

1. The first three terms of a geometric series are

18, 12 and p

respectively, where p is a constant.

Find

- (a) the value of the common ratio of the series, (1)
- (b) the value of p , (1)
- (c) the sum of the first 15 terms of the series, giving your answer to 3 decimal places. (2)

1 Q01aB
1 Q01bB
1 Q01cM
1 Q01cA

(a)

$$r = \frac{u_2}{u_1}$$

$$r = \frac{12}{18}$$

$$r = \frac{2}{3}$$

(b) $12 \times \frac{2}{3} = p = 8$

(c)
$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_n = \frac{18(1-\frac{2}{3}^{15})}{1-\frac{2}{3}} = 53.877 \text{ (3dp)}$$



2. (a) Use the binomial theorem to find all the terms of the expansion of

$$(2 + 3x)^4$$

Give each term in its simplest form.

(4)

(b) Write down the expansion of

$$(2 - 3x)^4$$

in ascending powers of x , giving each term in its simplest form.

(1)

1 Q02aB1
1 Q02aM1
1 Q02aA1
1 Q02aA2
1 Q02bB1

$$(a) \quad {}^4C_0 2^4 + {}^4C_1 2^3 (3x)^1 + {}^4C_2 2^2 (3x)^2 + {}^4C_3 2^1 (3x)^3 + {}^4C_4 (3x)^4$$

$$16 + 96x + 216x^2 + 216x^3 + 81x^4$$

$$(b) \quad 16 - 96x + 216x^2 - 216x^3 + 81x^4$$



3.

$$f(x) = 2x^3 - 5x^2 + ax + 18$$

where a is a constant.

Given that $(x - 3)$ is a factor of $f(x)$,

(a) show that $a = -9$

(2)

(b) factorise $f(x)$ completely.

(4)

Given that

$$g(y) = 2(3^{3y}) - 5(3^{2y}) - 9(3^y) + 18$$

(c) find the values of y that satisfy $g(y) = 0$, giving your answers to 2 decimal places where appropriate.

(3)

1 Q03aM1
1 Q03aA1
1 Q03bM1
1 Q03bA1
1 Q03bM2
1 Q03bA2

1 Q03cB1
1 Q03cM1
1 Q03cA1

(a) $f(3) = 0$

$$2(3)^3 - 5(3)^2 + 3a + 18 = 0$$

$$54 - 45 + 3a + 18 = 0$$

$$3a = -27$$

$$a = -9$$

(b)
$$\begin{array}{r} 2x^2 + \cancel{x} - 6 \\ x-3 \overline{) 2x^3 - 5x^2 - 9x + 18} \end{array}$$

$$\underline{2x^3 - 6x^2}$$

$$+ \cancel{x^2}$$

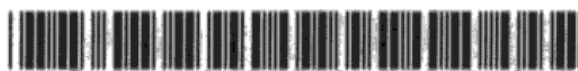
$$\bullet - 9x$$

$$\underline{x^2 - 3x}$$

$$- 6x + 18$$

$$\underline{- 6x + 18}$$

$$0$$



Question 3 continued

$$(2x^2 + x - 6)(x - 3) = 0$$

$$(2x - 3)(x + 2)(x - 3) = 0$$

$$(c) \quad g(y) = 2(3^y)^3 - 5(3^y)^2 - 9(3^y) + 18$$

$$3^y = x$$

$$x = -2$$

$$x = 3$$

$$x = 3/2$$

$$y = \log_3 -2 \quad y \neq$$

$$y = \log_3 3 \quad y = 1$$

$$y = \log_3 3^{3/2} \quad y = 0.37$$



4.

$$y = \frac{5}{(x^2 + 1)}$$

(a) Complete the table below, giving the missing value of y to 3 decimal places.

x	0	0.5	1	1.5	2	2.5	3
y	5	4	2.5	$\frac{20}{3}$	1	0.690	0.5

(1)

0 Q04aB

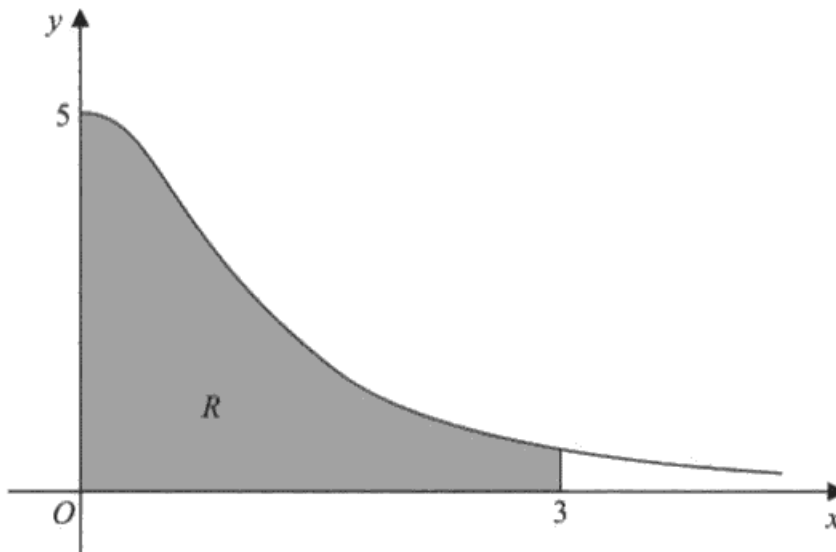


Figure 1

Figure 1 shows the region R which is bounded by the curve with equation $y = \frac{5}{(x^2 + 1)}$, the x -axis and the lines $x = 0$ and $x = 3$

(b) Use the trapezium rule, with all the values of y from your table, to find an approximate value for the area of R .

(4)

(c) Use your answer to part (b) to find an approximate value for

$$\int_0^3 \left(4 + \frac{5}{(x^2 + 1)} \right) dx$$

giving your answer to 2 decimal places.

(2)

1 Q04bB1
1 Q04bM1
1 Q04bA1
0 Q04bA2

1 Q04cM1
1 Q04cA1

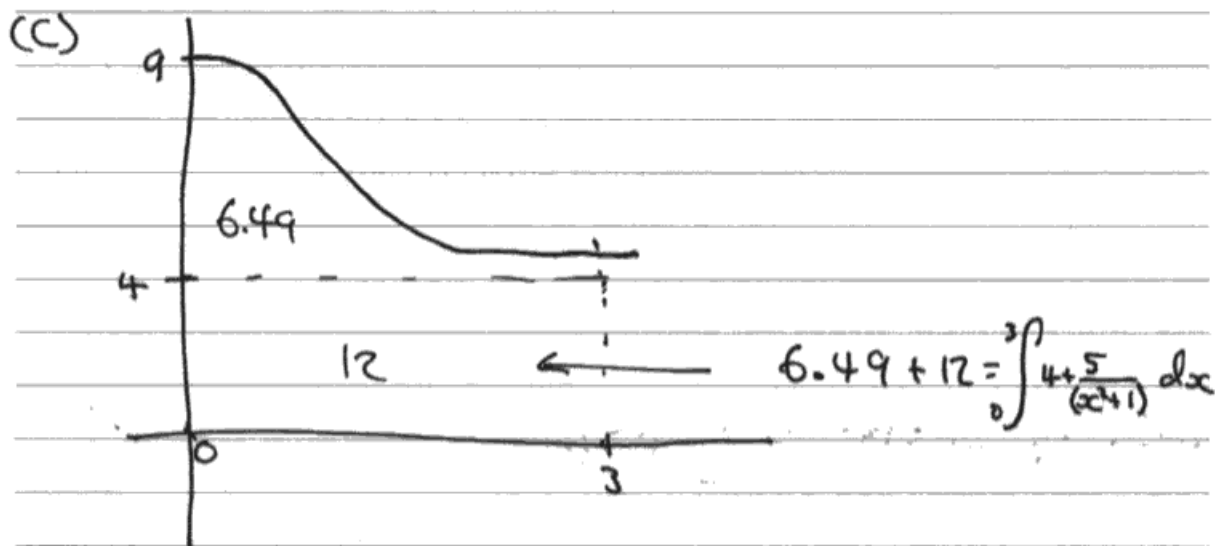


Question 4 continued

$$(b) \frac{h}{2} (y_0 + y_5 + 2(y_1 + y_2 + y_3 + y_4 + y_5))$$

$$\text{Area} = \frac{1}{4} (5 + 0.5 + 2(4 + 2.5 + \frac{20}{13} + 1 + 0.690))$$

$$\text{Area} = 6.49 \text{ units}^2$$



$$18.49 \text{ units}^2$$

(Total 7 marks)

Q4

5



5.

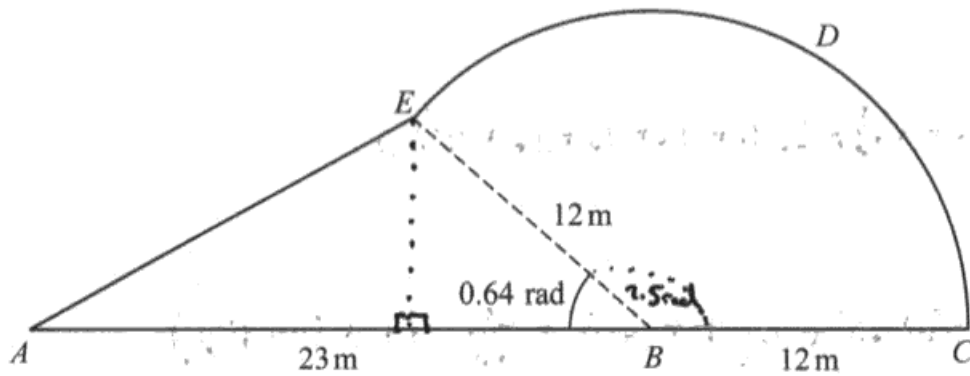


Figure 2

Figure 2 shows a plan view of a garden.

The plan of the garden $ABCDEA$ consists of a triangle ABE joined to a sector $BCDE$ of a circle with radius 12m and centre B .

The points A , B and C lie on a straight line with $AB = 23\text{m}$ and $BC = 12\text{m}$.

Given that the size of angle ABE is exactly 0.64 radians, find

- (a) the area of the garden, giving your answer in m^2 , to 1 decimal place, (4)
- (b) the perimeter of the garden, giving your answer in metres, to 1 decimal place. (5)

- 1 Q05aM1
- 1 Q05aA1
- 1 Q05aA2
- 1 Q05aA3
- 1 Q05bM1
- 1 Q05bA1
- 1 Q05bM2
- 1 Q05bM3
- 1 Q05bA2

(a) area of sector = $\frac{1}{2} r^2 \theta$

= $\frac{1}{2} (144)(2.5)$

area of sector = 180 m^2

area of triangle = $\frac{1}{2} ab \sin C$

= $\frac{1}{2} 12 \times 23 \times \sin 0.64$

= 82.4 m^2

$180 + 82.4 = 262.4 \text{ m}^2$



Question 5 continued

$$\begin{aligned} \text{(b) arc length} &= r\theta \\ &= 12 \times 2.5 \\ &= 30\text{m} \end{aligned}$$

$$30 + 12 + 23 = 65\text{m}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = 144 + 529 - 2 \times 12 \times 23 \times \cos 0.6$$

$$a^2 = 230.2431416$$

$$a = \sqrt{230.2431416}$$

$$a = 15.2\text{m}$$

$$15.2\text{m} + 65\text{m}$$

$$= 80.2\text{m}$$



6.

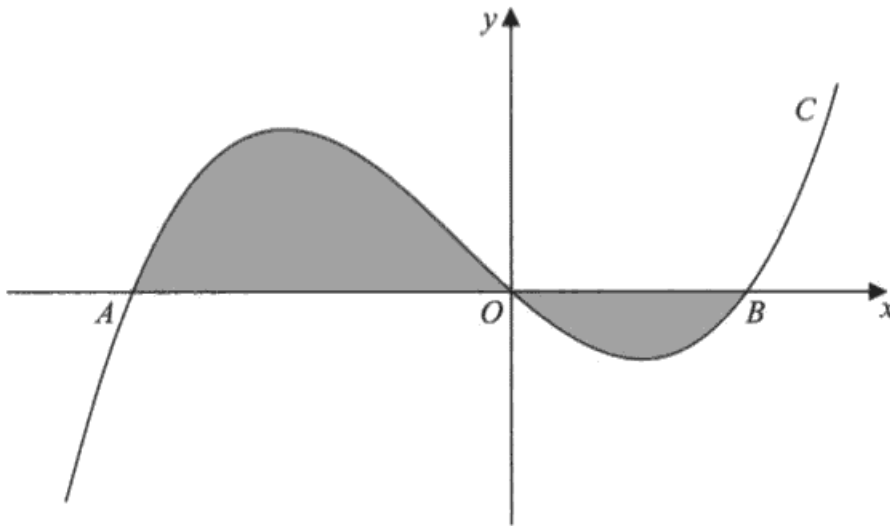


Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = x(x + 4)(x - 2)$$

The curve C crosses the x -axis at the origin O and at the points A and B .

(a) Write down the x -coordinates of the points A and B .

(1)

The finite region, shown shaded in Figure 3, is bounded by the curve C and the x -axis.

(b) Use integration to find the total area of the finite region shown shaded in Figure 3.

(7)

1 Q06aB1

1 Q06bB1

1 Q06bM1

1 Q06bA1

1 Q06bM2

1 Q06bA2

0 Q06bM3

0 Q06bA3

(a) $x = -4$ at A
 $x = 2$ at B

(b) $\int_{-4}^0 x(x+4)(x-2) dx + \int_0^2 x(x+4)(x-2) dx$

$x(x^2 + 2x - 8)$
 $x^3 + 2x^2 - 8x$



Question 6 continued

$$\int_0^2 x^3 + 2x^2 - 8x = \left[\frac{x^4}{4} + \frac{2x^3}{3} - 4x^2 \right]_0^2$$

$$= \frac{2^4}{4} + \frac{2(2)^3}{3} - 4(2)^2 - 0$$

$$= \frac{20}{3} \text{ units}^2 \quad \boxed{O \text{ to } B}$$

$$\int_{-4}^0 x^3 + 2x^2 - 8x = \left[\frac{x^4}{4} + \frac{2x^3}{3} - 4x^2 \right]_{-4}^0$$

$$= 0 - \left(\frac{-4^4}{4} + \frac{2(-4)^3}{3} - 4(-4)^2 \right)$$

$$= - \left(\frac{256}{4} + \frac{128}{3} - 64 \right)$$

$$= - \left(64 - \frac{128}{3} - 64 \right)$$

$$= \frac{128}{3} \text{ units}^2 \quad \boxed{A \text{ to } O}$$

$$\frac{128}{3} + \frac{20}{3} = \frac{148}{3} \text{ units}^2$$

total area.



7. (i) Find the exact value of x for which

$$\log_2(2x) = \log_2(5x + 4) - 3 \quad (4)$$

(ii) Given that

$$\log_a y + 3\log_a 2 = 5$$

express y in terms of a .
Give your answer in its simplest form.

(3)

1 Q07iM1
1 Q07iM2
1 Q07iM3
1 Q07iA1

1 Q7iiM1
1 Q7iiM2
1 Q7iiA1

(i) $\log_2 2x = \log_2 2 + \log_2 x$
 $= 1 + \log_2 x$

$$1 + \log_2 x = \log_2(5x+4) - 3$$

~~$$\log_2 x = \log_2(5x+4) - \log_2(8) - 4$$~~

~~$$\log_2 x = \log_2(5x+4) - \log_2 16$$~~

~~$$\log_2 x = \log_2 \left(\frac{5x+4}{16} \right)$$~~

~~$$x = \frac{5x+4}{16}$$~~

~~$$16x = 5x+4$$~~

~~$$11x = 4$$~~

~~$$x = \frac{4}{11}$$~~

$$\log_2 2x = \log_2(5x+4) - \log_2 8$$

$$\log_2 2x = \log_2 \frac{5x+4}{8}$$

$$2x = \frac{5x+4}{8}$$

$$16x = 5x+4$$

$$11x = 4$$

$$x = \frac{4}{11}$$



Question 7 continued

$$(ii) \log_a y + 3\log_a 2 = 5$$

$$\log_a y + \log_a 8 = 5$$

$$\log_a 8y = 5$$

$$a^5 = 8y$$

$$y = \frac{a^5}{8}$$

Q7

7

(Total 7 marks)



8. (i) Solve, for $-180^\circ \leq x < 180^\circ$,

$$\tan(x - 40^\circ) = 1.5$$

giving your answers to 1 decimal place.

(3)

(ii) (a) Show that the equation

$$\sin \theta \tan \theta = 3 \cos \theta + 2$$

can be written in the form

$$4 \cos^2 \theta + 2 \cos \theta - 1 = 0$$

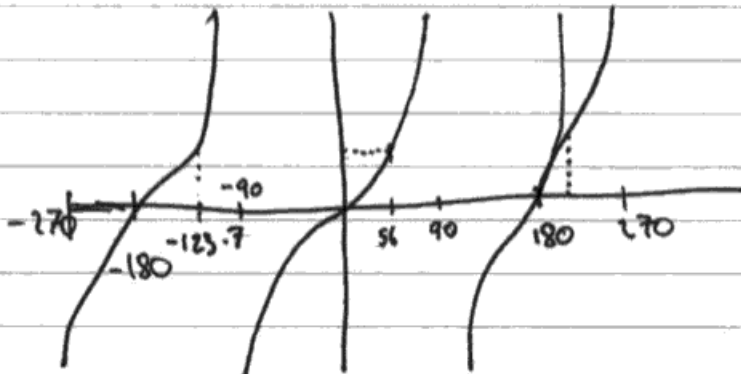
(3)

(b) Hence solve, for $0 \leq \theta < 360^\circ$,

$$\sin \theta \tan \theta = 3 \cos \theta + 2$$

showing each stage of your working.

(5)



$$-220^\circ \leq x - 40^\circ < 140^\circ$$

$$\tan^{-1}(1.5) = x - 40^\circ$$

$$56.3^\circ$$

$$-123.7^\circ$$

$$x = 96.3^\circ$$

$$x = -83.7^\circ$$

1 Q08iB1
1 Q08iM1
1 Q08iA1

1 8iiaM1
1 8iiaM2
1 8iiaA1

1 8iibM1
1 8iibA1
1 8iibA2
1 8iibM2
1 8iibA3



Question 8 continued

$$(ii) (a) \quad \sin \theta \tan \theta = 3 \cos \theta + 2$$

$$\sin \theta \frac{\sin \theta}{\cos \theta} = 3 \cos \theta + 2$$

$$\frac{\sin^2 \theta}{\cos \theta} = 3 \cos \theta + 2$$

$$\sin^2 \theta = 3 \cos^2 \theta + 2 \cos \theta$$

$$1 - \cos^2 \theta = 3 \cos^2 \theta + 2 \cos \theta$$

$$1 = 4 \cos^2 \theta + 2 \cos \theta$$

$$4 \cos^2 \theta + 2 \cos \theta - 1 = 0$$

$$(b) \quad \text{let } \cos \theta = t$$

$$4t^2 + 2t - 1 = 0$$

~~$$4t^2 + 2t - 1 = 0$$~~

$$\frac{-2 \pm \sqrt{4 + 16}}{8}$$

$$t = \cos \theta$$

so

$$\cos \theta = 0.309$$

~~$$\cos \theta = -0.809$$~~

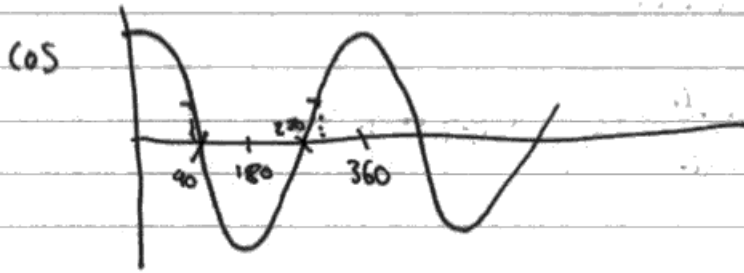
$$t = 0.309$$

$$t = -0.809 \quad \text{not a solution}$$



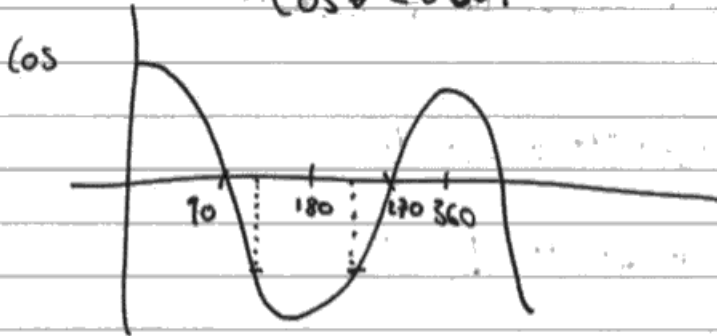
Question 8 continued

$$\cos \theta = 0.309$$



$$\theta = 72^\circ \quad \theta = 288^\circ$$

$$\cos \theta = 0.809$$



$$\theta = 14^\circ \quad \theta = 216^\circ$$



9. The curve with equation

$$y = x^2 - 32\sqrt{x} + 20, \quad x > 0$$

has a stationary point P .

Use calculus

(a) to find the coordinates of P ,

(6)

(b) to determine the nature of the stationary point P .

(3)

(a)

$$y = x^2 - 32x^{\frac{1}{2}} + 20$$

$$\frac{dy}{dx} = 2x - 16x^{-\frac{1}{2}}$$

$$2x - \frac{16}{\sqrt{x}} = 0$$

$$x = 4$$

$$y = 4^2 - 32\sqrt{4} + 20$$

$$P = (4, -28)$$

(b)

$$\frac{d^2y}{dx^2} = 2 + 8x^{-\frac{3}{2}}$$

$$\text{let } x=4 \quad * 2 + 8(4)^{-\frac{3}{2}} = 3$$

the second derivative is positive, \therefore it is a minimum point.

1 Q09aM1

1 Q09aA1

1 Q09aM2

1 Q09aA2

1 Q09aM3

1 Q09aA3

1 Q09bM1

1 Q09bA1

1 Q09bB1



10.

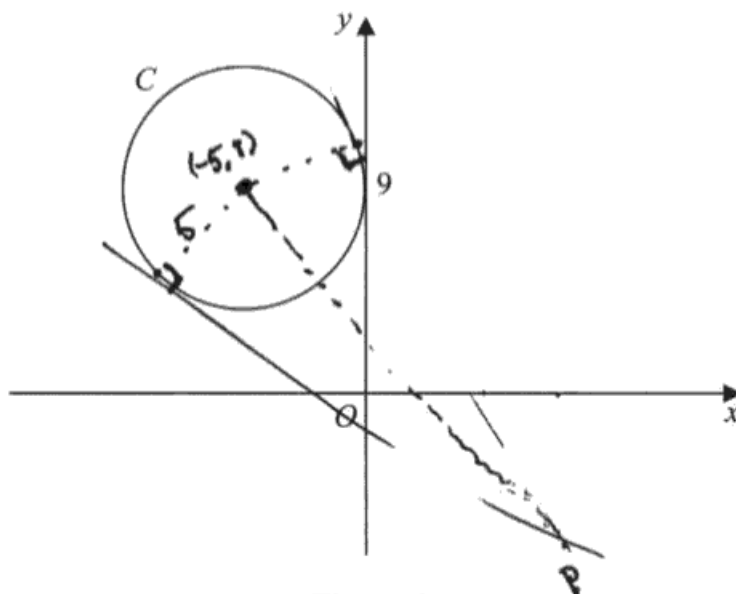


Figure 4

The circle C has radius 5 and touches the y -axis at the point $(0, 9)$, as shown in Figure 4.

- (a) Write down an equation for the circle C , that is shown in Figure 4. (3)

A line through the point $P(8, -7)$ is a tangent to the circle C at the point T .

- (b) Find the length of PT . (3)

Centre point = $(-5, 9)$

$$(x + 5)^2 + (y - 9)^2 = 25$$

length of $(-5, 9)$ to $(8, -7)$

$$\sqrt{13^2 + 16^2} = 5\sqrt{17}$$

$$PT = \sqrt{(5\sqrt{17})^2 - 25} = 20$$

1 Q10aM1

1 Q10aM2

1 Q10aA1

1 Q10bM1

1 Q10bM2

1 Q10bA1



