AQA

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ALLIANCE

General Certificate of Secondary Education

Additional Mathematics 9306

Pilot Specification
2008

PROBLEM–SOLVING QUESTIONS

Further copies of this resource are available from:

The GCSE Mathematics Department, AQA, Devas Street, Manchester, M15 6EX
Telephone: 0161 957 3852   Fax: 0161 957 3873

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Contents

1   Introduction

These questions have been written by Leeds University's Assessment and Evaluation Unit to support teachers in developing approaches to the type of problem-solving questions that will appear in the pilot GCSE in Additional Mathematics.

The problems are provided to assist teachers in their preparation for the delivery of courses based on the new AQA GCSE specification 9306. The questions in this document are available on a CD-Rom which is part of the Teacher's Guide and Teaching Resource for the specification. That document contains detailed guidance on using these questions as a teaching resource. The Specifications, Specimen Assessment materials and Teacher's Guide are available from the GCSE Mathematics Department, AQA, Devas Street, Manchester, M15 6EX, Telephone: 0161 957 3852, Fax: 0161 957 3873

2   The Problem-Solving Questions

This document contains 90 problem solving questions. These are presented alphabetically in PDF format.

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3Rex
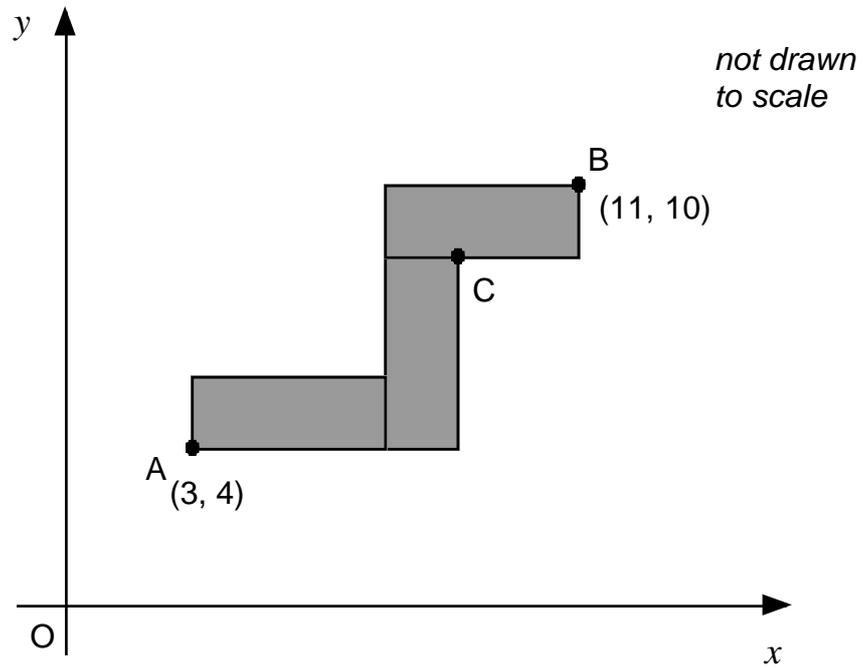
The diagram shows three identical rectangles that have their sides parallel to the axes.

(a)   What are the dimensions of each rectangle?

- - - - - - - - - - - - - - - -

(b)   Find the co-ordinates of point C.

- - - - - - - - - - - - - - - -

Abacus

The three points, A, B and C, on this graph are equally spaced.

not drawn accurately
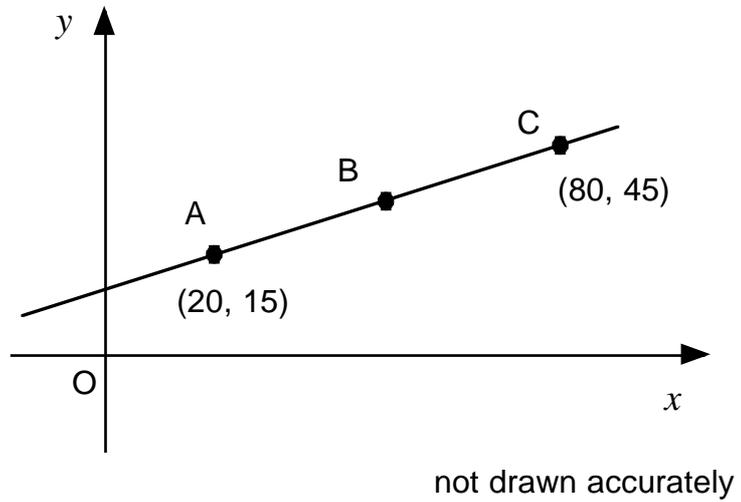
What are the co-ordinates of point B?

- - - - - - - - - - - - - - - -

Apple crumble
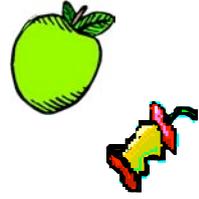
Lottie has a bag of apples.

She gives half of them to Fred.

Fred eats two and then has four left.

How many apples did Lottie have at the start?

April 1st

Explain why the 1st of April is always on the same day of the week as the 1st of July.

. . . .

. . . .

. . . .

April 1st

Arwick 40

40 members of Arwick Youth Club go on a trip to a leisure centre.

They go in minibuses that can each seat up to 15 people.

It costs £30 for each minibus and £150 for the group to have use of the leisure centre.

How much will the trip cost per person?

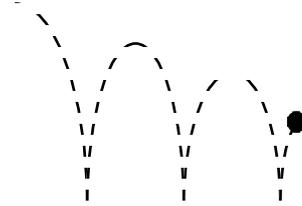£ _ _ _ _ _ _ _ _

Bouncy-bouncy

A ball is dropped and bounces up to a height that is 75% of the height from which it was dropped.

It then bounces again to a height that is 75% of the previous height and so on.

How many bounces does it make before it bounces up to less than 25% of the original height from which it was dropped?

Boxclever

A cube has edges of 10cm each.

Three slices, each of thickness x cm, are cut off the cube.
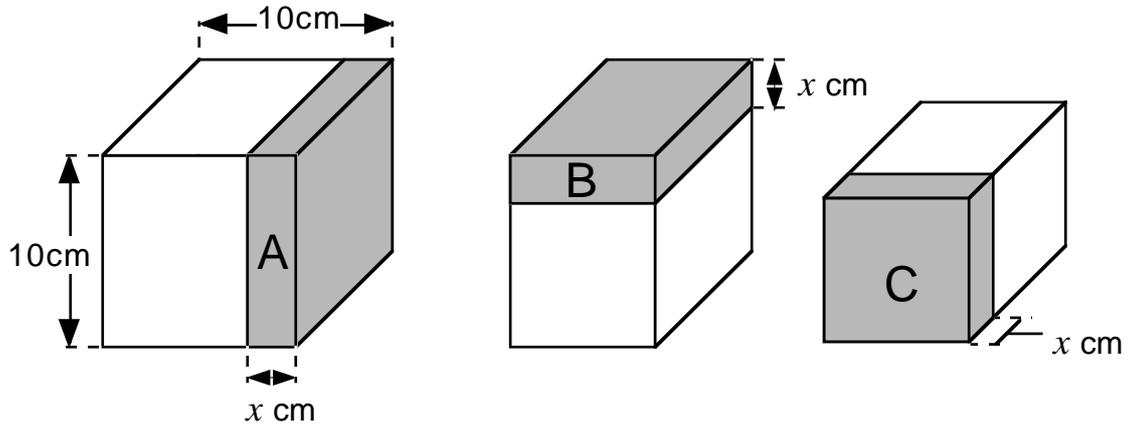
Slice A is cut off the side, slice B is cut off the top and slice C is cut off the front.

What is the volume of each slice in terms of x?

slice A   _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ cm³

slice B   _ _ _ _ _ _ _ _ _ _ _ _ _ _ cm³

slice C   _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ cm³

Bugeye

This hexagon has a perimeter of 24cm.

Three of the hexagons are used to make this shape.
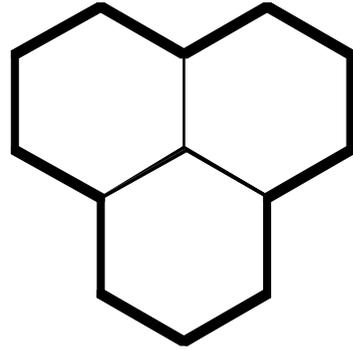
What is the perimeter of the shape?

_____ cm

Bunch of pens

Rulers cost 45p each.

Pens cost 35p each.
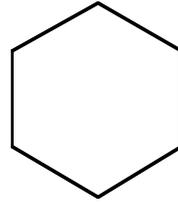
Danielle bought four rulers and a bunch of pens.

She paid with a £5 note and received 40p change.

How many pens did she buy?

- - - - - - - - - - - - - - - - ·

Charterly

Here is part of a number chart.

6	8	10
12	14	16
18	20	22
24	26	28
30	32	34
36	38	40

row 1
2
3
4
5
6

The chart continues.

(a)   What number comes at the start of row 50?

(b)   What is the number of the row that starts with 666?

(c)   What is the number of the row that contains the number 248?

Club sandwich

A tower of 30 identical wooden blocks is 4.5 metres tall.

What is the distance from the top of the
16th block to the top of the 24th block?

- - - - - - - - - - - - - - -
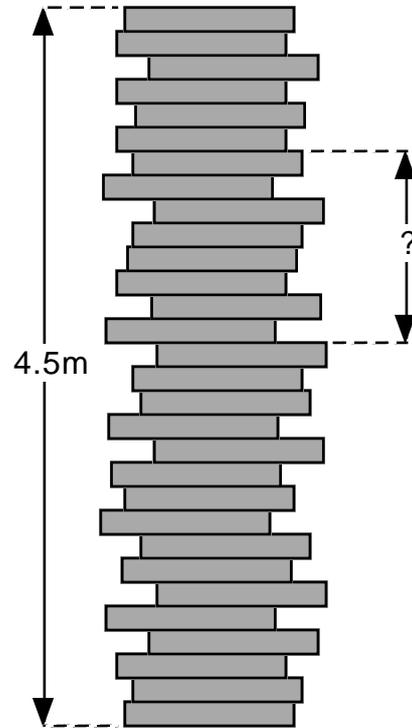
Coin double

Janice has three coins in her pocket, and they are all different from each other.

Jeremy has three coins in his pocket and they are all the same as each other.

Jeremy has twice as much money as Janice.

What are the coins they each have?

Janice   _ _ _ _ _   _ _ _ _ _   _ _ _ _ _

Jeremy   _ _ _ _ _   _ _ _ _ _   _ _ _ _ _

Coin double

Crate-ivity

12 of these cuboids are arranged in a block.
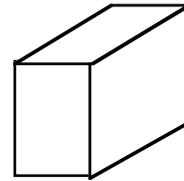
Two loops of tape are used to hold them together.

Each loop of tape goes around four sides of the block.
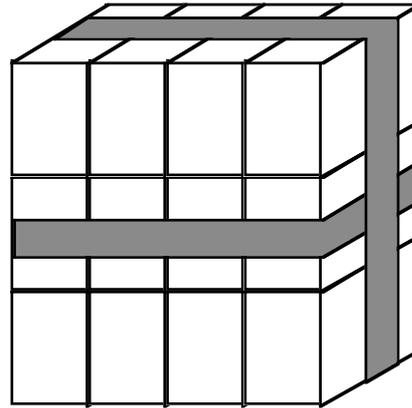
(a)   How many of the cuboids have got tape touching three faces?

- - - - - - - - - - - - - - -

(b)   How many of the cuboids have got tape touching two faces only?

- - - - - - - - - - - - - - -

Cubical

64 small cubes are used to build a larger cube.

How many of the small cubes
are still missing?

- - - - - - - - - - - - - - - - -
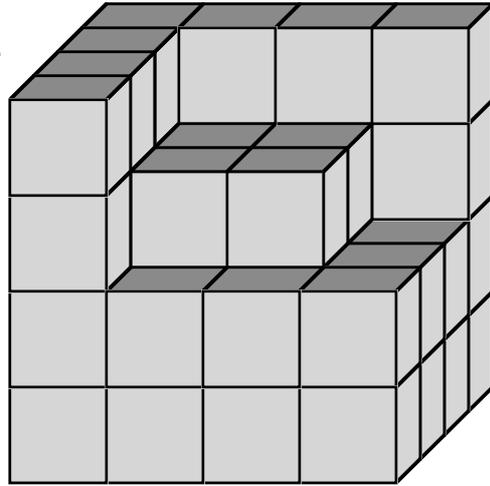
7 cubes are used to make this shape.

Shade squares on this grid to show
how the shape looks when seen from above.

One cube has already been marked on the grid
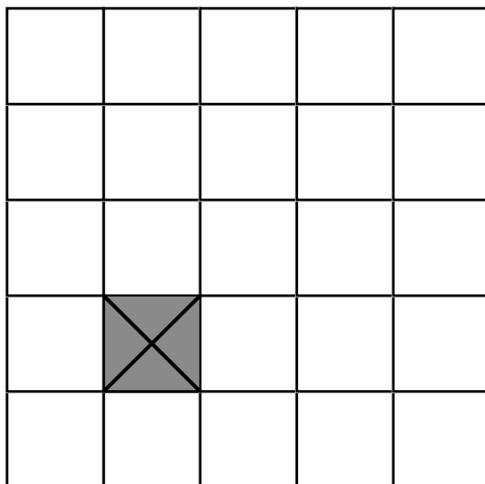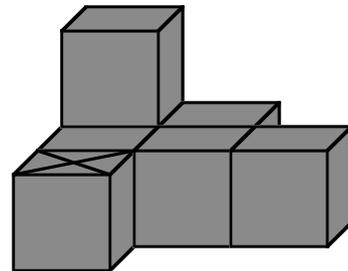
Cuboid ratio
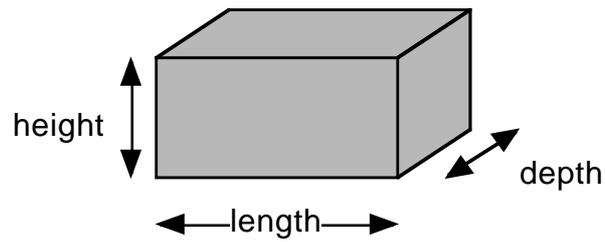
not drawn
to scale

The ratio of the length : height : depth of this cuboid is 1 : 2 : 3

The total surface area is 4950cm^2.

Find the length, height and depth of the cuboid.

length _ _ _ _ _ cm   height _ _ _ _ _ cm   depth _ _ _ _ _ cm

Cupid

p and *q* are two numbers each greater than zero.

$$\sqrt{(p^2 + 4q)} = 9$$

$$\sqrt{(p^2 - 3q)} = 5$$

Find the values of *p* and *q*.

p = _ _ _ _ _ _ _ _

q = _ _ _ _ _ _ _ _

Digitification

Use only the digits 1 to 9 (you can repeat digits if you wish).

Start with a three-digit number   497

Reverse the digits   794

Add the two numbers together   —————
                                    1291

Find the **largest** three-digit starting number that produces a total less than 1000

- - - - - - - - - - - - - - - - -

Double trouble

Use all the digits

0 1 5 0 1 5 0

to complete this multiplication:

□□□ x 2 = □□□□

Ex-cube-me

A cube is cut into three parts by two vertical slices.
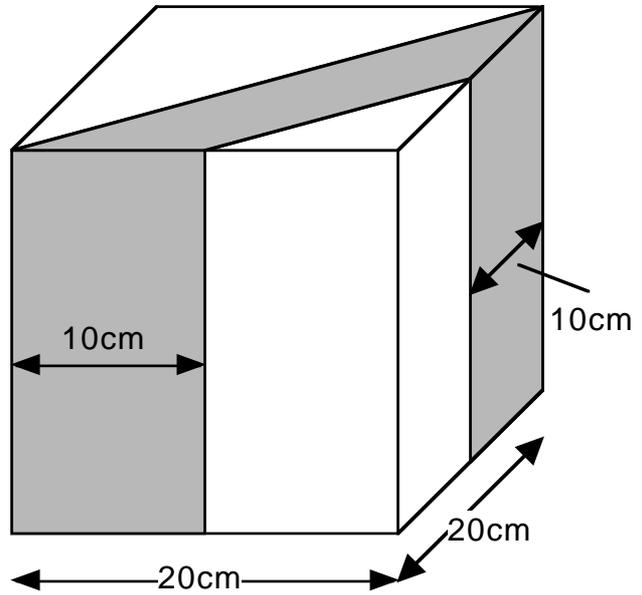
Find the volume of the shaded part.

_____ cm³

Ex-cube-me

Expand

Find all the pairs of values for a and b if

$(2x + a)(x + b)$ is equivalent to $2x^2 - 18$

Expand

Explain 7

Here is a flow chart.
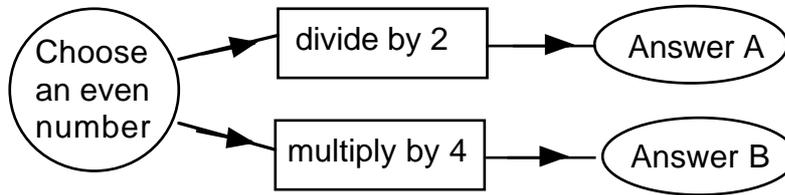
Explain why (B – A) is always a multiple of 7

- -

- -

- -

Eye test

The diagram shows a square of side length x
with two rectangles cut out of it.
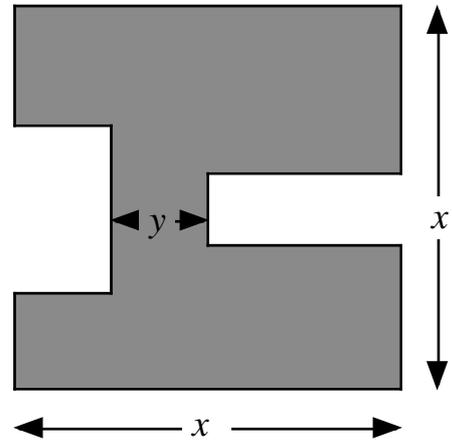
Find the perimeter of the shaded shape in terms of x and y.

Factory square

(a)   Find an odd factor of 840 greater than 10

- - - - - - - - - - - - - - - -

(b)   Find a square number greater than 200 but less than 1000

- - - - - - - - - - - - - - - -

Factory square

Fire rescue

'**Purple fire**' paint is made by mixing red and blue paint in the ratio 3 : 1

'**Purple sea**' paint is made by mixing red and blue paint in the ratio 1 : 3

1 litre of purple fire paint is mixed with 500 millilitres of purple sea by mistake.

How much red paint needs to be added to the mixture to make it purple fire again?

- - - - - - - - - - - - - - -

Five times

32

Five times a number gives the same answer as adding 24 to the number.

What is the number?

- - - - - - - - - - - - - -

Fivegrand

<div align="center">

7 6 5 4

</div>

Arrange these four digits to make the number that is the closest possible to 5000

<div align="center">

</div>

Fivegrand

Flight cost

The cost of a trip on a low-cost airline is given by this formula:

$$C = N(O + R + 2T)$$

C is the overall cost
N is the number of people travelling
O is the price of the outgoing flight, per person
R is the price of the return flight (the flight back), per person
T is the price of airport taxes for one flight, per person

Susan and her two friends went to Paris.

The return flight was £10 less than the outgoing flight, and the airport taxes were £21 for each flight for each person.

The overall cost was £294.

What was the price of the outgoing flight for each person?

£ _ _ _ _ _ _ _ _ _ _ _ _ _ _

Form

(a)   Find a quadratic equation that has solutions $x = 0$ and $x = 5$
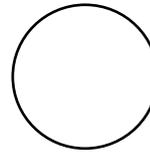
Give your answer without brackets.

(b)   Find a quadratic equation that has two solutions $x = 7$

Give your answer without brackets.
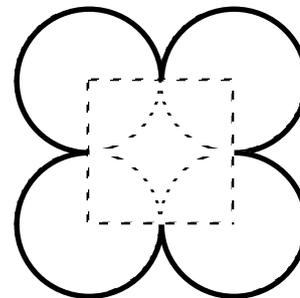

Gang of four

The circumference of this circle is 24cm.

Four of these circles are put together to make this shape.

The centres of the circles are at the vertices of a square.
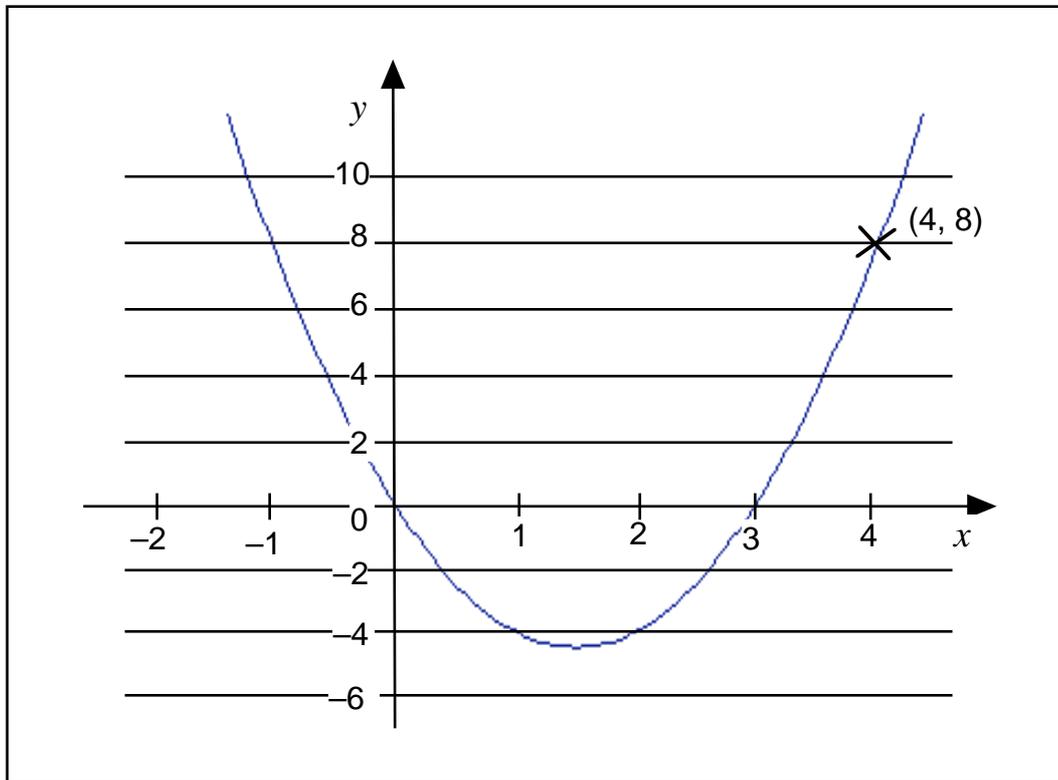
What is the perimeter of the shape?

_ _ _ _ _ _ _ _ _ _ _ _ _ _ _ cm

Gang of four

Graphy

Here is part of the graph of a quadratic function.

Find the equation of the graph.

$$y = \text{-------------------}$$

Half take

Marcus thinks of a number between 25 and 35

He divides the number by 2 and then subtracts 0.5

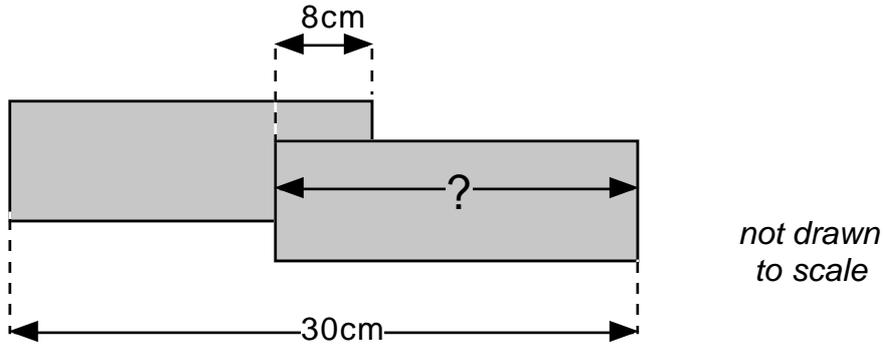He takes this answer, divides it by 2 and then subtracts 0.5

He repeats this process a number of times and gets zero.

What number did he start with?

- - - - - - - - - - - - - - -

Half take

Happylappy

Two identical rectangular tiles are arranged to overlap each other by 8cm.

The length of the whole arrangement is 30cm.

8cm

?

*not drawn
to scale*

30cm

Work out the length of a tile.

_ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ cm

Highroller
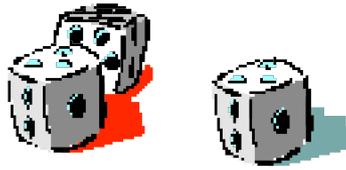
Three dice are each numbered 1 to 6

Two of them are red and one is blue.

All three dice are rolled.

What is the probability that the total on the two red dice will be equal to the score on the blue dice?
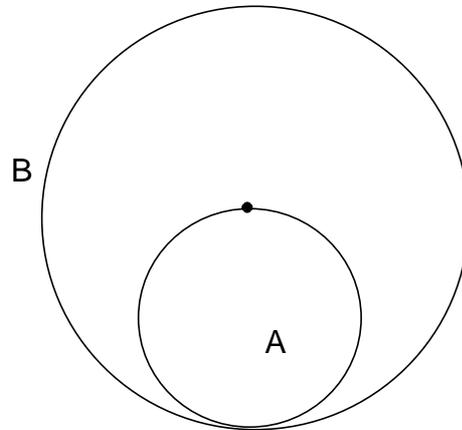
- - - - - - - - - - - - - - -

Hotel

A hotel charges £50 for a room for a single person per night and then £10 extra for each additional person per night.

A large family takes two rooms for a night and is charged £150 in total for the two rooms.

How many people are there in the family?

Inside circle

The circumference of circle A touches the edge of circle B and passes through its centre.
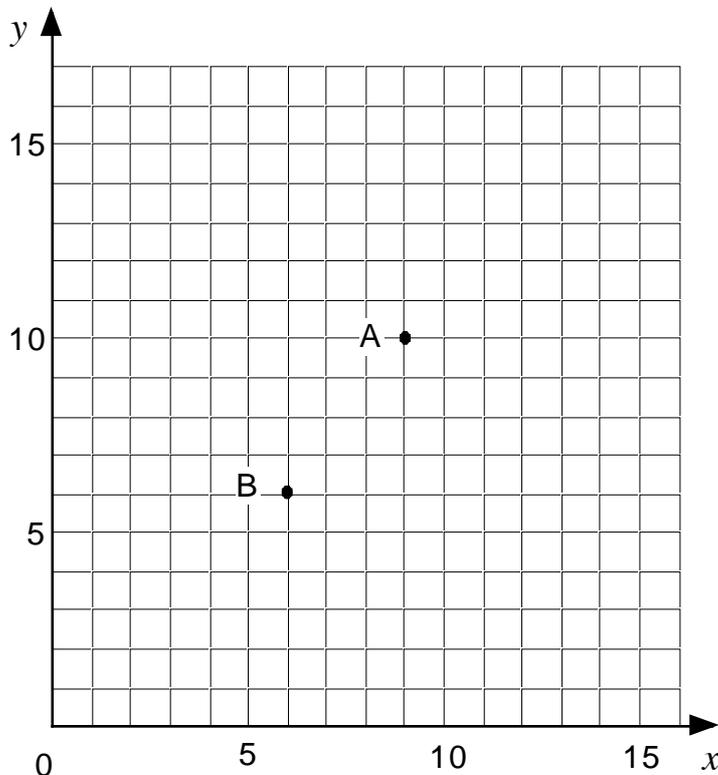
The area of circle A is 100cm^2

What is the area of circle B?

cm^2

- - - - - - - - - - - - - -

Isosceles grid

The two points A and B on the grid are the vertices of an isosceles triangle.

A is at (9, 10); B is at (6, 6).

(a)   The other vertex of the isosceles triangle is at a point with whole number co-ordinates.

What could be the co-ordinates of the other vertex?
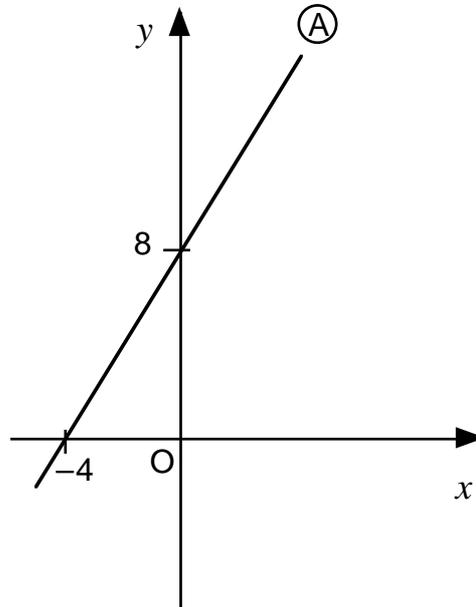
- - - - - - - -

(b)   There are several other points with whole number co-ordinates that could be the vertex of the isosceles triangle.

Give the co-ordinates of two more of these points.

- - - - - - - -   and   - - - - - - - -

Javelin A

Here is a graph.

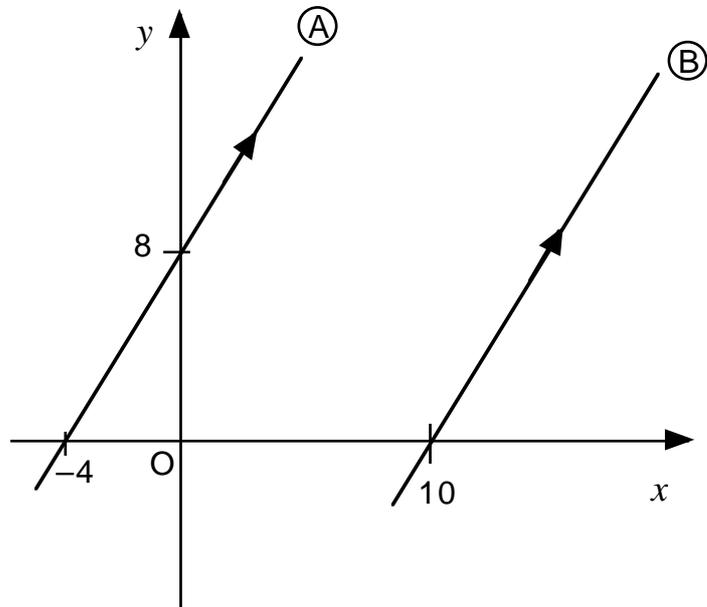not drawn to scale

What is the equation of line A?

Javelin B

The lines A and B are parallel.

not drawn to scale

What is the equation of line B?

- - - - - - - - - - - - - - - -

Last poster
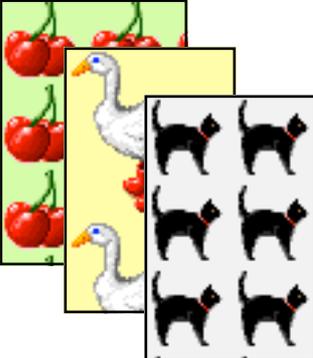
Posters cost £2.75 each.

You have to pay postage and packing charges as well.

These are:

postage and packing	
1 to 10 posters	£3.25
11 to 20 posters	£6.00
21 to 30 posters	£8.75
over 30 posters	£11.50

Zeke has £50 to spend.

How many posters can he get by post if he spends £50?

- - - - - - - - - - - - - - - -

Lineup

Four numbers are equally spaced on a number line.
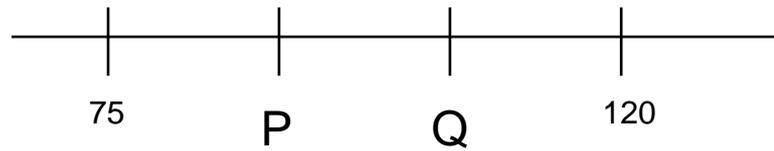
Find the numbers represented by P and Q

P   - - - - - -

Q   - - - - - - -

Loopy-do

A length of paper is 20cm long.

It has a 1.5 cm sticky strip at each end.
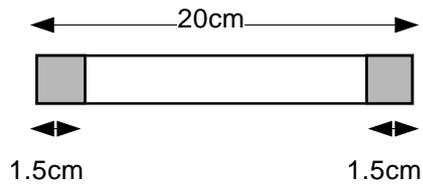
Four strips are stuck together, with the sticky parts overlapping exactly, to make a loop of paper.
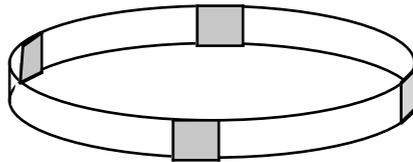
What is the circumference of the loop?

- - - - - - - - - - - . cm

Madbag

A bag contains only red counters and blue counters.

There are 90 red counters in the bag.

The probability of choosing a red counter from the bag is 0.3

How many blue counters are in the bag?

- - - - - - - - - - - - - - - -

Madbag

Mazy

Here is a block of squares.
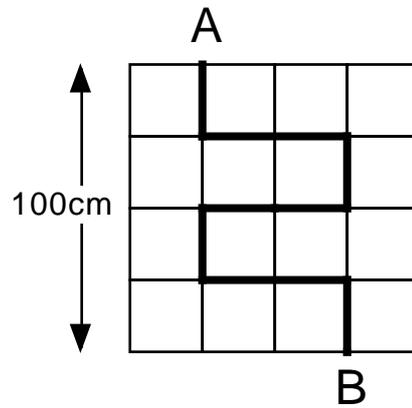
Find the length of the thick line that goes from A to B.

_____ cm

MeanN

A set of a thousand numbers has a mean of zero.

All but two of the numbers are 1

What is the mean of the other two numbers?

- - - - - - - - - - - - - -

MeanN

Meanset

A set of five numbers has:

      a mode of 12

      a median of 11

      a mean of 10

What could the numbers be?

------ ------ ------ ------ ------

Meanstreet

Three numbers have a mean of 23

Two of the numbers have a mean of 12

Two of the numbers have a mean of 30

What are the three numbers?

_____ and _____ and _____

Meet
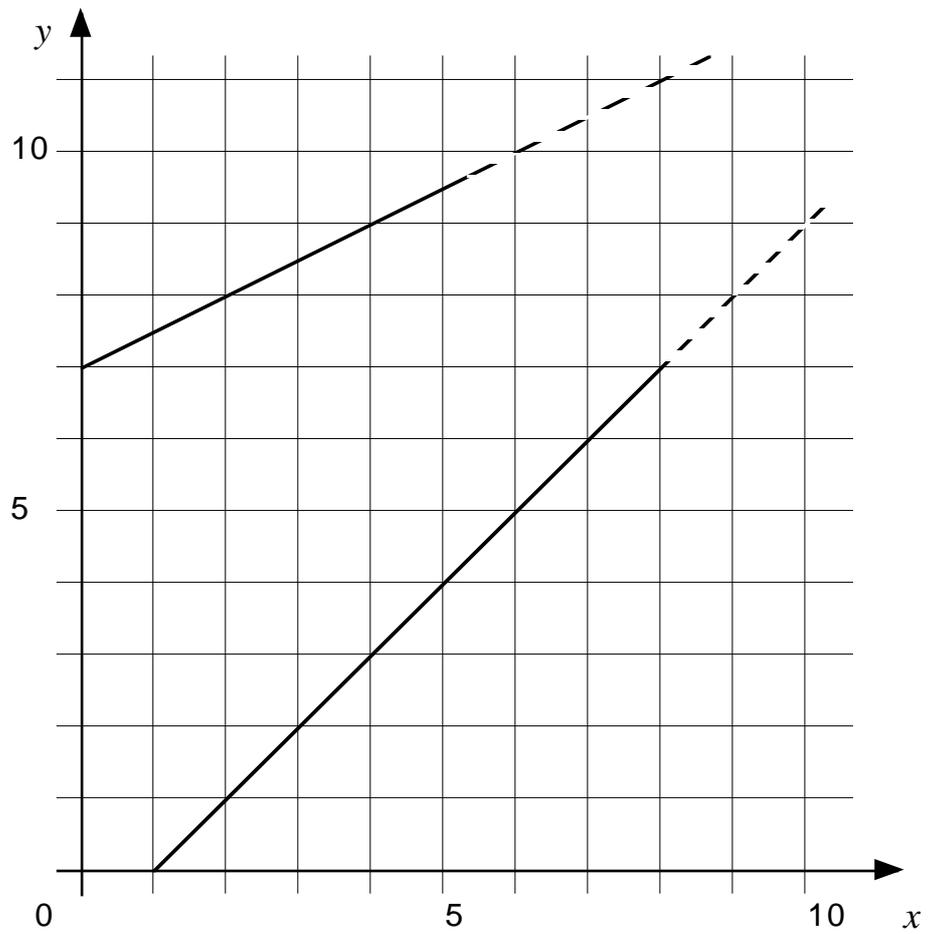
Find the co-ordinates of the point where these two lines meet if they are extended.

- - - - - - - - - - - - - -

Midseq

There are seven numbers in a sequence.

The difference between a term and the next one in the sequence is always the same amount.

The middle term of the sequence is m.

Find in terms of m the sum of the seven numbers.

Midseq

Moussey

Here is a recipe for chocolate mousse.

Chocolate Mousse

100g of chocolate
10g of butter
2   eggs

This makes enough chocolate mousse for two people.

I have 8 eggs, 45g of butter and 350g of chocolate.

What is the maximum number of people I can make chocolate mousse for?

Multitude

(a)   Find a multiple of 5 and a multiple of 6 that have a difference of 11

_ _ _ _ _ _ and _ _ _ _ _ _

(b)   Find a multiple of 7 and a multiple of 4 that add to make a total of 100

_ _ _ _ _ _ and _ _ _ _ _ _

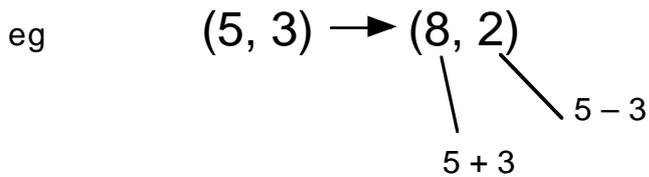### *Pair de deux*

The rule for a sequence of number pairs is

(first number, last number) ⟶

  (first number + last number, first number – last number)

eg $(5, 3) \longrightarrow (8, 2)$

$5 + 3$

$5 - 3$

Here is part of a sequence that follows this rule.

Write in the missing number pairs

(,   )   (,   )   (1, 2)   (3, −1)   (2, 4)   (,   )

Peculiar

p and q are two integers, each greater than zero.

$p > q$

$(p + q)^2 = 100$

$(p - q)^2 = 64$

Find the values of p and q.

$p =$ _____

$q =$ _____

Pecuniary

P and Q are two whole numbers.

P is greater than 10 and less than 20

Q is greater than 100 and less than 200

(a)   What is the largest value that (P + Q) could have?

- - - - - - - - - - - - - - -

(b)   What is the smallest difference there could be between P and Q?

- - - - - - - - - - - - - - -

Perp perp

The diagram shows two right-angled triangles ABC and DEB.
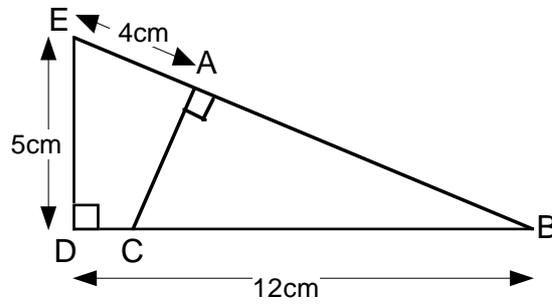
Find the length of the line AC.
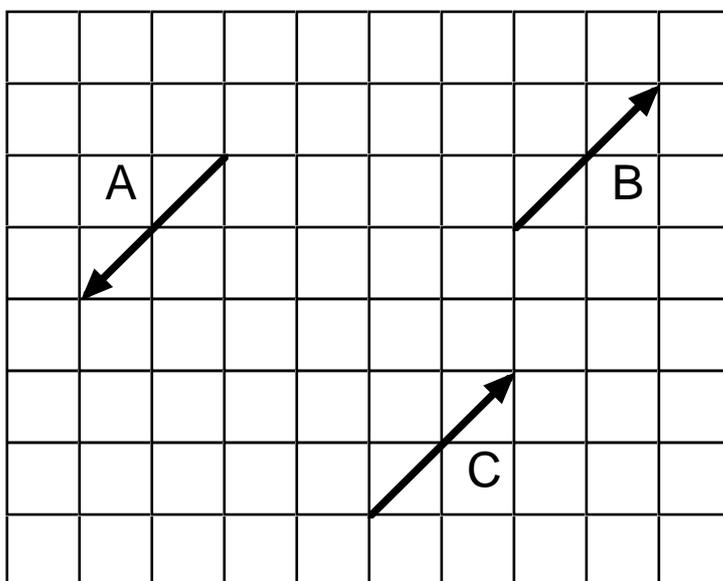
- - - - - - - - - - - - - - - -

Pointillism

(a)   The arrow in position A is rotated into position B.

Mark the point P that is the centre of this rotation.

(b)   The arrow in position A is rotated into position C.

Mark the point Q that is the centre of this rotation.

pqr

p, q and *r* are numbers, each greater than 1

$$p > q > r$$

$$\frac{q + r}{p} = \frac{3}{4}$$

$$p - q - r = 2$$

If *p, q* and *r* are each single digits, find their values.

$p =$ _____ $\qquad q =$ _____ $\qquad r =$ _____

Put the numbers in

Write four different numbers in the spaces to make the number sentence correct.

(___ − ___) − (___ − ___) = 35

Write the following four numbers in the spaces to make this number sentence correct.

80 60 50 20

(___ − ___) − (___ − ___) = 10

Repeater
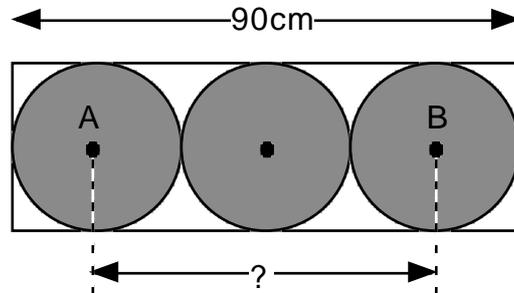
556, 484 and 333 are examples of numbers with repeated digits.

How many of the whole numbers from 1 to 201 have repeated digits?

- - - - - - - - - - - - - - -

Repeater

Roller

Three identical circles fit inside a rectangle.

The length of the rectangle is 90cm.

Find the distance between the two centres, A and B.

_ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ cm

Roller

Rollover

Three circles overlap each other as shown in the diagram.
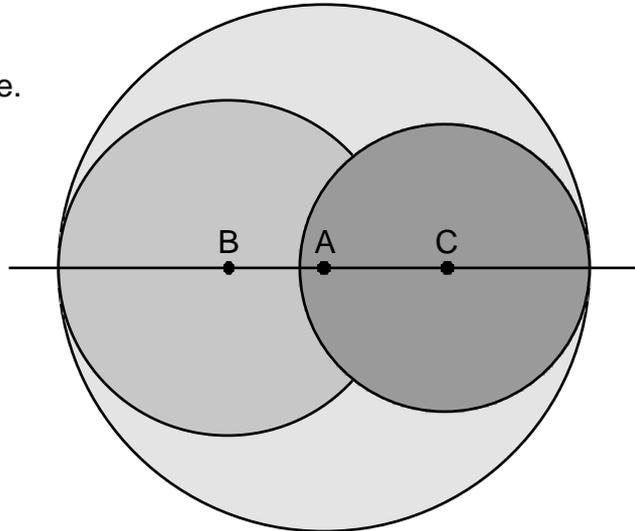
The centres of the circles are all on the same straight line.

A is the centre of the largest circle.

B is the centre of the middle-sized circle.

C is the centre of the smallest circle.

The diameters of the circles are
22cm, 16cm and 13cm.

Calculate the lengths BA and AC.

*not drawn
to scale*

BA _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ cm

AC _ _ _ _ _ _ _ _ _ _ _ _ _ _ cm

Rooting range

$$\sqrt{20\ 000} = 141.4 \text{ (correct to 1 decimal place)}$$

What is the smallest whole number that has a square root equal to 141.4 (correct to 1 decimal place)?

- - - - - - - - - - - - - -

Rooting range

Scalefactor

On this grid are two shapes, A and B.

Shape B is an enlargement of shape A, but some parts of B are missing.

The centre of enlargement is on the dotted line.
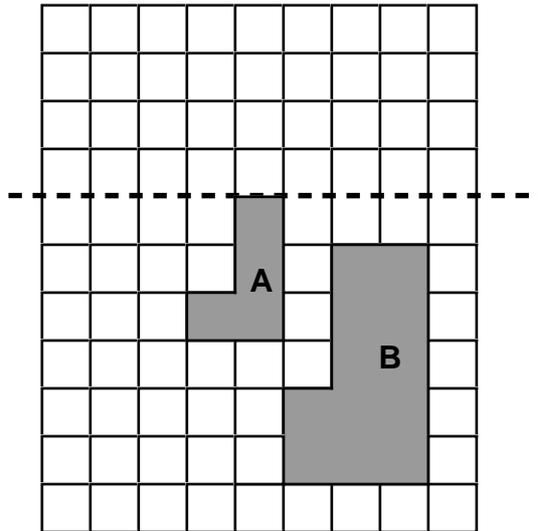
(a) Shade in squares to complete shape B.

(b) Find the centre of enlargement and mark it on the diagram with an 'X'.

(c) What is the scale factor of the enlargement?

Seesaw

The table below shows the change in the value of Seesaw plc shares over the last three years.

year	2004	2005	2006
change in value	+25%	−40%	+40%

Note: the percentage change each year is based upon the value at the start of that year and the value at the end of that year.

Calculate the percentage change in Seesaw plc shares from the start of 2004 to the end of 2006.

_____ %

Sevendiff

Three whole numbers have a total of 100
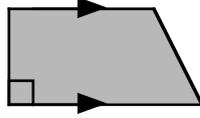
Two of the numbers have a difference of 7

Two of the numbers are the same.

Find the numbers.

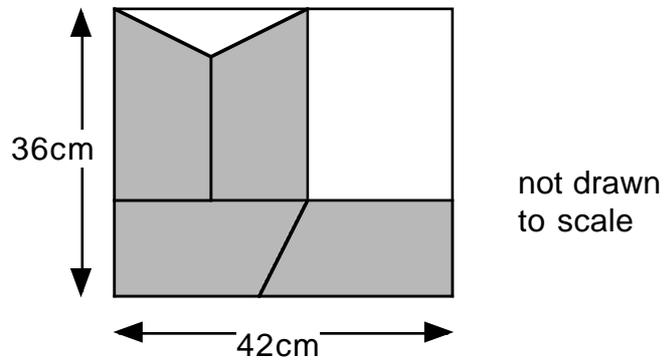...... and .... and ......

Shaperone

Here is a trapezium-shaped tile.

Four of these tiles are arranged inside a
rectangle that measures 36cm by 42cm.

36cm

42cm

not drawn
to scale

Calculate the area of one trapezium tile.

_____cm^2

Shares

Petra and Stephan share out £240 so that Petra gets one third of what Stephan gets.

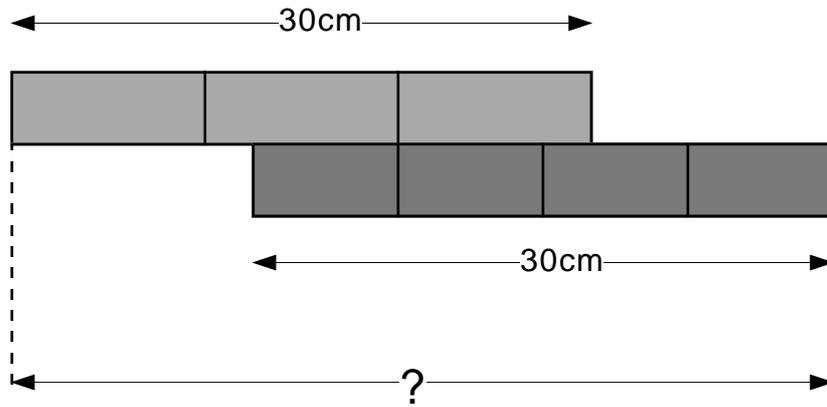How much do they each get?

Petra ⎯ ⎯ ⎯ ⎯ ⎯ ⎯ ⎯

Stephan ⎯ ⎯ ⎯ ⎯ ⎯ ⎯ ⎯

Side by side

Here are two 30cm strips of card.

One is divided into thirds and the other is divided into quarters.

What is the total length of this arrangement?

.... cm

Skywalker

Luke has £3.20 and Lottie has £4.50

How much will they each have if they share their money equally?

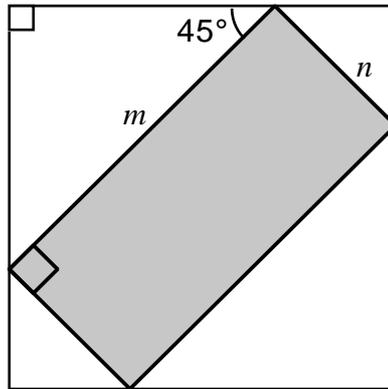£ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _

Skywalker

Smallfry

Three different two-digit numbers add to a total of 286

What is the smallest that any of the numbers could be?

Sold out

A rectangle is placed symmetrically inside a square.

The rectangle has sides of length m and n.

Find the area of the square in terms of m and n.

- - - - - - - - - - - - - - - - - - -

Spinalot

Spinner A has 6 equal sections and spinner B has 8 equal sections.

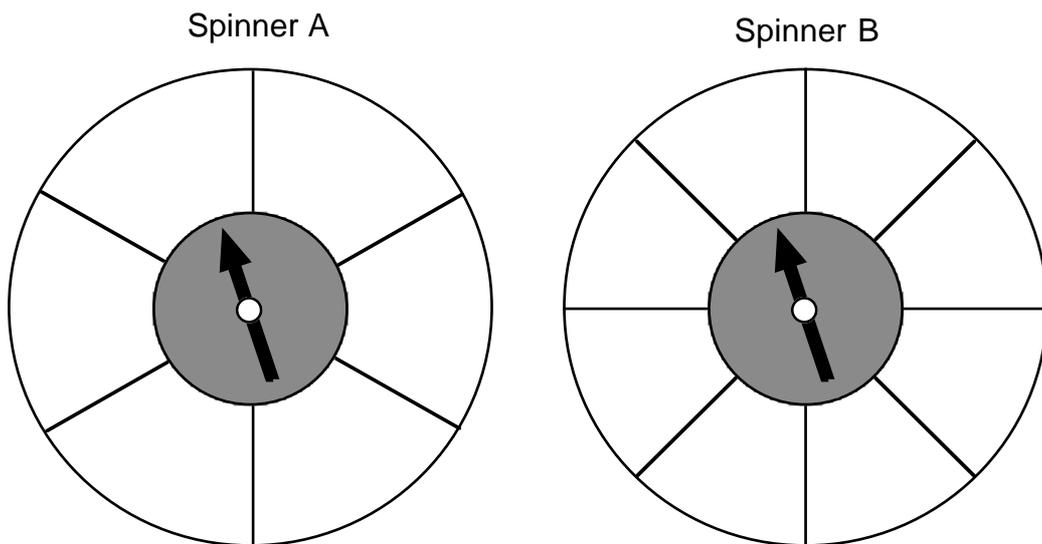Each section of the spinners contains the number 1, 2 or 3

All three numbers appear on each spinner.

Write numbers in the spinner sections so that:

a score of 1 is more likely on spinner A than spinner B,

a score of 2 is more likely on spinner B than spinner A,

a score of 3 is equally likely on either spinner.

Spinner A   Spinner B

Stamper

A letter needs 85p postage.

You have only got 15p and 20p stamps.

How many of each do you need to make exactly 85p?

_ _ _ _ _ _ _ _ 15p stamps

_ _ _ _ _ _ _ _ 20p stamps

Stamper

Stretcher

The diagram shows a square of side length x, and a triangle with a vertex at a perpendicular distance y from one side of the square.
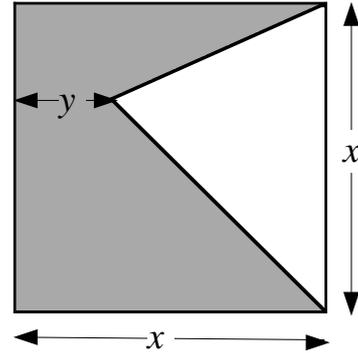
(a)   Find an expression for the shaded area in terms of x and y.

- -

(b)   If $y = \frac{1}{2} x$ calculate the percentage of the square that is shaded.

                                                                %
                                                  - - - - - - - -

(c)   What is the minimum percentage area of the square that can be shaded?

                                                  - - - - - - - - %

Explain your answer.

- -

- -

- -

Sum and difference

(a)   Find two three-digit odd numbers that add to make 204

_ _ _ _ _ _ and . _ _ _ _ _ _

(b)   Find two numbers, each less than 200, that differ by 150

. _ _ _ _ _ _ and _ _ _ _ _ _ .

Summertime

Find three numbers that add to make a total of 10 and which multiply together to make 30

- - - - -   - - - - -   - - - - ·

Sweet rapper

If the mean of a thousand numbers is zero, and all but one of the numbers are each 1, the other number is –999

The mean of n numbers is m, and all but one of the numbers are each one more than m.

What is the other number in terms of n and m?

Tape length

A length of tape is 135 centimetres long.

It is cut into two pieces.

The first piece is twice as long as the second piece.
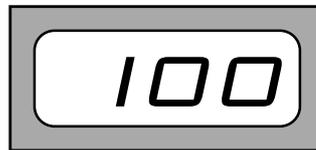
How long is the shorter of the two pieces of tape?

- - - - - - - - - - - - - - - - cm

Tape length

Tendency

Three different numbers multiply together to make 1000

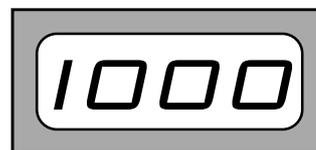Explain why at least one of the numbers must be less than 10

Tendency

Terms

A digital counter is set to count up in tens starting from 100, once a second.

    100, 110, 120, 130, ...

Another digital counter is set to count down in tens starting from 1000 once a second.

    1000, 990, 980, 970, ...

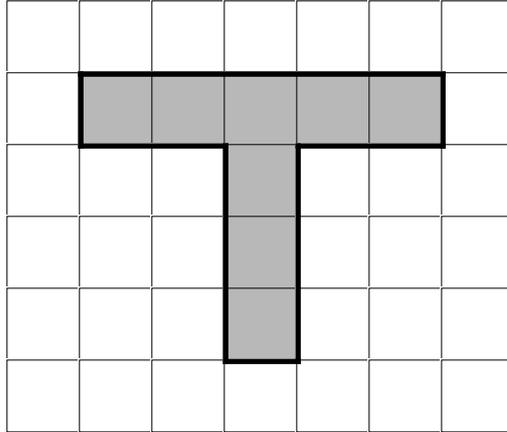Both counters start at exactly the same time.

(a)   After how many seconds do they each display the same number?

(b)   What number is this?

Tgrid

The letter 'T' on this square grid has an area of 200cm^2

Calculate the perimeter of the 'T'.

.‑ ‑ ‑ ‑ ‑ ‑ ‑ ‑ ‑ ‑ ‑ ‑ ‑ . cm

Three, four, five

(a)   Find a multiple of 3, greater than 100, that is also a multiple of 4

_ _ _ _ _ _ _ _ _   and   _ _ _ _ _ _ _ _

(b)   Give a number greater than 5 that is a multiple of 5 but is not a multiple of 2 or a multiple of 3

_ _ _ _ _ _ _ _

Threesquare

A piece of wire is 60cm long.

It is bent into a shape that consists of three identical squares.

How long is the side of a square?
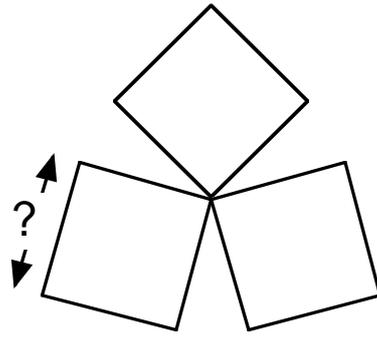
_ _ _ _ _ _ _ _ _ _ _ _ _ _ _ cm

Toto

Three numbers have a total of 30

Two of the numbers are equal.

The third number is half the size of the other two.

What are the numbers?

_____ and _____ and _____

Towerism

These towers are made of identical hexagons and identical rectangles.
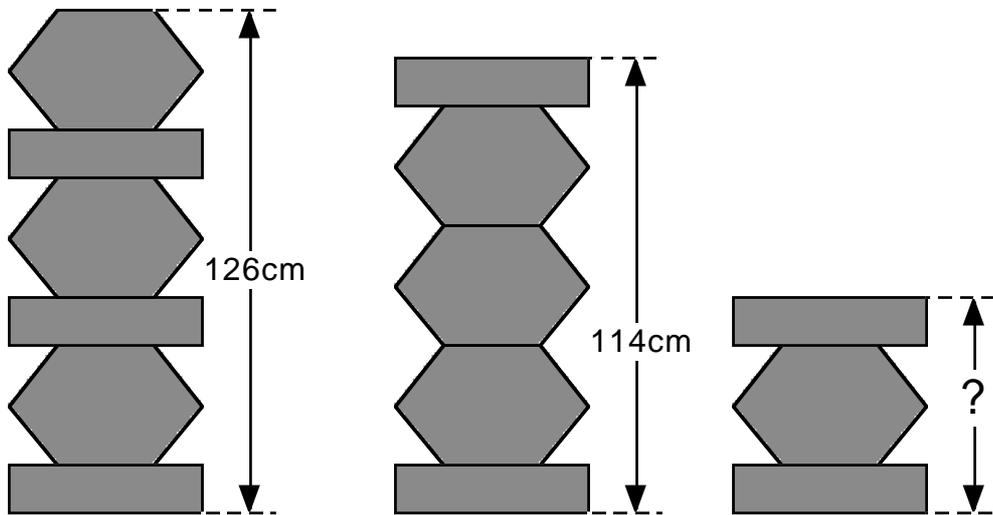
Calculate the height of the smallest tower.

_ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ cm

Tribubble

The diagram shows 15 identical circles, arranged as a rectangle, and a shaded triangle.

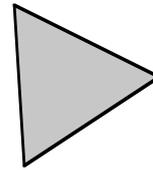The vertices of the triangle are at the centres of circles.
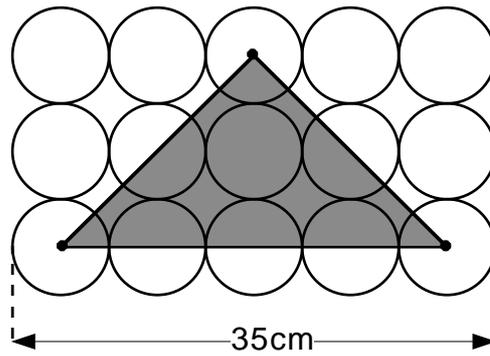
Calculate the area of the shaded triangle.

cm^2

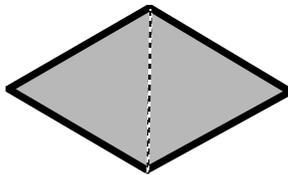$\cdots\!-\!-\!-\!-\!-\!-\!-\!-\!-\!-\!-\!-\!-\!-\!-\cdot$

Two-tri

An equilateral triangle has a perimeter of 12 cm.

perimeter = 12 cm

Two of the triangles are joined together, edge to edge.

What is the new perimeter?

_ _ _ _ _ _ _ _ _ _ _ cm

V-boats

The cost of hiring a boat is
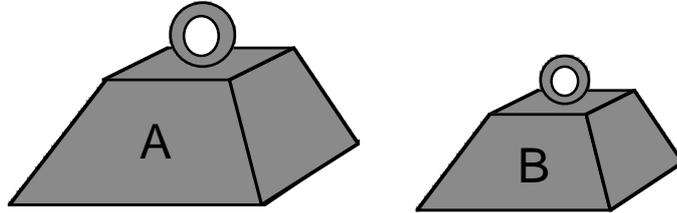
£4.50 for the first hour and then £2.50 for each hour after that.

Vicky and her friends want to hire a boat.

They can afford £12 at most.

How many hours can they hire the boat for?

- - - - - - - - - - - - - - - - - - - -

Weighup

A and B are two weights.

A is five times as heavy as B.

The difference between the weights is 6kg.
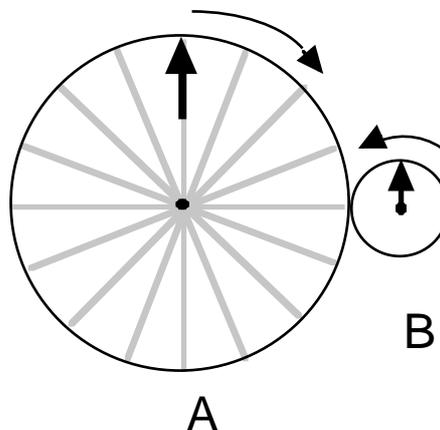
Find the weight of A.

. _ _ _ _ _ _ _ _ _ _ _ _ _ _ .kg

Wheelie bin

The two wheels A and B turn together, in opposite directions.
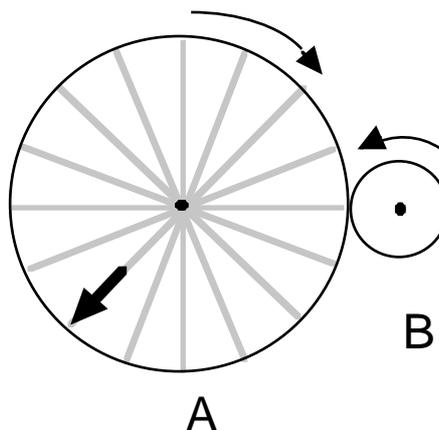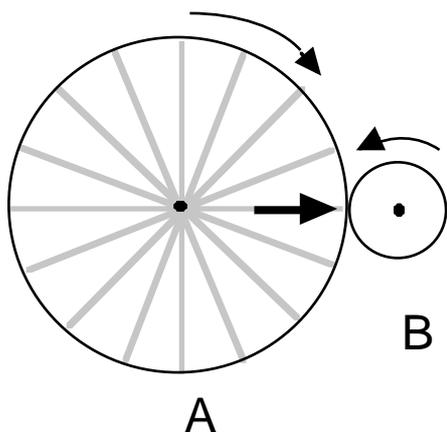
As wheel A makes one complete turn clockwise,
wheel B makes four complete turns anticlockwise.

This diagram shows how the wheels look
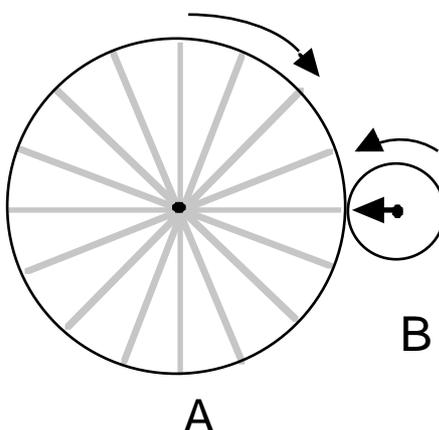at the start.

The diagrams below show new positions after turning.

In each case, draw in the missing arrow on wheel B.

In this diagram, draw **all** the possible positions for the arrow on wheel A.

Yogourtician

A supermarket sells 500g pots of yoghourt.

There is a special offer on yoghourt:

Buy 2 pots
and get a
3rd one free!

A week later, the price of a single pot of yoghourt
is still the same, but the offer changes to:

Buy 1 pot
and get a
2nd one
for half price!

Is the second offer better than the first?

Show working to justify your answer.

_____

_____

_____

_____