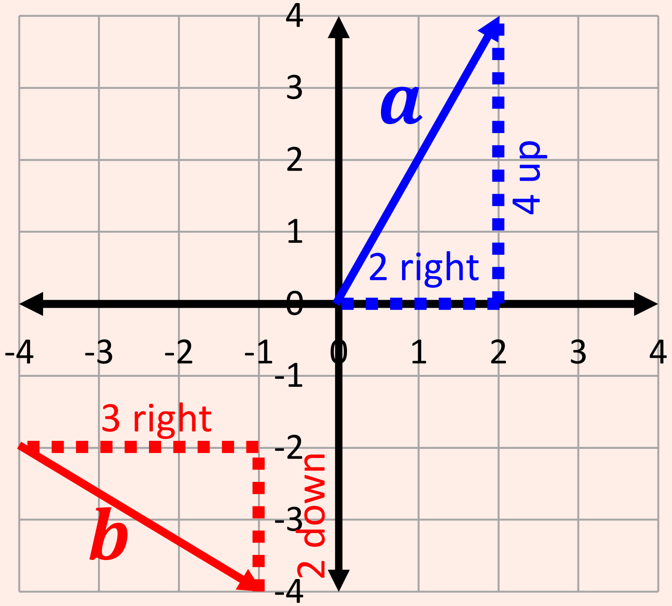
Magnitude = Length of Arrow

Direction = Where arrow is pointing

Represented by arrows

$\boldsymbol{a} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

$\boldsymbol{b} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$

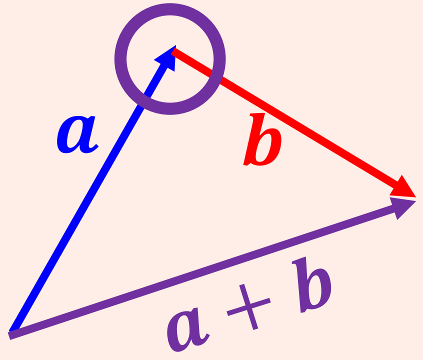


Adding and Subtracting

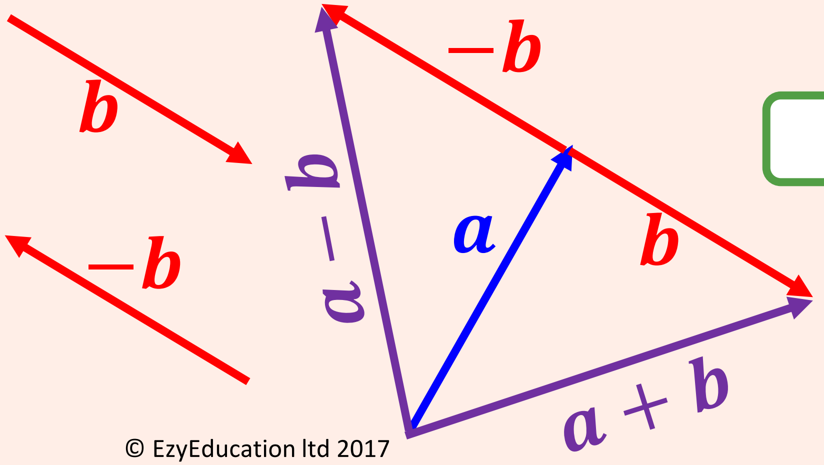
Start arrow at end of previous vector

b starts at the end of ***a***

$\boldsymbol{a} + \boldsymbol{b}$



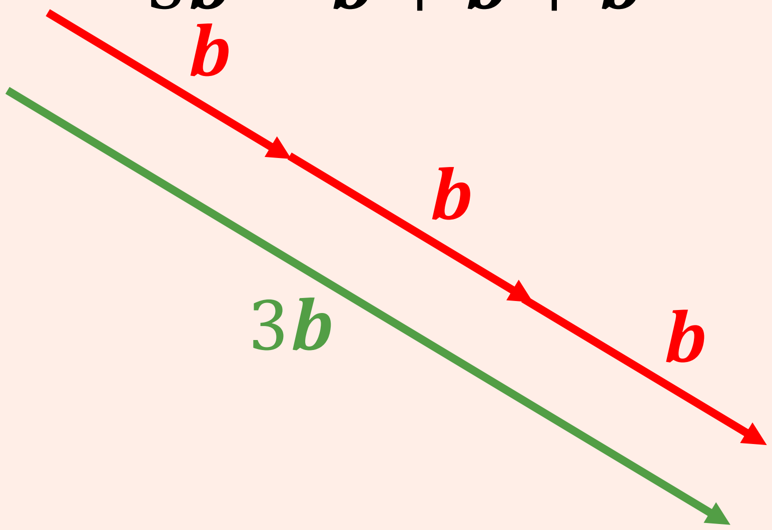
Negative vector goes in opposite direction



Multiplying

Only affects magnitude, not direction

$3\boldsymbol{b} = \boldsymbol{b} + \boldsymbol{b} + \boldsymbol{b}$



Parallel Vectors

Same direction. May have different magnitude.

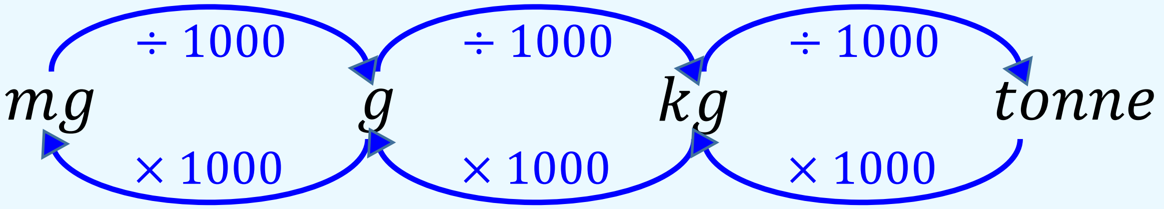
Same Direction

$\boldsymbol{a} - 3\boldsymbol{b}$ $3(\boldsymbol{a} - 3\boldsymbol{b})$

Number – Units – Mass, Length, Area and Volume

EZY MATHS

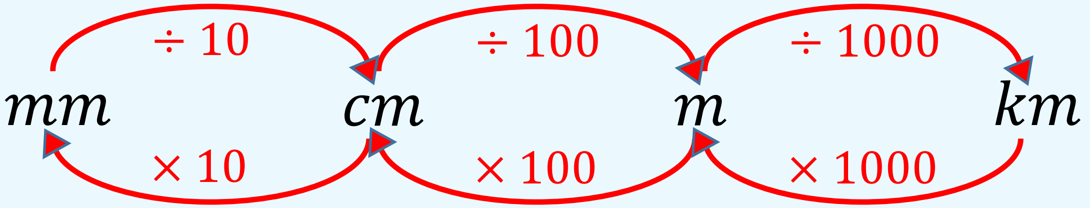
Mass



Standard Metric Conversions

$$1{,}000mg = 1g$$
$$1{,}000g = 1kg$$
$$1{,}000kg = 1\ tonne$$

Length



Standard Metric Conversions

$$10mm = 1\ cm$$
$$100cm = 1m$$
$$1{,}000m = 1\ km$$

Area and Volume

Area is a 2D measurement formed by multiplying two lengths

$$100mm^2 = 1cm^2$$
$$10{,}000cm^2 = 1m^2$$
$$1{,}000{,}000m^2 = 1km^2$$

Squared units results in squared conversion factors

Volume is a 3D measurement formed by multiplying three lengths

$$1000mm^3 = 1cm^3$$
$$1{,}000{,}000cm^3 = 1m^3$$
$$1{,}000{,}000{,}000m^3 = 1km^3$$

Cubed units have cubed conversion factors

Number – Units – Time and Money

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Time

Conversions between units are not decimal

1 Week	7 days
1 Day	24 hours
1 Hour	60 minutes
1 Minute	60 seconds

Convert 350 seconds into minutes and seconds

1 Minute	60 seconds
2 Minutes	120 seconds
3 Minutes	180 seconds
4 Minutes	240 seconds
5 Minutes	300 seconds ✓
6 Minutes	360 seconds ✗

5 minutes and 50 seconds

Add $6^1/_4$ hours to $4{:}23\ am$

$4{:}23\ am \xrightarrow{+6\ hours} 10{:}23\ am \xrightarrow{+15\ mins} 10{:}38\ am$

Money

$$\begin{array}{r} £\,65.20 \\ £\,10.00 \\ +\ £\quad 0.65 \\ \hline £\,75.85 \end{array}$$

Adding and Subtracting money requires lining up the decimal point

Exchange rate calculations

GBP:USD = 1: 1.34

£1 is worth and can be exchanged for $1.34

$GBP \xrightarrow{\times 1.34} USD$, $USD \xrightarrow{\div 1.34} GBP$

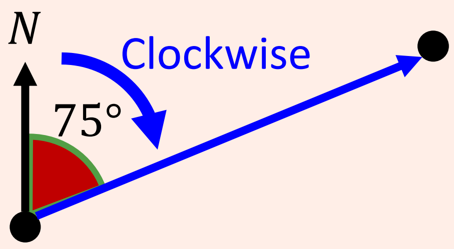
Money comes in specific denominations

Geometry – Bearings

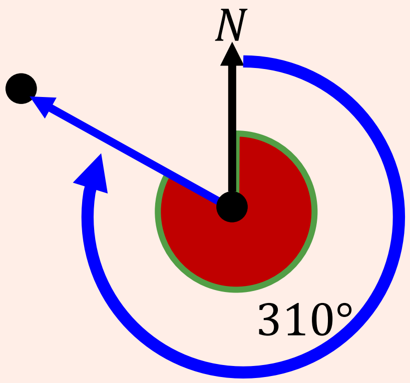
EZY MATHS

Angle clockwise from North

075°



310°



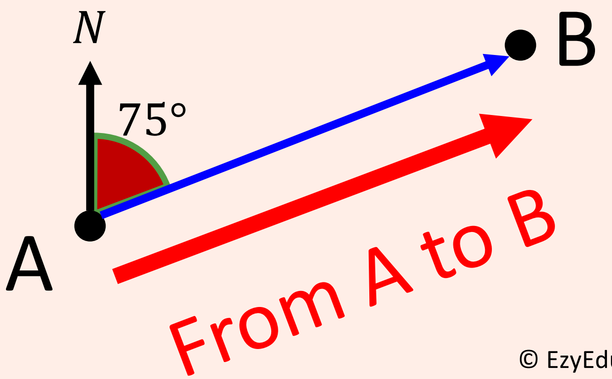
Always 3 digits

$75° \rightarrow 075°$

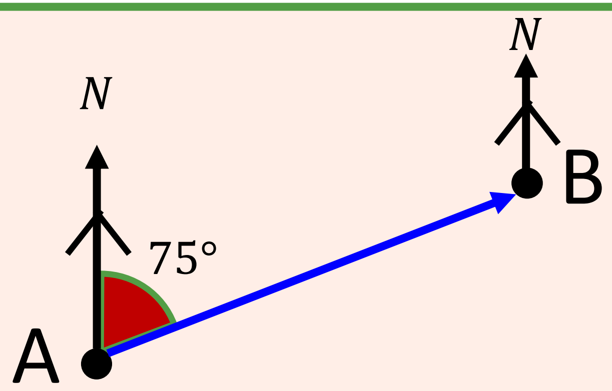
$4° \rightarrow 004°$

Sentence Structure Important

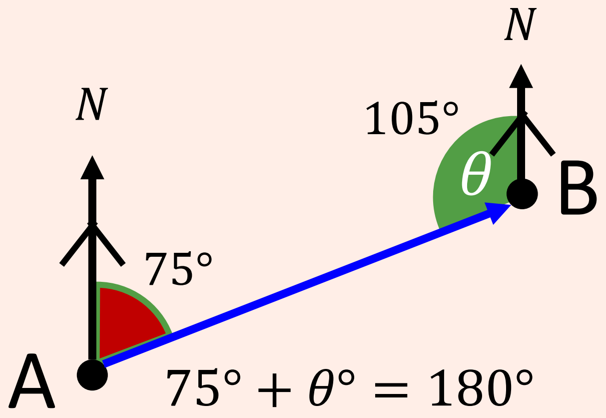
The bearing of B from A is 075°



Lines North are Parallel



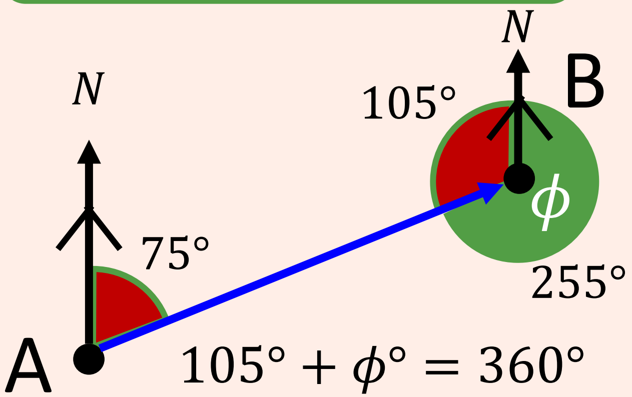
Co-Interior Angles



$$75° + \theta° = 180°$$

$$\theta° = 180° - 75° = 105°$$

Angles around a point



$$105° + \phi° = 360°$$

$$\phi° = 360° - 105° = 255°$$

The bearing of A from B is 255°