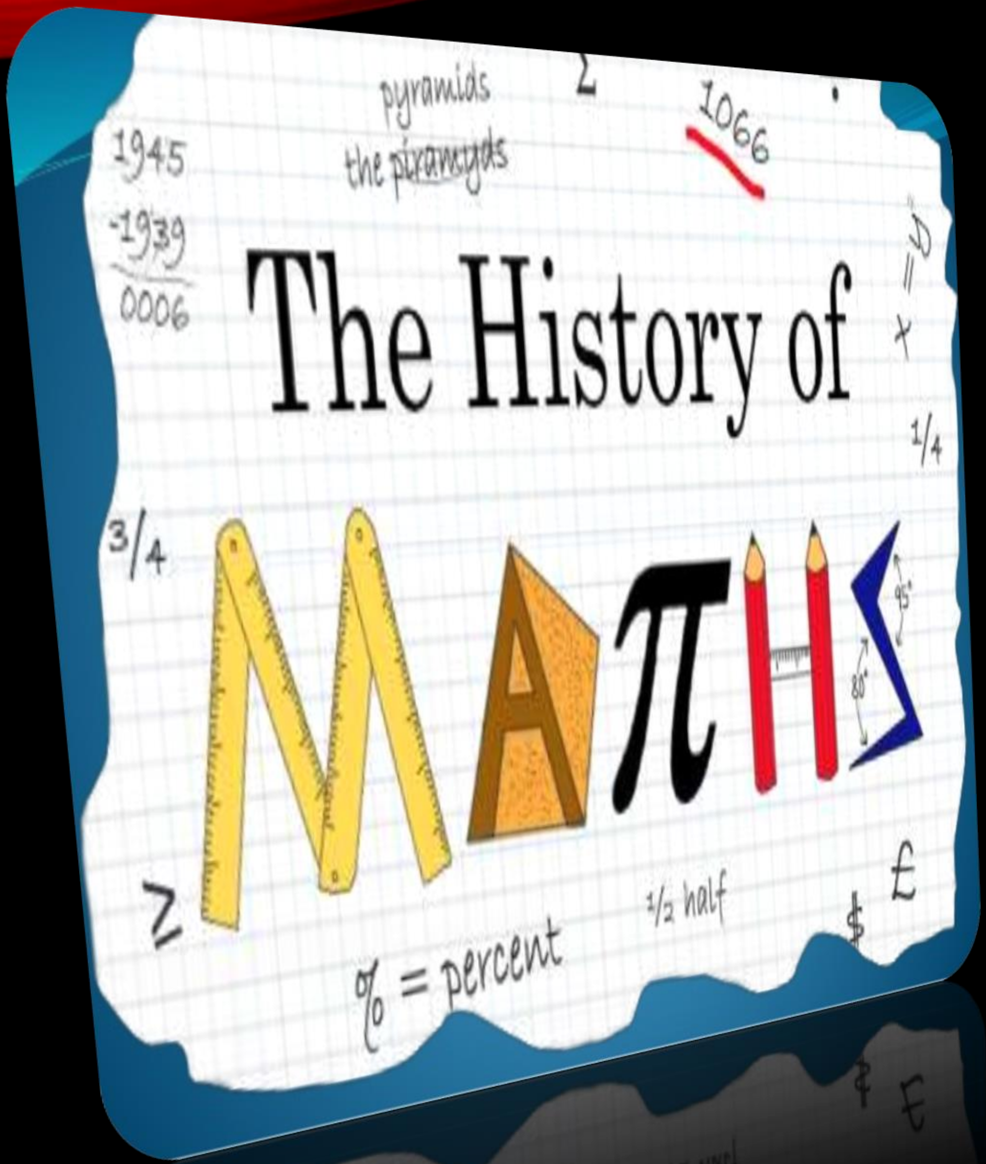
BETH'S PRESENTATION

Mathamatics



The history of mathematics is nearly as old as humanity itself. Since antiquity, mathematics has been fundamental to advances in science, engineering, and philosophy. It has evolved from simple counting, measurement and calculation, and the systematic study of the shapes and motions of physical objects, through the application of abstraction, imagination and logic, to the broad, complex and often abstract discipline we know today.

From the notched bones of early man to the mathematical advances brought about by settled agriculture in Mesopotamia and Egypt and the revolutionary developments of ancient Greece and its Hellenistic empire, the story of mathematics is a long and impressive one.

The East carried on the baton, particularly China, India and the medieval Islamic empire, before the focus of mathematical innovation moved back to Europe in the late Middle Ages and Renaissance. Then, a whole new series of revolutionary developments occurred in 17th Century and 18th Century Europe, setting the stage for the increasing complexity and abstraction of 19th Century mathematics, and finally the audacious and sometimes devastating discoveries of the 20th Century.

FRACTIONS

A fraction (from Latin fractus, "broken") represents a part of a whole or, more generally, any number of equal parts. When spoken in everyday English, a fraction describes how many parts of a certain size there are, for example, one-half, eight-fifths, three-quarters. A common, vulgar, or simple fraction (examples:

1 2 {\display style {\ tfrac {1}{2}}}

and 17/3) consists of an integer numerator displayed above a line (or before a slash), and a non-zero integer denominator, displayed below (or after) that line. Numerators and denominators are also used in fractions that are not common, including compound fractions, complex fractions, and mixed numerals.

The numerator represents a number of equal parts, and the denominator, which cannot be zero, indicates how many of those parts make up a unit or a whole. For example, in the fraction 3/4, the numerator, 3, tells us that the fraction represents 3 equal parts, and the denominator, 4, tells us that 4 parts make up a whole. The picture to the right illustrates

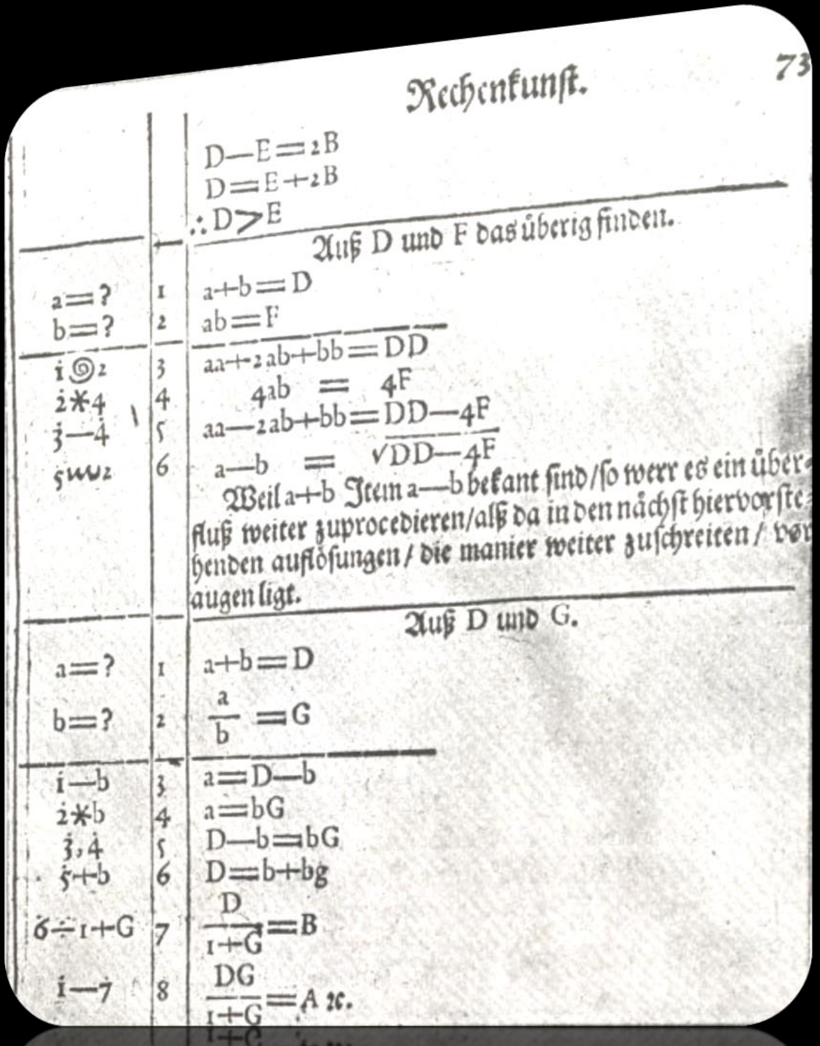
3 4 {\display style {\ tfrac {3}{4}}}

or ¾ of a cake.

Fractional numbers can also be written without using explicit numerators or denominators, by using decimals, percent signs, or negative exponents (as in 0.01, 1%, and 10−2 respectively, all of which are equivalent to 1/100). An integer such as the number 7 can be thought of as having an implicit denominator of one: 7 equals 7/1.

Other uses for fractions are to represent ratios and to represent division.[1] Thus the fraction ¾ is also used to represent the ratio 3:4 (the ratio of the part to the whole) and the division 3 ÷ 4 (three divided by four).

In mathematics the set of all numbers that can be expressed in the form a/b, where a and b are integers and b is not zero, is called the set of rational numbers and is represented by the symbol Q, which stands for quotient. The test for a number being a rational number is that it can be written in that form (i.e., as a common fraction). However, the word fraction is also used to describe mathematical expressions that are not rational numbers, for example algebraic fractions (quotients of algebraic expressions), and expressions that contain irrational numbers, such as √2/2 (see square root of 2) and π/4 (see proof that π is irrational).


Rechenkunst. 73

D—E=2B
D=E+2B
∴ D>E

Auß D und F das übrig finden.

a=?	1	a+b=D
b=?	2	ab=F
1 ⊗ 2	3	aa+2ab+bb=DD
2*4	4	4ab = 4F
3—4	5	aa—2ab+bb=DD—4F
5 ⋓ 2	6	a—b = √DD—4F

Weil a+b Item a—b bekant sind/so werr es ein über-fluß weiter zuprocedieren/alß da in den nächst hiervorste-henden auflösungen/die manier weiter zuschreiten/vor augen ligt.

Auß D und G.

a=?	1	a+b=D
b=?	2	a/b =G
1—b	3	a=D—b
2*b	4	a=bG
3,4	5	D—b=bG
5+b	6	D=b+bg
6÷1+G	7	D/(1+G)=B
1—7	8	DG/(1+G)=A &c.

- The history of algebra began in ancient Egypt and Babylon, where people learned to solve linear (ax = b) and quadratic (ax2 + bx = c) equations, as well as indeterminate equations such as x2 + y2 = z2, whereby several unknowns are involved. The ancient Babylonians solved arbitrary quadratic equations by essentially the same procedures taught today. They also could solve some indeterminate equations.

  The Alexandrian mathematicians Hero of Alexandria and Diophantus continued the traditions of Egypt and Babylon, but Diophantus's book Arithmetica is on a much higher level and gives many surprising solutions to difficult indeterminate equations. This ancient knowledge of solutions of equations in turn found a home early in the Islamic world, where it was known as the "science of restoration and balancing." (The Arabic word for restoration, al-jabru, is the root of the word algebra.) In the 9th century, the Arab mathematician al-Khwarizmi wrote one of the first Arabic algebras, a systematic exposé of the basic theory of equations, with both examples and proofs. By the end of the 9th century, the Egyptian mathematician Abu Kamil had stated and proved the basic laws and identities of algebra and solved such complicated problems as finding x, y, and z such that x + y + z = 10, x2 + y2 = z2, and xz = y2.

  Ancient civilizations wrote out algebraic expressions using only occasional abbreviations, but by medieval times Islamic mathematicians were able to talk about arbitrarily high powers of the unknown x, and work out the basic algebra of polynomials (without yet using modern symbolism). This included the ability to multiply, divide, and find square roots of polynomials as well as a knowledge of the binomial theorem. The Persian mathematician, astronomer, and poet Omar Khayyam showed how to express roots of cubic equations by line segments obtained by intersecting conic sections, but he could not find a formula for the roots. A Latin translation of Al-Khwarizmi's Algebra appeared in the 12th century. In the early 13th century, the great Italian mathematician Leonardo Fibonacci achieved a close approximation to the solution of the cubic equation x3 + 2x2 + cx = d. Because Fibonacci had travelled in Islamic lands, he probably used an Arabic method of successive approximations.

TIMES TABLES

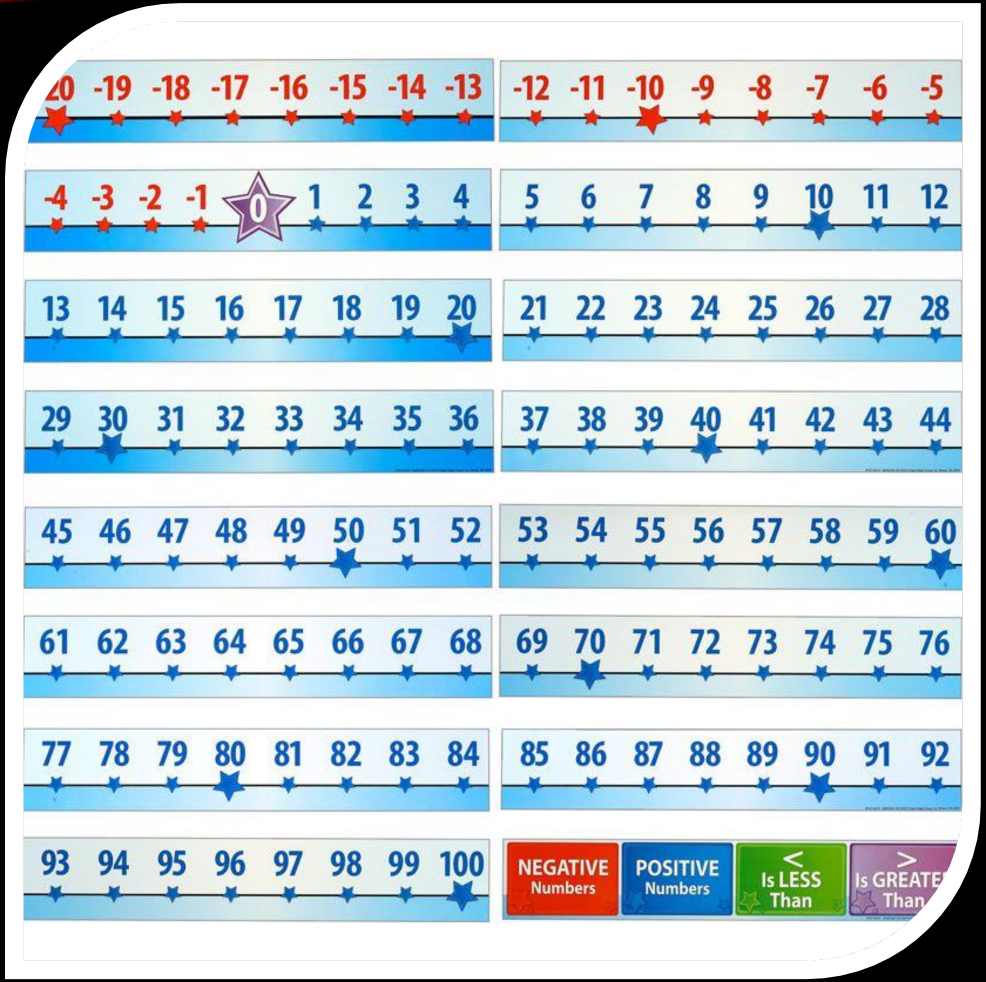
The oldest known multiplication tables were used by the Babylonians about 4000 years ago. However, they used a base of 60. The oldest known tables using a base of 10 are the Chinese decimal multiplication table on bamboo strips dating to about 305 BC, during China's Warring States period.

"Table of Pythagoras" on Napier's bones

The multiplication table is sometimes attributed to the ancient Greek mathematician Pythagoras (570–495 BC). It is also called the Table of Pythagoras in many languages (for example French, Italian and at one point even Russian), sometimes in English. The Greco-Roman mathematician Nichomachus (60–120 AD), a follower of Neopythagoreanism, included a multiplication table in his Introduction to Arithmetic, whereas the oldest surviving Greek multiplication table is on a wax tablet dated to the 1st century AD and currently housed in the British Museum.

In 493 AD, Victorius of Aquitaine wrote a 98-column multiplication table which gave (in Roman numerals) the product of every number from 2 to 50 times and the rows were "a list of numbers starting with one thousand, descending by hundreds to one hundred, then descending by tens to ten, then by ones to one, and then the fractions down to 1/144."[

In his 1820 book The Philosophy of Arithmetic, mathematician John Leslie published a multiplication table up to 99 × 99, which allows numbers to be multiplied in pairs of digits at a time. Leslie also recommended that young pupils memorize the multiplication table up to 25 × 25. The illustration below shows a table up to 12 × 12, which is a common size to teach in schools



In mathematics, a negative number is a real number that is less than zero. Negative numbers represent opposites. If positive represents movement to the right, negative represents movement to the left. If positive represents above sea level, then negative represents below level. If positive represents a deposit, negative represents a withdrawal. They are often used to represent the magnitude of a loss or deficiency. A debt that is owed may be thought of as a negative asset, a decrease in some quantity may be thought of as a negative increase. If a quantity may have either of two opposite senses, then one may choose to distinguish between those senses—perhaps arbitrarily—as positive and negative. In the medical context of fighting a tumor, an expansion could be thought of as a negative shrinkage. Negative numbers are used to describe values on a scale that goes below zero, such as the Celsius and Fahrenheit scales for temperature. The laws of arithmetic for negative numbers ensure that the common sense idea of an opposite is reflected in arithmetic. For example, − − 3 = 3 because the opposite of an opposite is the original thing.

Negative numbers are usually written with a minus sign in front. For example, −3 represents a negative quantity with a magnitude of three, and is pronounced "minus three" or "negative three". To help tell the difference between a subtraction operation and a negative number, occasionally the negative sign is placed slightly higher than the minus sign (as a superscript). Conversely, a number that is greater than zero is called positive; zero is usually thought of as neither positive nor negative. The positivity of a number may be emphasized by placing a plus sign before it, e.g. +3. In general, the negativity or positivity of a number is referred to as its sign.

Square roots

THE YALE BABYLONIAN COLLECTION YBC 7289 CLAY TABLET WAS CREATED BETWEEN 1800 BC AND 1600 C, SHOWING √2 AND 30√2 AS 1;24,51,10 AND 42;25,35 BASE 60 NUMBERS ON A SQUARE CROSSED BY TWO DIAGONALS.
THE RHIND MATHEMATICAL PAPYRUS IS A COPY FROM 1650 BC OF AN EARLIER BERLIN PAPYRUS AND OTHER TEXTS – POSSIBLY THE KAHUN PAPYRUS – THAT SHOWS HOW THE EGYPTIANS EXTRACTED SQUARE ROOTS BY AN INVERSE PROPORTION METHOD.
IN ANCIENT INDIA, THE KNOWLEDGE OF THEORETICAL AND APPLIED ASPECTS OF SQUARE AND SQUARE ROOT WAS AT LEAST AS OLD AS THE SULBA SUTRAS, DATED AROUND 800–500 BC (POSSIBLY MUCH EARLIER).[CITATION NEEDED] A METHOD FOR FINDING VERY GOOD APPROXIMATIONS TO THE SQUARE ROOTS OF 2 AND 3 ARE GIVEN IN THE BAUDHAYANA SULBA SUTRA. ARYABHATA IN THE ARYABHATIYA (SECTION 2.4), HAS GIVEN A METHOD FOR FINDING THE SQUARE ROOT OF NUMBERS HAVING MANY DIGITS.
IT WAS KNOWN TO THE ANCIENT GREEKS THAT SQUARE ROOTS OF POSITIVE WHOLE NUMBERS THAT ARE NOT PERFECT SQUARES ARE ALWAYS IRRATIONAL NUMBERS: NUMBERS NOT EXPRESSIBLE AS A RATIO OF TWO INTEGERS (THAT IS TO SAY THEY CANNOT BE WRITTEN EXACTLY AS M/N, WHERE M AND N ARE INTEGERS). THIS IS THE THEOREM EUCLID X, 9 ALMOST CERTAINLY DUE TO THEAETETUS DATING BACK TO CIRCA 380 BC. THE PARTICULAR CASE √2 IS ASSUMED TO DATE BACK EARLIER TO THE PYTHAGOREANS AND IS TRADITIONALLY ATTRIBUTED TO HIPPASUS. [CITATION NEEDED] IT IS EXACTLY THE LENGTH OF THE DIAGONAL OF A SQUARE WITH SIDE LENGTH 1.

IN THE CHINESE MATHEMATICAL WORK WRITINGS ON RECKONING, WRITTEN BETWEEN 202 BC AND 186 BC DURING THE EARLY HAN DYNASTY, THE SQUARE ROOT IS APPROXIMATED BY USING AN "EXCESS AND DEFICIENCY" METHOD, WHICH SAYS TO "...COMBINE THE EXCESS AND DEFICIENCY AS THE DIVISOR; (TAKING) THE DEFICIENCY NUMERATOR MULTIPLIED BY THE EXCESS DENOMINATOR AND THE EXCESS NUMERATOR TIMES THE DEFICIENCY DENOMINATOR, COMBINE THEM AS THE DIVIDEND."
MAHĀVĪRA, A 9TH-CENTURY INDIAN MATHEMATICIAN, WAS THE FIRST TO STATE THAT SQUARE ROOTS OF NEGATIVE NUMBERS DO NOT EXIST.
A SYMBOL FOR SQUARE ROOTS, WRITTEN AS AN ELABORATE R, WAS INVENTED BY REGIOMONTANUS (1436–1476). AN R WAS ALSO USED FOR RADIX TO INDICATE SQUARE ROOTS IN GEROLAMO CARDANO'S ARS MAGNA.
ACCORDING TO HISTORIAN OF MATHEMATICS D.E. SMITH, ARYABHATA'S METHOD FOR FINDING THE SQUARE ROOT WAS FIRST INTRODUCED IN EUROPE BY CATANEO IN 1546

Cube roots

Squares	Square Roots	Cubes
$1^2=1$	$\sqrt{1}=1$	$1^3=1$
$2^2=4$	$\sqrt{4}=2$	$2^3=8$
$3^2=9$	$\sqrt{9}=3$	$3^3=27$
$4^2=16$	$\sqrt{16}=4$	$4^3=64$
$5^2=25$	$\sqrt{25}=5$	$5^3=125$
$6^2=36$	$\sqrt{36}=6$	$6^3=216$
$7^2=49$	$\sqrt{49}=7$	$7^3=343$
$8^2=64$	$\sqrt{64}=8$	$8^3=512$
$9^2=81$	$\sqrt{81}=9$	$9^3=729$
$10^2=100$	$\sqrt{100}=10$	$10^3=1000$
$11^2=121$	$\sqrt{121}=11$	$11^3=1331$
$12^2=144$	$\sqrt{144}=12$	$12^3=1728$

- *The calculation of cube roots can be to traced back to Babylonian mathematicians from as early as 1800 BCE. In the fourth century BCE Plato posed the problem of doubling the cube, which required a compass-and-straightedge construction of the edge of a cube with twice the volume of a given cube; this required the construction, now known to be impossible, of the length $\sqrt[3]{2}$.*
- *A method for extracting cube roots appears in The Nine Chapters on the Mathematical Art, a Chinese mathematical text compiled around the 2nd century BCE and commented on by Liu Hui in the 3rd century CE. The Greek mathematician Hero of Alexandria devised a method for calculating cube roots in the 1st century CE. His formula is again mentioned by Eutokios in a commentary on Archimedes. In 499 CE Aryabhata, a mathematician-astronomer from the classical age of Indian mathematics and Indian astronomy, gave a method for finding the cube root of numbers having many digits in the Aryabhatiya (section 2.5).*

DECIMALS

Modern methods of writing decimals were invented less than 500 years ago. Some use of decimal fractions was made in ancient China, medieval Arabia and in Renaissance Europe. By about 1500, decimals were well accepted by professional mathematicians but not widely used.

Thousands of years earlier, the Babylonians had used a place-value number system based on 60 (whereas ours is based on 10) and had been extended to deal with numbers less than one. This was still in use in the 1500s in Europe, but the advantages of a thoroughly base ten system were becoming apparent. In his book Canon-mathematicus (1579) the Italian/French mathematician Francois Viete called for the use of base ten decimal fractions rather than base 60 sexagesimal fractions when he wrote:

"Sexagesimals and sixties are to be used sparingly or never in mathematics, and thousandths and thousands, hundredths and hundreds, tenths and tens, and similar progressions, ascending and descending, are to be used frequently or exclusively"

Viete used decimals in this work, but wrote them differently to how we now write them. For example, for the length of the side of a square inscribed in a circle of diameter of 200 000 he wrote which we would write as 141421.35624. (This is a very accurate approximation to 100 000 times the square root of 2). Later he used for 100 000 times pi, the number we would write as 314 159.26535. He also used boldface type to indicate the whole number part of this number, writing it as 314,159,265,36. Sometimes he also included a vertical stroke to separate the whole number and fractional parts. For example, he wrote 99946.45875 as 99,946|458,75.

The person who is credited with making decimal fractions widely known and understood among common people and practical users of mathematics in Europe is Simon Stevin of Bruges, in the Flemish Netherlands. He sought to teach everyone "how to perform with an ease unheard of, all computations necessary between men by integers without fractions ". In a book entitled De thiende ("The Tenth") published in 1585, he wrote decimal expressions for fractions by writing the power of ten assumed as a divisor in a circle above or after each digit.

For example, the approximate value of pi which we write as 3.1416 appeared as

3 (0) 1 (1) 4(2) 1(3) 6(4)

- The history of numbers Presently, the earliest known archaeological evidence of any form of writing or counting are scratch marks on a bone from 150,000 years ago. But the first really solid evidence of counting,in the form of the number one, is from a mere twenty-thousand years ago. An ishango bone was found in the Congo with two identical markings of sixty scratches each and equally numbered groups on the back.These markings are a certain indication of counting and they mark a defining moment in western civilization.
- Anthropologists tell us that in Suma, in about 4,000 BCE, Sumerians used tokens to represent numbers, an improvement over notches in a stick or bone. A very important development from using tokens to represent numbers was that in addition to adding tokens you can also take away, giving birth to arithmetic, an event of major significance.The Sumerian's tokens made possible the arithmetic required for them to assess wealth, calculate profit and loss and even more importantly, to collect taxes, as well as keep permanent records. The standard belief is that in this way numbers became the world's first writings and thus accounting was born.
- More primitive societies, such as the Wiligree of Central Australia, never used numbers, nor felt the need for them.We may ask, why then did the Sumerians on the other side of the world feel the need for simple mathematics? The answer of course, was because they lived in cities which required organizing. For example, grain needed to be stored and determining how much each citizen received required arithmetic